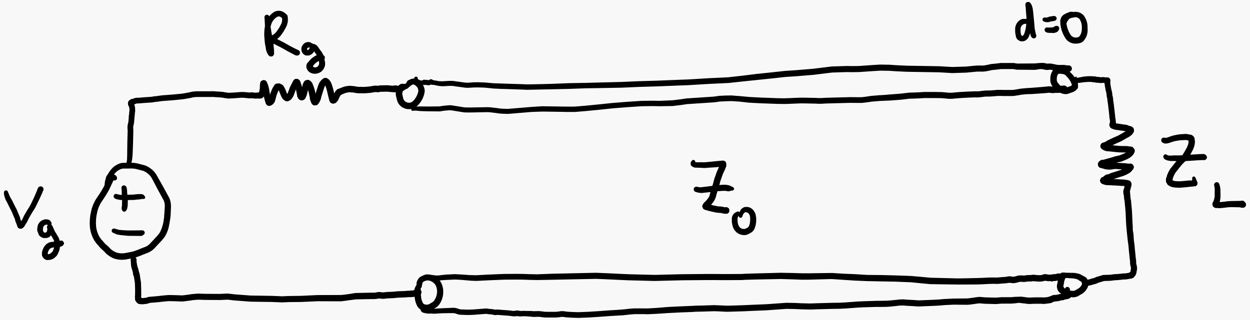
# ECE329: Tutorial Session 12

November 19th, 2024

# Basic TL: Time Domain, Not Steady State




Generator injection coefficient: $\tau_g = \frac{Z_0}{R_g+Z_0} \neq 1 + \Gamma_g$

Load reflection coefficient: $\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$

Generator reflection coefficient: $\Gamma_g = \frac{R_g - Z_0}{R_g + Z_0}$

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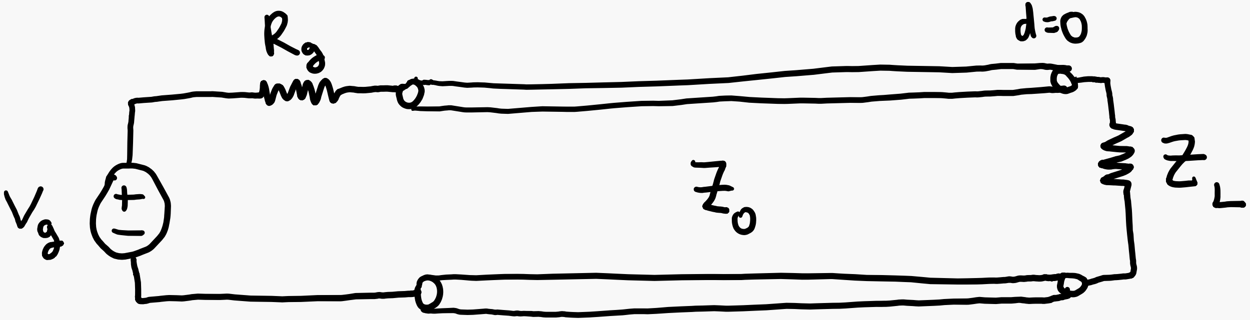
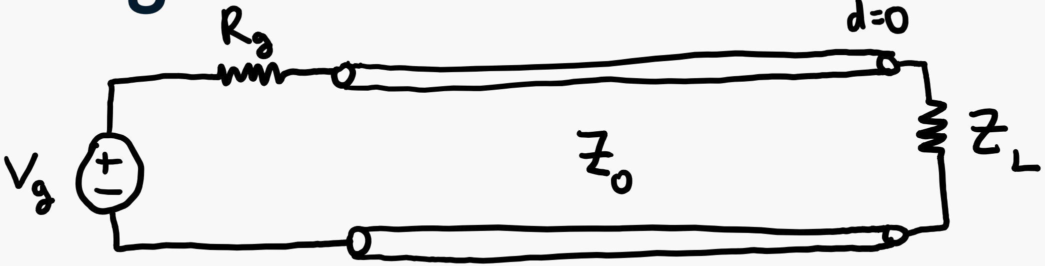



Special cases:

- What if $Z_L = 0$?
- What if $Z_L = \infty$?
- What if $Z_L = Z_0$?

# Problem 1: Bounce Diagrams

Input: $V_g = 5\delta(t)$ [V]
$Z_L = 50\Omega, R_g = 50\Omega, Z_0 = 100\Omega$
$v = c$, TL length = 3 [m].




Create a voltage bounce diagram for the first 45 nanoseconds.
Create a current bounce diagram for the first 45 nanoseconds.
Plot $V(0.25\text{m}, t)$ for the first 45 nanoseconds.

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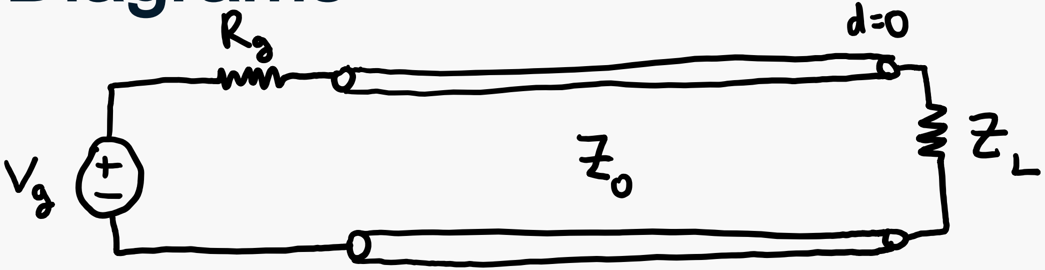

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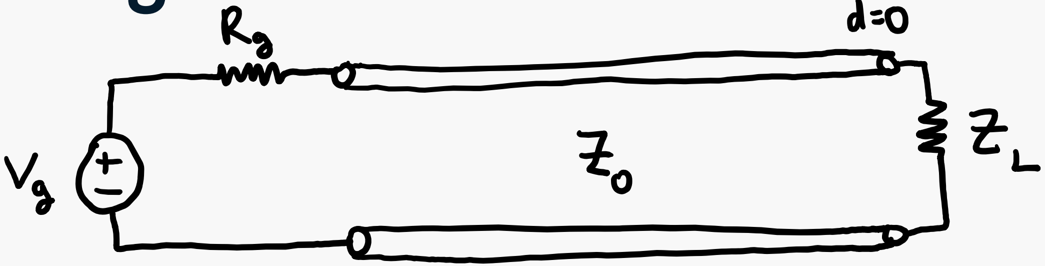


Plot $V(0.25\text{m}, t)$ for the first 45 nanoseconds.

