ECE329: Tutorial Session 2

September 10th, 2024

Share any thoughts on anything

1.

Math (again ☹)

Divergence & Divergence Theorem

Divergence = How much the field is DIVERGING at a certain point.

- Notation: $\nabla \cdot \vec{D}$
- Input: Vector field
- Output: Scalar field

Divergence Theorem: $\oiint \vec{D} \cdot d\vec{S} = \iiint \nabla \cdot \vec{D} dV$

Curl & Stoke's Theorem

Curl = How much the field is CURLING around a certain point.

- Notation: $\nabla \times \vec{E}$
- Input: Vector field
- Output: Vector field, with direction indicating how the field right-hand curls around.

Stoke's Theorem: $\oint \vec{E} \cdot d\vec{l} = \iint \left(\nabla \times \vec{E}\right) \cdot d\vec{S}$

Gradient & Gradient Theorem

Gradient = How much the scalar function changes (or its GRADE).

- Notation: ∇V
- Input: Scalar field
- Output: Vector field, with direction indicating steepest **uphill**.

Fundamental theorem of calculus: $\int_a^b \mathrm{f}'(\mathrm{t}) \cdot dt = f(b) - f(a)$

Similarly:

Gradient theorem: $\int_a^b \nabla V \cdot d\vec{l} = V(b) - V(a)$

Laplacian

Laplacian (scalar) = How much the scalar function LAPLACES.

Laplacian (scalar) = How much the rate of change of the scalar function varies (aka stress).

- Notation: $\nabla^2 V = \nabla \cdot \nabla V$
- Input: Scalar field
- Output: Scalar field

Problem 1

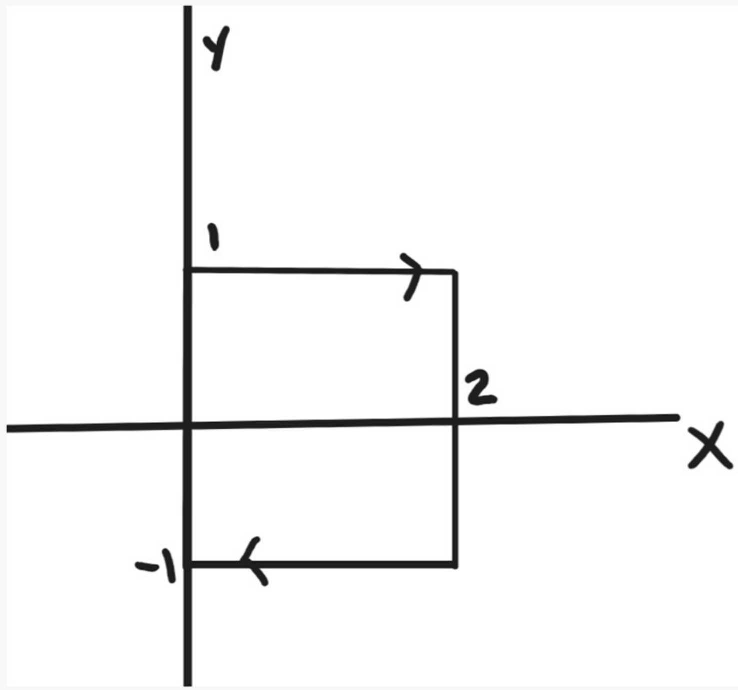
Find the divergence and curl of $\vec{E} = xy\hat{x} + yz\hat{y} + xz\hat{z}$.

Problem 2

Find the gradient and Laplacian of $f = x^2 + y^2 + z^2$.

Problem 3

Suppose $\vec{E} = \frac{x^2}{2}\hat{y} - x\hat{z}$ V/m^2. What is the circulation for the closed square loop picture below?



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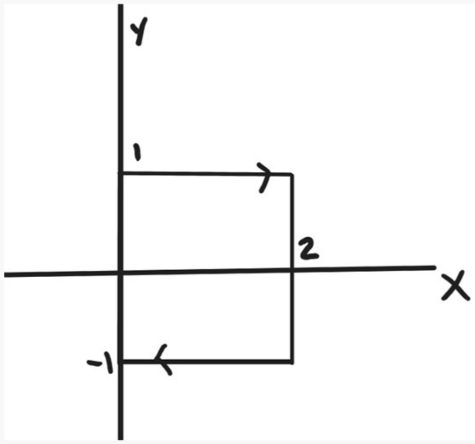

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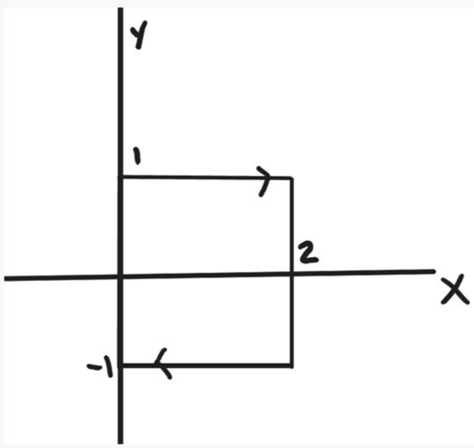

2.

Physics ☺

Fluxes

Recall electric and current flux equations from last time.

$$\oiint \overrightarrow{D} \cdot d\vec{S} = \iiint \rho_{\text{enclosed}} dV \qquad \oiint \vec{J} \cdot d\vec{S} = -\frac{\partial}{\partial t} \iiint \rho_{\text{enclosed}} dV$$

Fluxes

$$\oiint \vec{D} \cdot d\vec{S} = \iiint \rho_{\text{enclosed}} dV \qquad \oiint \vec{J} \cdot d\vec{S} = -\frac{\partial}{\partial t} \iiint \rho_{\text{enclosed}} dV$$

Problem 4

A point charge of 4C sits at the origin in free space. Find the divergence of the electric field.

