ECE329: Tutorial Session 14

December 10th, 2024

VSWR

$$V(d) = V^{+}e^{j\beta d}\left(1 + \Gamma_L e^{-j2\beta d}\right)$$

What is the maximum magnitude of $V(d)$?
What is the minimum magnitude of $V(d)$?

VSWR

$$V(d) = V^+ e^{j\beta d}\left(1 + \Gamma_L e^{-j2\beta d}\right)$$

VSWR

Where is VSWR on a Smith Chart?

VSWR Demo

VSWR

What are the extreme values that VSWR can take on?
What situations do they represent in real life?

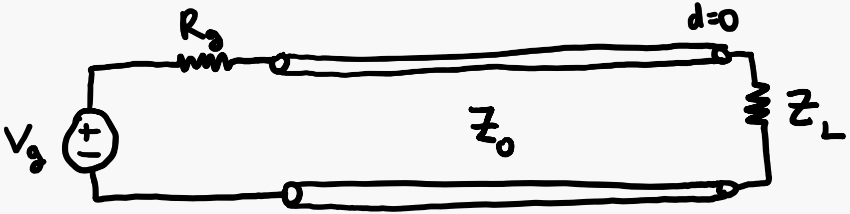
Impedance Matching: Motivation

$$V(d) = V^+ e^{j\beta d} + V^- e^{-j\beta d}$$

$$V(d) = V^+ e^{j\beta d}\left(1 + \Gamma_L e^{-j2\beta d}\right)$$

Impedance Matching: Quarter -Wave Transformer

Assume $v = c$, $f = 150\text{MHz}$.
$Z_0 = 50\Omega$, $Z_L = 25 + j50\Omega$



A quarter-wave transformer will be inserted on the TL at some distance d away from the load. Find d and the optimal intrinsic impedance of the line to be inserted Z_q such that the load is a perfect match.

Impedance Matching: Quarter-Wave Transformer

This method arises from how the Smith Chart works!!

While you can write down the exact steps you should follow to solve this and regurgitate on the exam, it is highly recommended that you instead understand why you are doing what you are doing.

Each step has a purpose. Understanding this purpose will help you deal with the countless variations that they could throw at you.

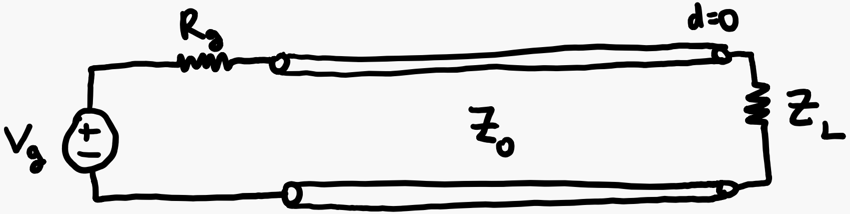
- See HW14 Extra Credit P2

Impedance Matching: Quarter -Wave Transformer

1. Normalize the load impedance and plot it on the Smith Chart. The Smith Chart deals with normalized impedances.
2. Walk towards the horizontal axis of the Smith Chart. This gives us a real z_{out}, which is useful for the next step.
3. Denormalize z_{out} to get Z_{out}. The quarter-wave transform formulae deal with denormalized impedances. Both Z_{in} and Z_{out} are now known!
4. Use the quarter-wave transform formula to find Z_q: $Z_q^2 = Z_{in}Z_{out}$.

Impedance Matching: Quarter -Wave Transformer

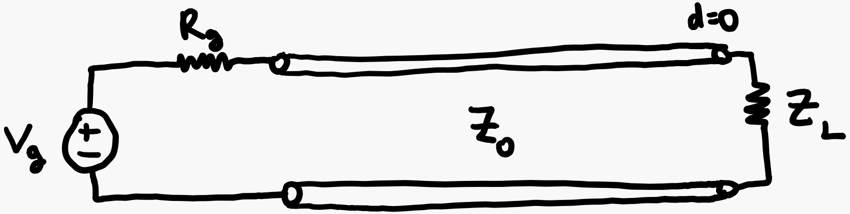
Assume $v = c$, $f = 150\text{MHz}$.
$Z_0 = 50\Omega$, $Z_L = 25 + j50\Omega$



A quarter-wave transformer will be inserted on the TL at some distance d away from the load. Find d and the optimal intrinsic impedance of the line to be inserted Z_q such that the load is a perfect match.