# Problem 2: Bounce Diagrams

Input: $V_g = 5u(t)$ [V]
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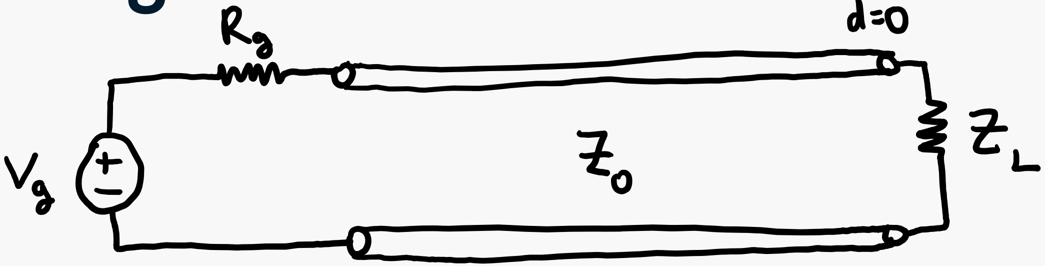


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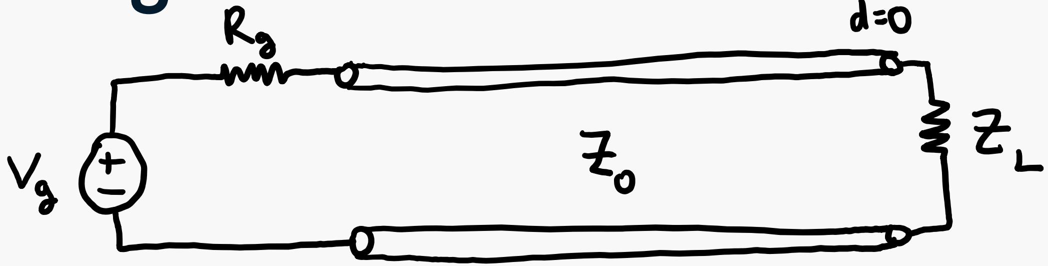
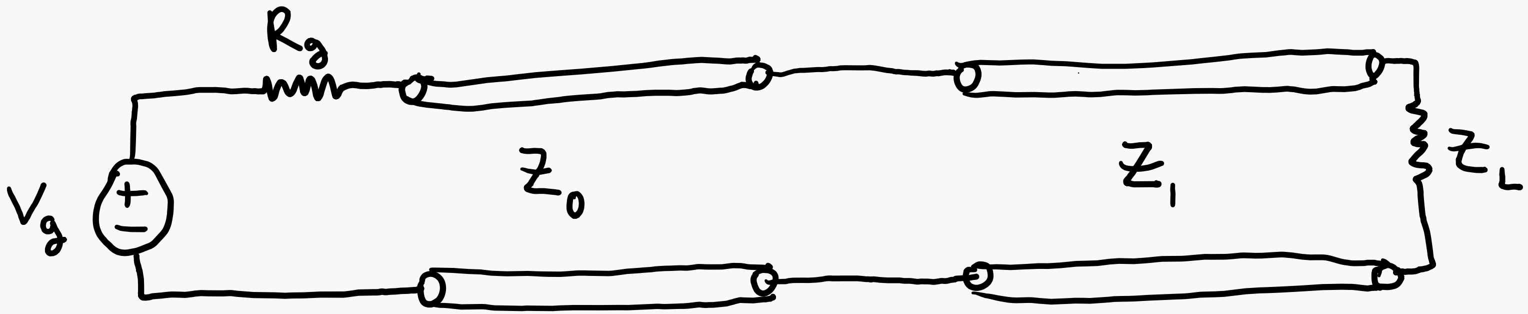



What is the steady-state voltage over the load?

What is the steady state current through the load?

# Multiline TL Circuits




When we travel FROM line $j$ TO line $k$:

$$\Gamma_{jk} = \frac{Z_k - Z_j}{Z_k + Z_j}$$

$$\tau_{jk} = 1 + \Gamma_{jk}$$

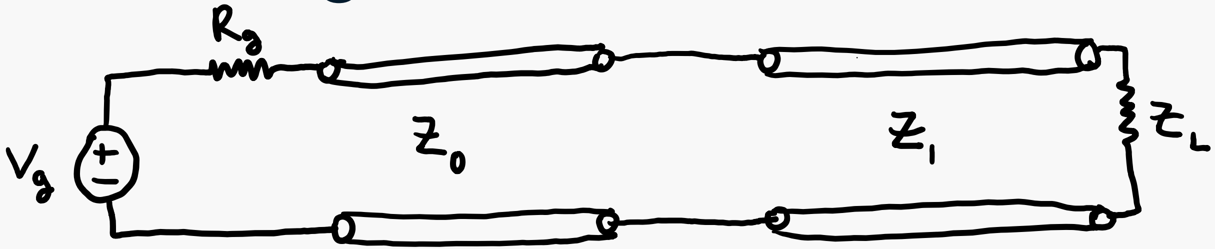
# Problem 3: Bounce Diagrams w/ Multiline

Input: $V_g = 5\delta(t)$ [V]

$Z_L = 50\Omega, R_g = 50\Omega$

$Z_0 = 100\Omega, Z_1 = 200\Omega$

$v = c$




First TL length = 3 [m]. Second TL length = 4.5 [m].

Create a voltage bounce diagram for the first 45 nanoseconds.
Create a current bounce diagram for the first 45 nanoseconds.