Conservative Fields

The following are equivalent for vector field $\vec{E}$:

- $\nabla \times \vec{E} = 0$
- $\vec{E}$ is conservative
- $\oint \vec{E} \cdot d\vec{l} = 0$
- $\int_a^b \vec{E} \cdot d\vec{l}$ is path-independent
- $\vec{E} = \nabla V$ for some scalar field V.

Electrostatic Potential

Work done by you to move a charge from a to b, causing change in electrostatic potential U:

$$W = U(b) - U(a) = \int_a^b \vec{F}_{\text{applied}} \cdot d\vec{l}$$

$$= -\int_a^b \vec{F}_E \cdot d\vec{l} = -\int_a^b q\vec{E} \cdot d\vec{l}$$

Volts = work per unit charge (take $q = 1$).
U/q = electrostatic potential energy per unit charge = V = **electrostatic potential**:

$$V_{ab} = V(b) - V(a) = -\int_a^b \vec{E} \cdot d\vec{l}$$

V to E

$$V_{ab} = V(b) - V(a) = -\int_a^b \vec{E} \cdot d\vec{l}$$

Problem 5

Suppose $\vec{E} = 2(x-1)\hat{x} + 3(z+y^2+1)\hat{y} + 3(y+1)\hat{z}$ V/m². Let the point $(1,1,1)$ be grounded with $V = 0$. Find the voltage difference from the origin to point B at $(3,2,0)$.

Problem 5: Blank slide

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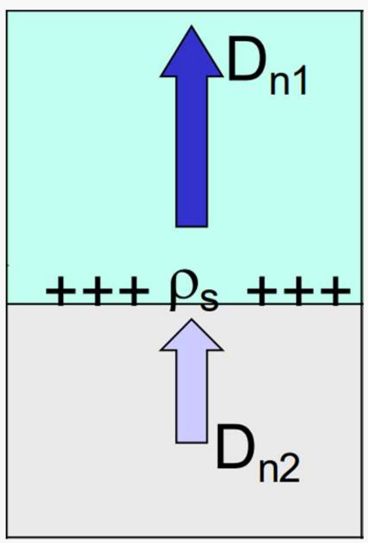
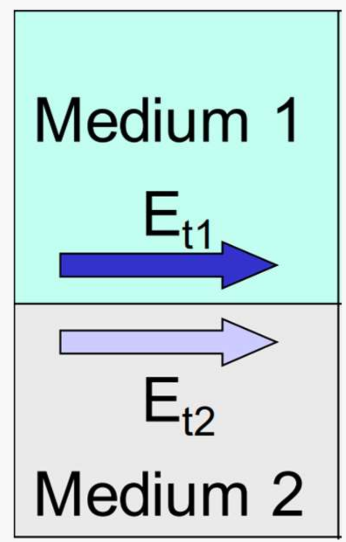
Boundary Conditions

$$\hat{n} \cdot \left(\vec{D}_1 - \vec{D}_2\right) = \rho_s$$



$$\hat{n} \times \left(\vec{E}_1 - \vec{E}_2\right) = 0$$



Problem 6

Suppose the yz-plane holds a surface charge density of $\rho_s = 5$ C/m^2. The electric displacement field on the $-x$ side is given as $\vec{D} = \hat{x} + \hat{y} + \hat{z}$. Find the electric displacement field on the $+x$ side using boundary conditions.

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Jumping Between Quantities

Q

ρ

$\vec{D}$

V

$\vec{E}$

P.S.: Helmholtz Theorem

A vector field $\vec{E}$ is specified completely by its divergence and curl.

Week 2 equations, in one place

$$\vec{F} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\vec{F} = q_1\vec{E} + q_1(\vec{v}_1 \times \vec{B})$$

$$\vec{E} = \frac{q_2}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\hat{n}\cdot\left(\vec{D}_1 - \vec{D}_2\right) = \rho_s$$

$$\hat{n}\times\left(\vec{E}_1 - \vec{E}_2\right) = 0$$

$$\epsilon_0 \oiint \vec{E}\cdot d\vec{S} = Q_{\text{enclosed}}$$

$$\vec{D} = \epsilon_0\vec{E}$$

$$\oiint \vec{D}\cdot d\vec{S} = Q_{\text{enclosed}}$$

$$\iiint \rho dV = Q_{\text{enclosed}}$$

$$\oiint \vec{B}\cdot d\vec{S} = 0$$

$$I = \oiint \vec{J}\cdot d\vec{S} = -\frac{\partial Q_{\text{enclosed}}}{\partial t}$$

$$\nabla\cdot\vec{D} = \rho$$

$$\nabla\cdot\vec{J} = -\frac{\partial\rho}{\partial t}$$

$$\nabla\times\vec{E} = 0$$

$$\vec{E} = -\nabla V$$

$$\oint \vec{E}\cdot d\vec{l} = 0$$

$$V_{ab} = V(b) - V(a) = -\int_a^b \vec{E}\cdot d\vec{l}$$

$$\oiint \vec{D}\cdot d\vec{S} = \iiint \nabla\cdot\vec{D}dV$$

$$\oint \vec{E}\cdot d\vec{l} = \iint \left(\nabla\times\vec{E}\right)\cdot d\vec{S}$$

$$\int_a^b \nabla V\cdot d\vec{l} = V(b) - V(a)$$

Office Hours

Any questions?

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