Impedance Matching: Open -Stub

Assume $v = c$, $f = 150\text{MHz}$.
$Z_0 = 50\Omega$, $Z_L = 25 + j50\Omega$



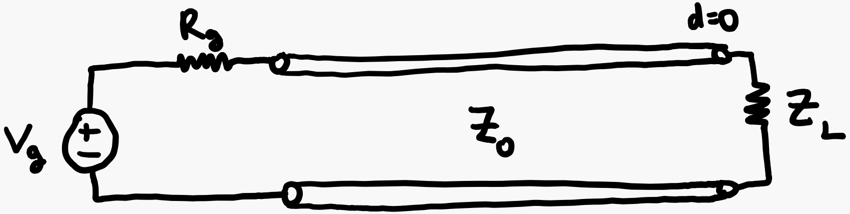
An open stub with $Z_1 = 50\Omega$ will be inserted on the TL at some distance d away from the load. Find d and the length of an open-circuited stub required such that the load is a perfect match.

Impedance Matching: Open -Stub

1. Normalize the load impedance and plot it on the Smith Chart. The Smith Chart deals with normalized impedances.
2. Convert to normalized admittance and plot it on the Smith Chart. We're adding a stub in parallel. Admittances simply add when combined in parallel. This step is unnecessary but makes the math easier.
3. Find the intersection between the constant gamma circle and the r = 1 circle. There are two points where this happens. Both are valid. Pick one.
4. Calculate distance from admittance to intersection point found in step 3. This is the distance to insertion point.
5. Denormalize to yield denormalized admittance. The complex part needs to be cancelled out.
6. On another Smith Chart, start at the load end of the stub. Rotate until the denormalized admittance cancels the denormalized admittance found in step 5 when adding the two denormalized admittances together.
7. Calculate distance from the stub's load to the point in step 6. This is the length of the stub.

Impedance Matching: Open -Stub

Assume $v = c$, $f = 150\text{MHz}$.
$Z_0 = 50\Omega$, $Z_L = 25 + j50\Omega$



An open stub with $Z_1 = 50\Omega$ will be inserted on the TL at some distance d away from the load. Find d and the length of an open-circuited stub required such that the load is a perfect match.

End-Of-Semester Logistics

HW14 is due Wednesday at 11:59pm.

- No-penalty late submissions accepted until Thursday 5:59pm.

All office hours on Thursday will run as usual.

HKN will host a review session on Thursday from 5:30–7:30pm. Details here: https://hkn.illinois.edu/services

- HKN also has a worksheet w/ solutions. I **strongly** recommend problems 1–8 as practice.

Final Exam Notes

The final will cover Smith Charts heavily.

- You do not need to bring your own Smith Charts.

Completely understanding HWs 13 and 14 is a must.

- Extra credit problems on HW14 serve as good practice for possible variations of Smith Chart problems you could see.

Review past midterms as well, particularly the biggest concepts from the past midterms.

- Midterm 1: Electrostatics.
- Midterm 2: Magnetostatics.
- Midterm 3: Wave theory.

Final Exam Notes

You are allowed one 8.5”x11” cheatsheet, both sides.
It must be handwritten.

- The total amount of handwritten area you bring cannot exceed this area. If it does, WE will choose what part of your cheatsheet to keep.
- If your cheatsheet has any printed portion, we will take away the printed portion the cheatsheet. Anything handwritten on the back will be lost too.

Bring:

- A pencil (pen not recommended)
- A compass (to draw circles)
- A protractor
- A straightedge

Midterm 1 equations, in one place

$$\vec{F} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\vec{F} = q_1\vec{E} + q_1(\vec{v}_1 \times \vec{B})$$

$$\vec{E} = \frac{q_2}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\epsilon \oiint \vec{E} \cdot d\vec{S} = Q_{\text{enclosed}}$$

$$\oiint \vec{D} \cdot d\vec{S} = Q_{\text{enclosed}}$$

$$\iiint \rho dV = Q_{\text{enclosed}}$$

$$\oiint \vec{B} \cdot d\vec{S} = 0$$

$$I = \oiint \vec{J} \cdot d\vec{S} = -\frac{\partial Q_{\text{enclosed}}}{\partial t}$$