# Problem 3: Bounce Diagrams w/ Multiline

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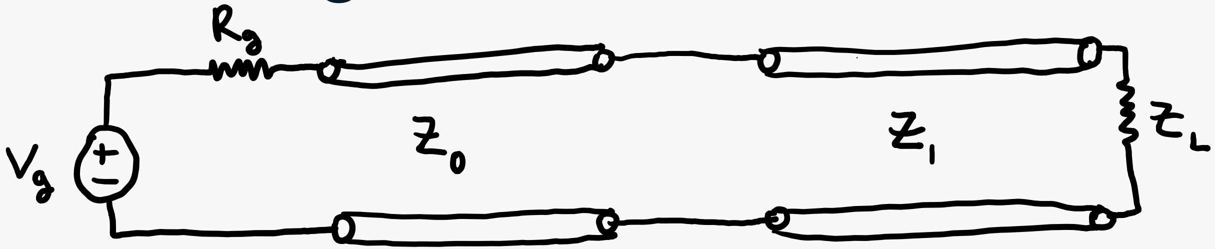
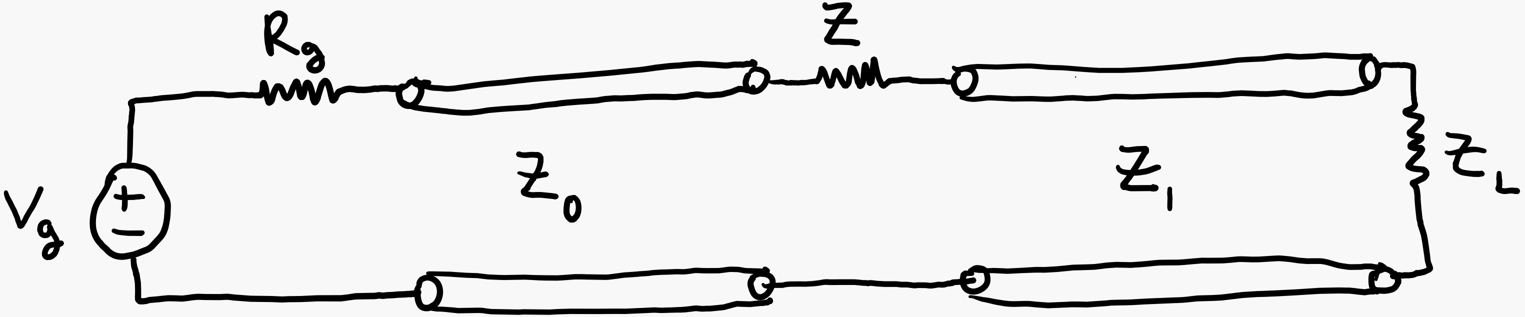
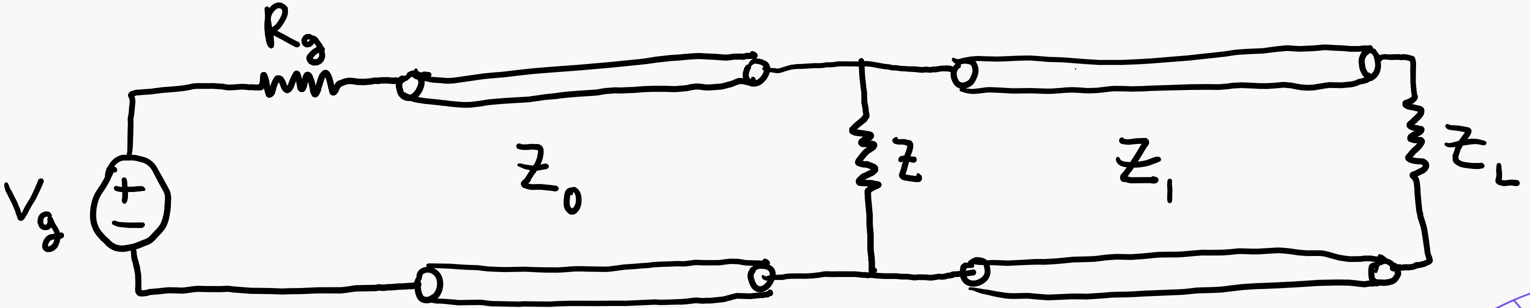


# Multiline TL Circuits

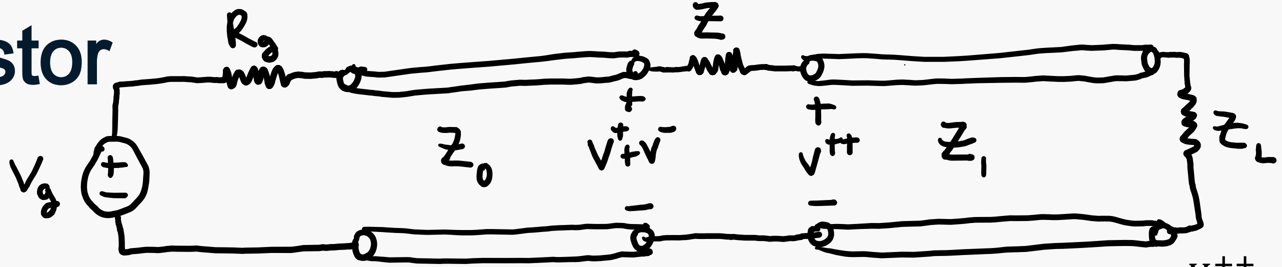
Series resistor (harder to deal with ☹):




Parallel resistor (easier to deal with ☺):



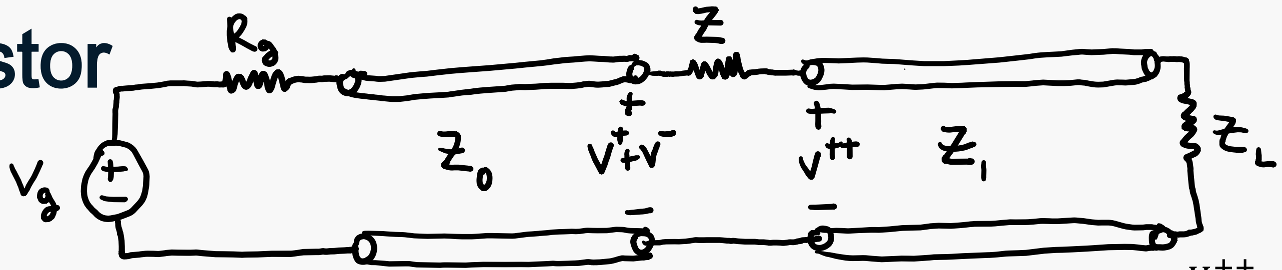
# Problem 4: Multiline Circuit w/ Series Resistor




Find the transmission coefficient across the junction, or $\tau_{01} = \frac{V^{++}}{V^{+}}$.

Find the reflection coefficient across the junction, or $\Gamma_{01} = \frac{V^{-}}{V^{+}}$.

# Problem 4: Multiline Circuit w/ Series Resistor

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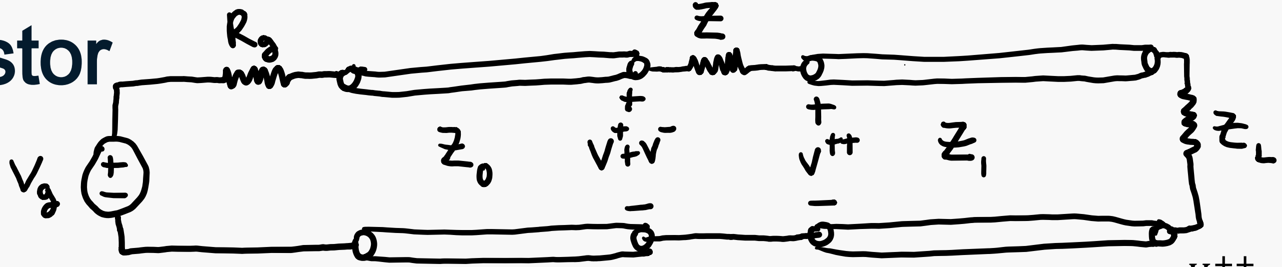



Find the transmission coefficient across the junction, or $\tau_{01} = \frac{V^{++}}{V^{+}}$.

Find the reflection coefficient across the junction, or $\Gamma_{01} = \frac{V^{-}}{V^{+}}$.

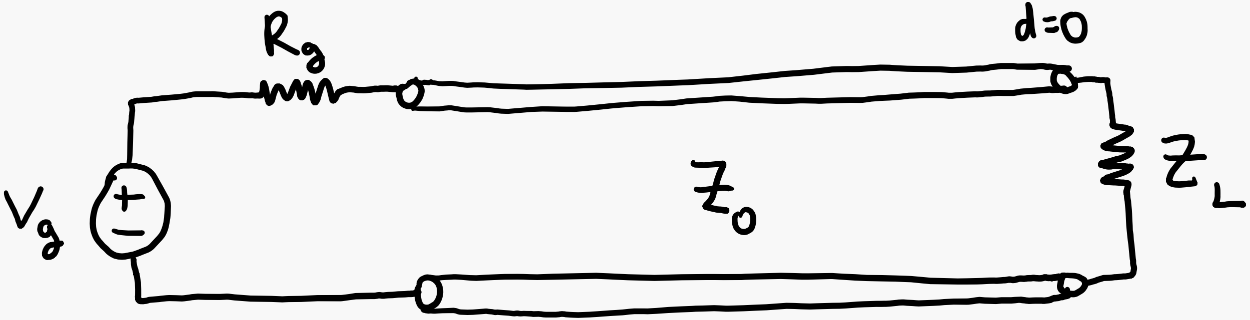
# Bounce Diagrams: General Formulation

This is an LTI system!
Input: $\delta(t)$
Output (at position $d$): $V(t)$
Time to travel down the line: $t_0$.

$$V(d,t) = \tau_g \sum_{n=0}^{\infty} \left(\Gamma_L \Gamma_g\right)^n \delta\left(t + \frac{d}{v} - n t_0\right) + \tau_g \Gamma_L \sum_{n=0}^{\infty} \left(\Gamma_L \Gamma_g\right)^n \delta\left(t - \frac{d}{v} - (n+1) t_0\right)$$

For any general input: convolve!

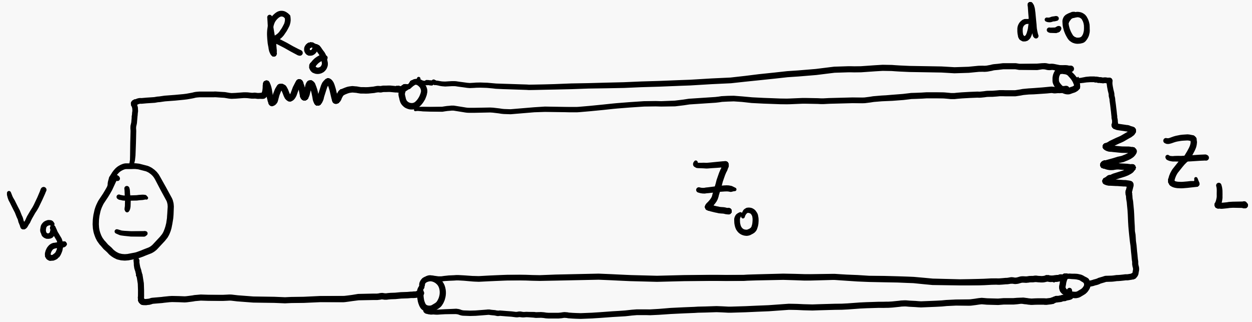
# Basic TL: Phasor Domain, Steady State




On TL: Forward-going voltage wave + backward-going voltage wave

On TL: Forward-going current wave + backward-going current wave

# Basic TL: Phasor Domain, Steady State

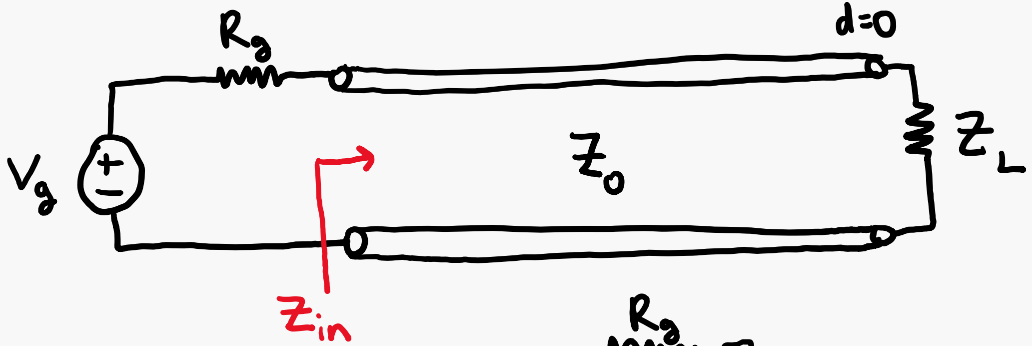



Impedance as a function of position?

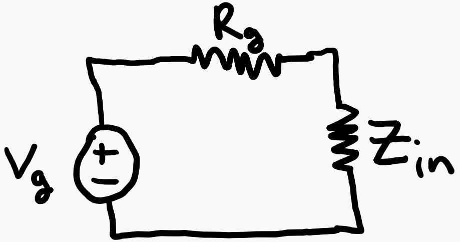
# Impedance Looking In: Steady State

The remainder of the semester deals with ‘Input Impedance’, i.e. $Z(d)$ at some interesting position $d$.