$$\nabla \times \vec{E} = 0$$

$$\vec{E} = -\nabla V$$

$$\oint \vec{E} \cdot d\vec{l} = 0$$

$$V_{ab} = V(b) - V(a) = -\int_a^b \vec{E} \cdot d\vec{l}$$

$$\hat{n} \cdot (\vec{D}_1 - \vec{D}_2) = \rho_s$$

$$\hat{n} \times (\vec{E}_1 - \vec{E}_2) = 0$$

$$\hat{n} \cdot (\vec{P}_1 - \vec{P}_2) = -\rho_{b,s}$$

$$\epsilon = \epsilon_0(1 + \chi_e)$$

$$\vec{P} = \epsilon_0 \chi_e \vec{E}$$

$$\vec{D} = \epsilon_0\vec{E} + \vec{P} = \epsilon\vec{E}$$

$$\vec{J} = \sigma\vec{E}$$

$$\rho_b = -\nabla \cdot \vec{P}$$

$$\nabla \cdot \epsilon_0\vec{E} = \rho_f + \rho_b$$

$$\nabla \cdot \vec{D} = \rho$$

$$\nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$$

$$-\nabla^2 V = \frac{\rho}{\epsilon}$$

$$\oiint \vec{D} \cdot d\vec{S} = \iiint \nabla \cdot \vec{D} dV$$

$$\oint \vec{E} \cdot d\vec{l} = \iint (\nabla \times \vec{E}) \cdot d\vec{S}$$

$$\int_a^b \nabla V \cdot d\vec{l} = V(b) - V(a)$$

Midterm 2 equations, in one place

$$\vec{B} = \frac{\mu I}{2\pi r}\hat{\phi}$$

$$d\vec{B} = \frac{\mu I d\vec{\ell} \times \hat{r}}{4\pi r^2}$$

$$\oint_C \vec{H} \cdot d\vec{\ell} = \iint_S \vec{J} \cdot d\vec{S}$$

$$\oint_C \vec{B} \cdot d\vec{\ell} = \mu I_{\text{encl}}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0$$

$$\Psi = \iint_s \vec{B} \cdot d\vec{S}$$

$$-\frac{d}{dt}\iint_s \vec{B} \cdot d\vec{S} = \oint_c \vec{E} \cdot d\vec{l}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\oint_c \vec{E} \cdot d\vec{l} = \varepsilon$$

$$\varepsilon = \frac{W}{q} = \oint_C \frac{\vec{F}}{q} \cdot d\vec{l}$$

$$\Psi = LI$$

$$\varepsilon = IR$$

$$v = \frac{\omega}{\beta} = \lambda f = \frac{1}{\sqrt{\mu\epsilon}}$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$\eta = \frac{\sqrt{\mu}}{\sqrt{\epsilon}}$$

$$\beta = \omega\sqrt{\mu\epsilon}$$

$$\nabla^2 \vec{E} = \mu\epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\vec{S} = \vec{E} \times \vec{H}$$

$$\tilde{S} = \tilde{E} \times \tilde{H}^*$$

$$< \vec{S} > = \frac{1}{2}\text{Re}\{\tilde{E} \times \tilde{H}^*\}$$

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\epsilon\vec{E} \cdot \vec{E} + \frac{1}{2}\mu\vec{H} \cdot \vec{H}\right) + \nabla \cdot \vec{S} + \vec{J} \cdot \vec{E} = 0$$

$$Q = CV$$

$$G = \frac{\sigma}{\epsilon}C \qquad R = \frac{1}{G}$$

$$\vec{J}_b = \frac{\partial \vec{P}}{\partial t} + \nabla \times \vec{M}$$

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

$$\vec{M} = \chi_m \vec{H}$$

$$\vec{B} = \mu_0\mu_r\vec{H} = \mu\vec{H}$$

$$A\cos(\omega t - \beta x)\hat{z} \leftrightarrow Ae^{-j\beta x}\hat{z}$$

$$\hat{n} \cdot (\vec{B}_1 - \vec{B}_2) = 0$$

$$\hat{n} \times (\vec{H}_1 - \vec{H}_2) = \vec{J}_s$$

$$\hat{n} \times (\vec{M}_1 - \vec{M}_2) = \vec{J}_{b,s}$$

Midterm 3 equations, in one place

Waves:

	Condition	β	α	$\lvert\eta\rvert$	τ	$\lambda = \frac{2\pi}{\beta}$	$\delta = \frac{1}{\alpha}$
Perfect dielectric	$\sigma = 0$	$\omega\sqrt{\epsilon\mu}$	0	$\sqrt{\frac{\mu}{\epsilon}}$	0	$\frac{2\pi}{\omega\sqrt{\epsilon\mu}}$	∞
Imperfect dielectric	$\frac{\sigma}{\omega\epsilon} \ll 1$	$\sim \omega\sqrt{\epsilon\mu}$	$\beta\frac{1}{2}\frac{\sigma}{\omega\epsilon} = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}$	$\sim \sqrt{\frac{\mu}{\epsilon}}$	$\sim \frac{\sigma}{2\omega\epsilon}$	$\sim \frac{2\pi}{\omega\sqrt{\epsilon\mu}}$	$\frac{2}{\sigma}\sqrt{\frac{\epsilon}{\mu}}$
Good conductor	$\frac{\sigma}{\omega\epsilon} \gg 1$	$\sim \sqrt{\pi f\mu\sigma}$	$\sim \sqrt{\pi f\mu\sigma}$	$\sqrt{\frac{\omega\mu}{\sigma}}$	$45°$	$\sim \frac{2\pi}{\sqrt{\pi f\mu\sigma}}$	$\sim \frac{1}{\sqrt{\pi f\mu\sigma}}$
Perfect conductor	$\sigma = \infty$	∞	∞	0	-	0	0

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \qquad \tau = \frac{2\eta_2}{\eta_2 + \eta_1} = 1 + \Gamma$$

$$v = \frac{\omega}{\beta} = \lambda f$$

$$\nabla^2 \tilde{E} = (j\omega\mu)(\sigma + j\omega\epsilon)\tilde{E}$$

TLs:

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\tau_g = \frac{Z_0}{R_g + Z_0}$$

$$\Gamma_g = \frac{R_g - Z_0}{R_g + Z_0}$$

$$\Gamma_{jk} = \frac{Z_k - Z_j}{Z_k + Z_j}$$

$$\tau_{jk} = 1 + \Gamma_{jk}$$

Half-wave:

$$V_{in} = -V_{out}$$
$$I_{in} = -I_{out}$$
$$Z_{in} = Z_{out}$$

Quarter-wave:

$$V_{in} = jI_{out}Z_0$$
$$I_{in} = \frac{jV_{out}}{Z_0}$$
$$Z_{in} = \frac{Z_0^2}{Z_{out}}$$

$$\cos(\theta) = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$\sin(\theta) = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

Final exam equations, in one place

$$V(d) = V^+ e^{j\beta d}(1 + \Gamma_L e^{-j2\beta d})$$

$$\Gamma(d) = \Gamma_L e^{-j2\beta d}$$

$$z(d) = \frac{1 + \Gamma(\mathrm{d})}{1 - \Gamma(\mathrm{d})}$$

$$y(d) = \frac{1}{z(d)}$$

$$z(d) = \frac{Z(d)}{Z_0}$$

$$\mathrm{VSWR} = \frac{1+|\Gamma(\mathrm{d})|}{1-|\Gamma(\mathrm{d})|}$$

Resonance:

Type	Input	Output	T.L. Length
Series	Short	Short	$\frac{n\lambda}{2}$, n integer
Series	Short	Open	$\frac{n\lambda}{4}$, n odd integer
Parallel	Open	Short	$\frac{n\lambda}{4}$, n odd integer
Parallel	Open	Open	$\frac{n\lambda}{2}$, n integer

Units

Charge Q: C
Current I: A
Electric field strength $\vec{E}$: N/C or V/m
Electric flux density $\vec{D}$: C/m^2
Polarization field $\vec{P}$: C/m^2
Electric potential V: V
Capacitance C: F
Magnetic flux density $\vec{B}$: T or Wb/m^2
Magnetic field strength $\vec{H}$: A/m
Magnetic flux Ψ: Wb
Electromotive force ε: V
Inductance L: H

Electric permittivity ϵ: F/m
Magnetic permeability μ: H/m
Conductivity σ: Si/m

Charge density ρ: C/m^3
Surface charge density ρ_S: C/m^2
Current density $\vec{J}$: A/m^2

Intrinsic impedance η: Ohm
Wave number β: rad/m

Characteristic impedance Z: Ohm

Office Hours

Any questions?