**Input impedance**: the impedance of the Thevenin resistor if the entire TL were replaced by this resistor.

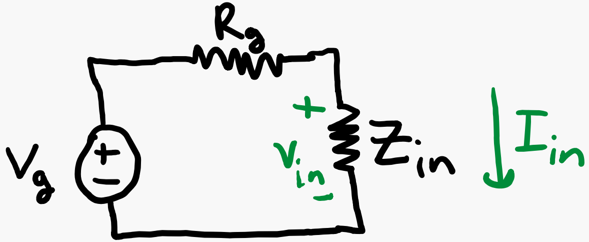



Behavior of $R_g$ and $V_g$ is the same for both circuits!!!

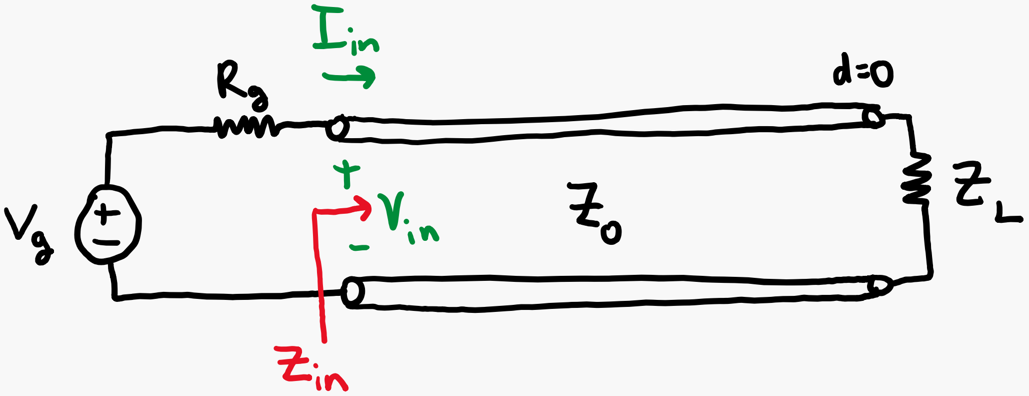


# Impedance Looking In: Steady State

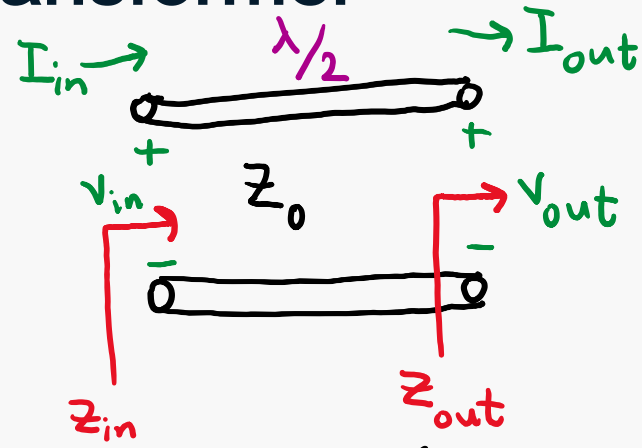
In Thevenin-equivalent circuit, impedance $Z_{in}$ has some voltage $V_{in}$ over it and some current $I_{in}$ running through it.




This corresponds to voltage and current at the input end of the TL!



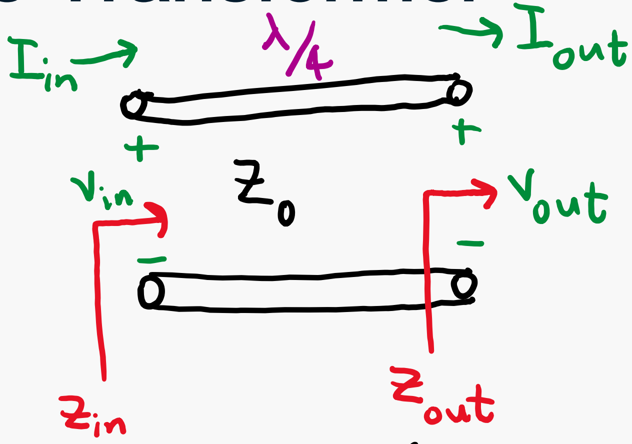
# Half - Wave Transformer




Allows you to ‘crawl’ up (or down) a TL in $\frac{\lambda}{2}$ steps.

$$V_{in} = -V_{out}$$
$$I_{in} = -I_{out}$$
$$Z_{in} = Z_{out}$$

# Quarter -Wave Transformer




Allows you to 'crawl' up (or down) a TL in $\frac{\lambda}{4}$ steps.

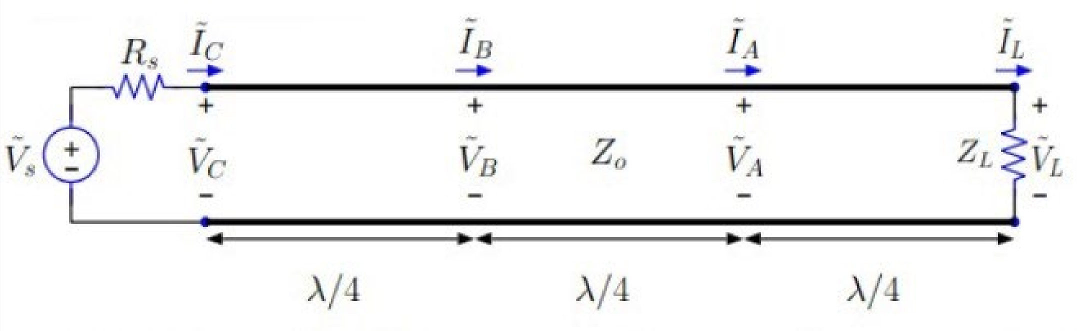
$$V_{in} = jI_{out}Z_0$$

$$I_{in} = \frac{jV_{out}}{Z_0}$$

$$Z_{in} = \frac{Z_0^2}{Z_{out}}$$

# Problem 5: Transformers

$\tilde{V}_s = 5e^{j\frac{\pi}{6}}$, $R_s = 75\Omega$
$Z_0 = 25\Omega$, $Z_L = 50 + j50\Omega$




Find $Z_B = \frac{\tilde{V}_B}{\tilde{I}_B}$ and $Z_C = \frac{\tilde{V}_C}{\tilde{I}_C}$.
Find $\tilde{V}_C$, $\tilde{V}_A$, and $\tilde{I}_L$.
Find average power absorbed by $Z_L$.

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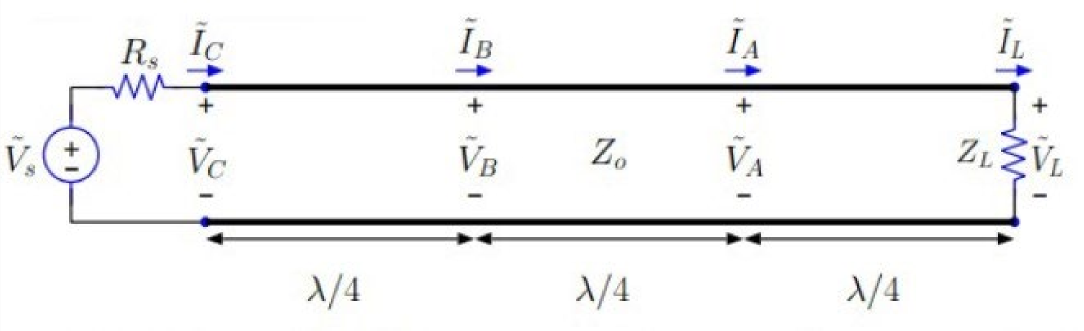


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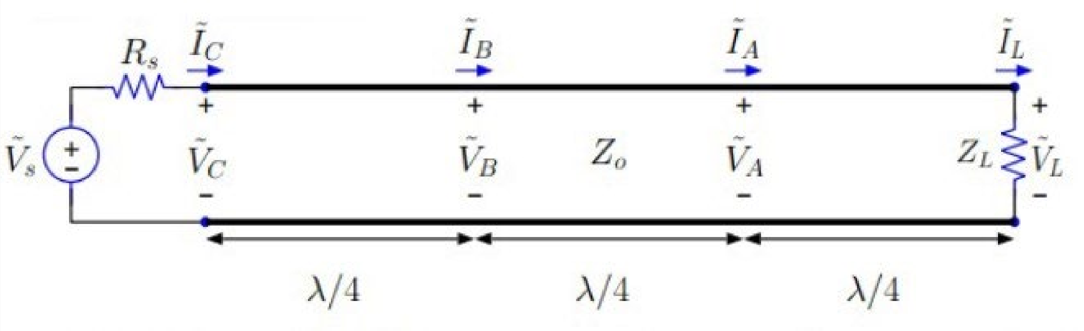


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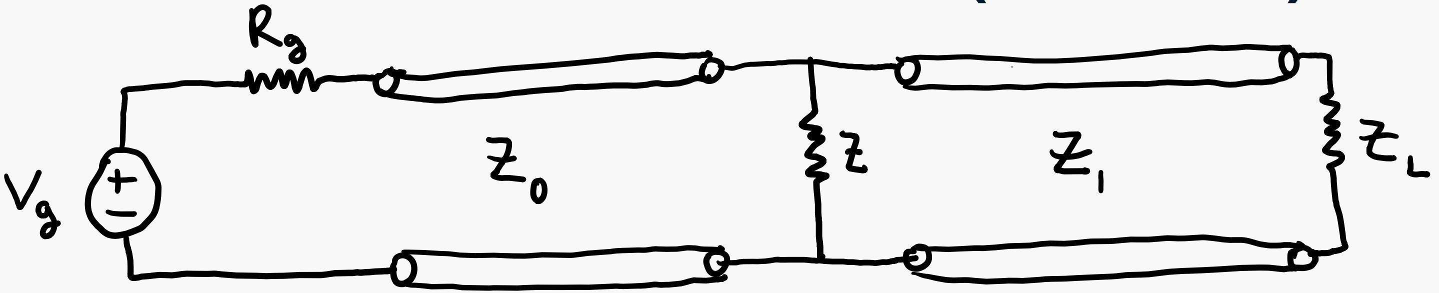
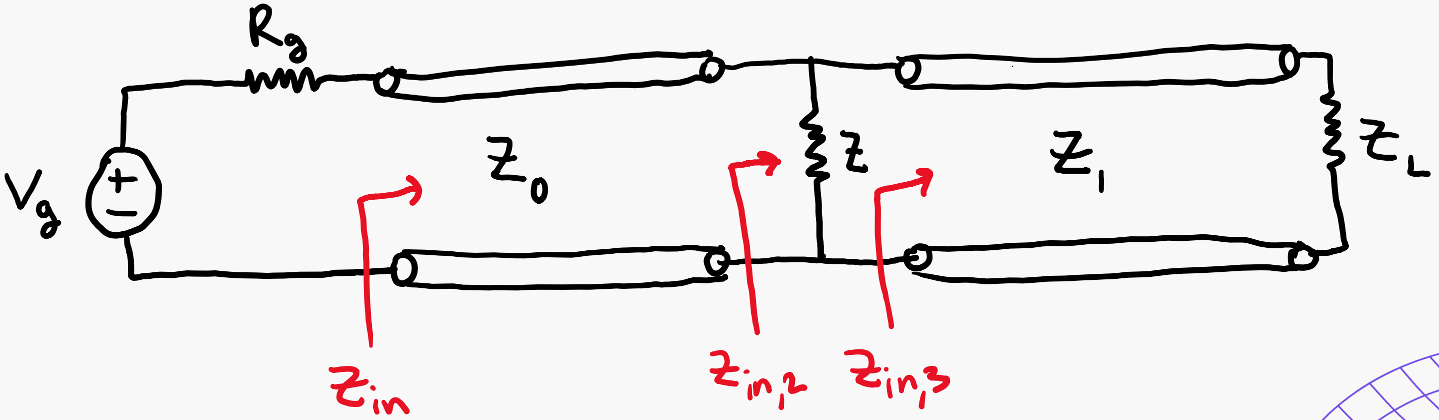
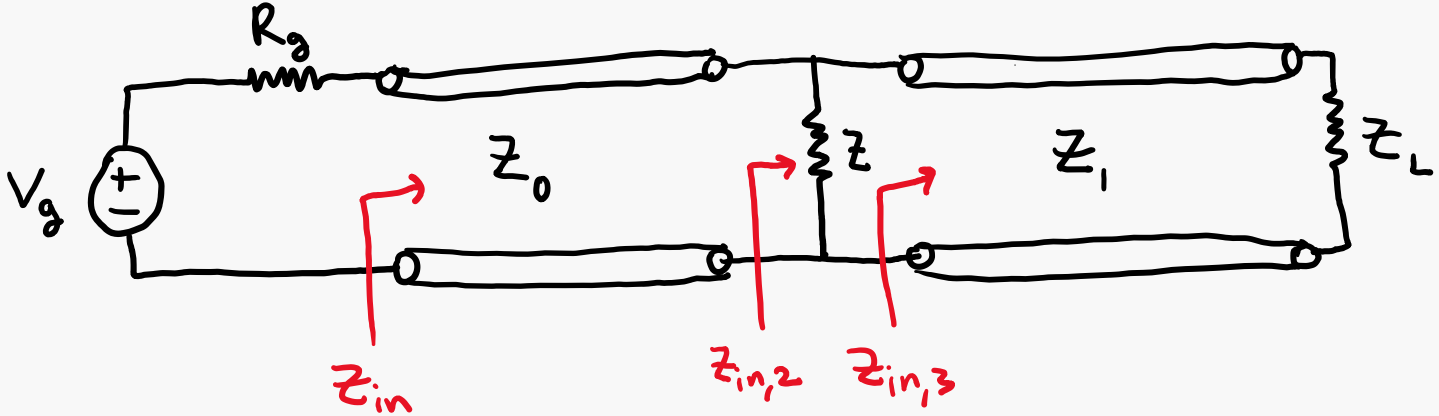
Find average power absorbed by $Z_L$.

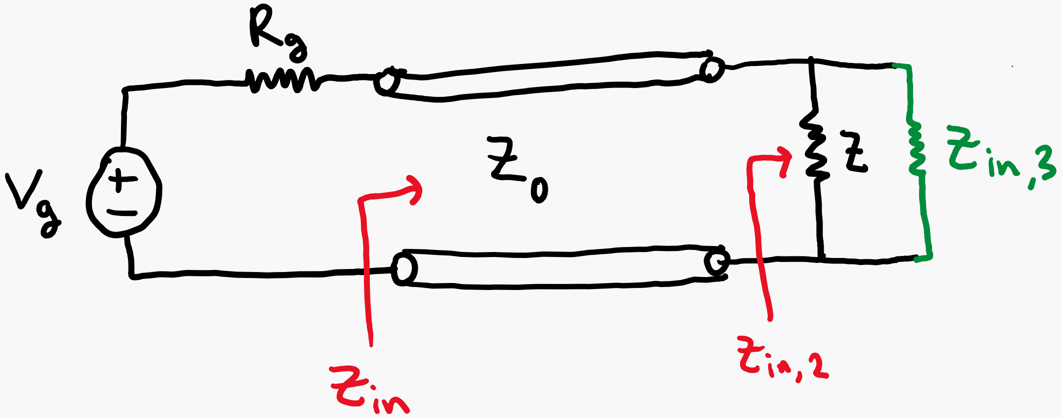
# Multiline TL: Parallel Resistor (HW Hint)



# Multiline TL: Parallel Resistor



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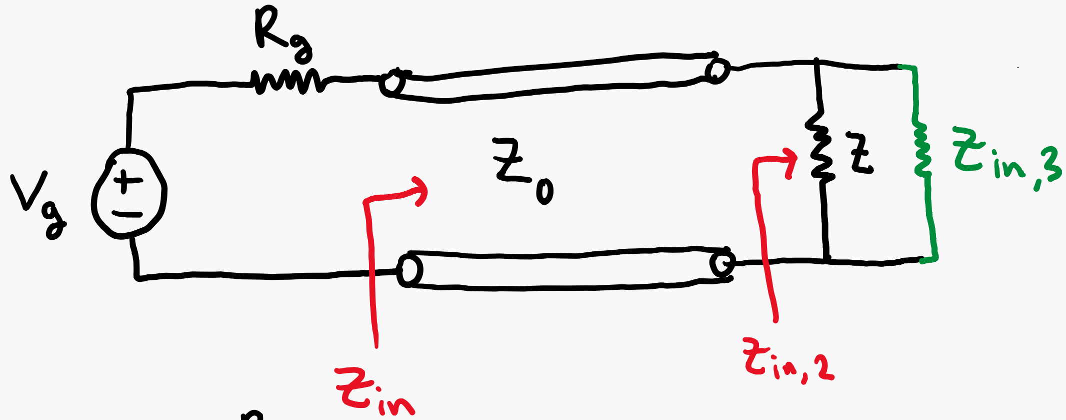


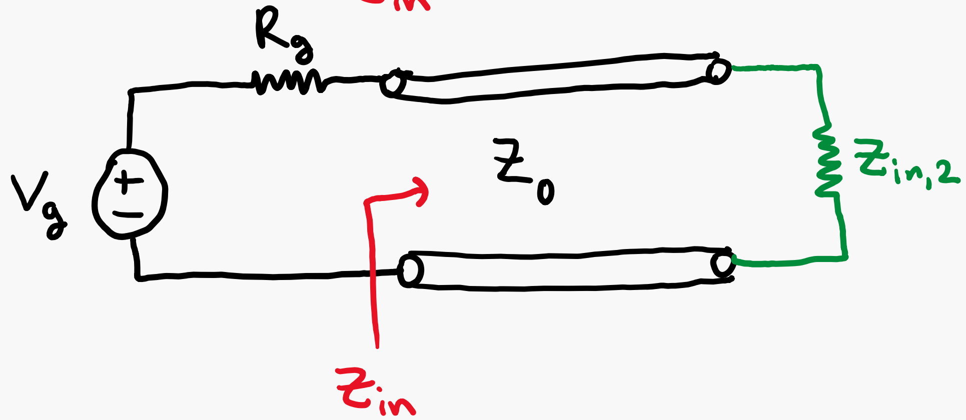

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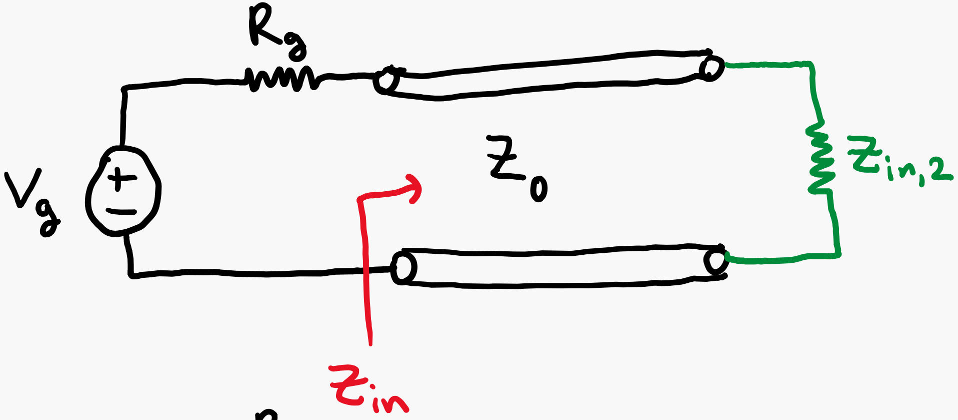


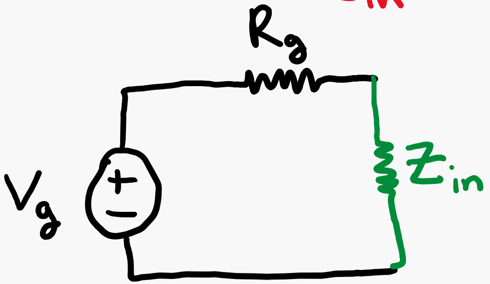

# Midterm 1 equations, in one place

$$\vec{F} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\vec{F} = q_1\vec{E} + q_1(\vec{v}_1 \times \vec{B})$$

$$\vec{E} = \frac{q_2}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\epsilon \oiint \vec{E} \cdot d\vec{S} = Q_{\text{enclosed}}$$

$$\oiint \vec{D} \cdot d\vec{S} = Q_{\text{enclosed}}$$

$$\iiint \rho dV = Q_{\text{enclosed}}$$

$$\oiint \vec{B} \cdot d\vec{S} = 0$$

$$I = \oiint \vec{J} \cdot d\vec{S} = -\frac{\partial Q_{\text{enclosed}}}{\partial t}$$

$$\nabla \times \vec{E} = 0$$

$$\vec{E} = -\nabla V$$

$$\oint \vec{E} \cdot d\vec{l} = 0$$

$$V_{ab} = V(b) - V(a) = -\int_a^b \vec{E} \cdot d\vec{l}$$

$$\hat{n} \cdot \left(\vec{D}_1 - \vec{D}_2\right) = \rho_s$$

$$\hat{n} \times \left(\vec{E}_1 - \vec{E}_2\right) = 0$$

$$\hat{n} \cdot \left(\vec{P}_1 - \vec{P}_2\right) = -\rho_{b,s}$$

$$\epsilon = \epsilon_0(1 + \chi_e)$$

$$\vec{P} = \epsilon_0\chi_e\vec{E}$$

$$\vec{D} = \epsilon_0\vec{E} + \vec{P} = \epsilon\vec{E}$$

$$\vec{J} = \sigma\vec{E}$$

$$\rho_b = -\nabla \cdot \vec{P}$$

$$\nabla \cdot \epsilon_0\vec{E} = \rho_f + \rho_b$$

$$\nabla \cdot \vec{D} = \rho$$

$$\nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$$

$$-\nabla^2 V = \frac{\rho}{\epsilon}$$

$$\oiint \vec{D} \cdot d\vec{S} = \iiint \nabla \cdot \vec{D} dV$$

$$\oint \vec{E} \cdot d\vec{l} = \iint \left(\nabla \times \vec{E}\right) \cdot d\vec{S}$$

$$\int_a^b \nabla V \cdot d\vec{l} = V(b) - V(a)$$

# Midterm 2 equations, in one place

$$\vec{B} = \frac{\mu I}{2\pi r}\hat{\phi}$$

$$d\vec{B} = \frac{\mu I d\vec{\ell} \times \hat{r}}{4\pi r^2}$$

$$\oint_C \vec{H} \cdot d\vec{\ell} = \iint_S \vec{J} \cdot d\vec{S}$$

$$\oint_C \vec{B} \cdot d\vec{\ell} = \mu I_{\text{encl}}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0$$

$$\Psi = \iint_s \vec{B} \cdot d\vec{S}$$

$$-\frac{d}{dt}\iint_s \vec{B} \cdot d\vec{S} = \oint_c \vec{E} \cdot d\vec{l}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\oint_c \vec{E} \cdot d\vec{l} = \varepsilon$$

$$\varepsilon = \frac{W}{q} = \oint_C \frac{\vec{F}}{q} \cdot d\vec{l}$$

$$\Psi = LI$$

$$\varepsilon = IR$$

$$v = \frac{\omega}{\beta} = \lambda f = \frac{1}{\sqrt{\mu\epsilon}}$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$\eta = \frac{\sqrt{\mu}}{\sqrt{\epsilon}}$$

$$\beta = \omega\sqrt{\mu\epsilon}$$

$$\nabla^2\vec{E} = \mu\epsilon\frac{\partial^2\vec{E}}{\partial t^2}$$

$$\vec{S} = \vec{E} \times \vec{H}$$

$$\tilde{S} = \tilde{E} \times \tilde{H}^*$$

$$< \vec{S} > = \frac{1}{2}\text{Re}\{\tilde{E} \times \tilde{H}^*\}$$

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\epsilon\vec{E} \cdot \vec{E} + \frac{1}{2}\mu\vec{H} \cdot \vec{H}\right) + \nabla \cdot \vec{S} + \vec{J} \cdot \vec{E} = 0$$

$$Q = CV$$

$$G = \frac{\sigma}{\epsilon}C \qquad R = \frac{1}{G}$$

$$\vec{J}_b = \frac{\partial \vec{P}}{\partial t} + \nabla \times \vec{M}$$

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

$$\vec{M} = \chi_m \vec{H}$$

$$\vec{B} = \mu_0\mu_r\vec{H} = \mu\vec{H}$$

$$A\cos(\omega t - \beta x)\hat{z} \leftrightarrow Ae^{-j\beta x}\hat{z}$$

$$\hat{n} \cdot (\vec{B}_1 - \vec{B}_2) = 0$$

$$\hat{n} \times (\vec{H}_1 - \vec{H}_2) = \vec{J}_s$$

$$\hat{n} \times (\vec{M}_1 - \vec{M}_2) = \vec{J}_{b,s}$$

# Midterm 3 equations, in one place

***Waves***:

| | Condition | $\beta$ | $\alpha$ | $\lvert\eta\rvert$ | $\tau$ | $\lambda = \frac{2\pi}{\beta}$ | $\delta = \frac{1}{\alpha}$ |
|---|---|---|---|---|---|---|---|
| Perfect dielectric | $\sigma = 0$ | $\omega\sqrt{\epsilon\mu}$ | $0$ | $\sqrt{\frac{\mu}{\epsilon}}$ | $0$ | $\frac{2\pi}{\omega\sqrt{\epsilon\mu}}$ | $\infty$ |
| Imperfect dielectric | $\frac{\sigma}{\omega\epsilon} \ll 1$ | $\sim \omega\sqrt{\epsilon\mu}$ | $\beta\frac{1}{2}\frac{\sigma}{\omega\epsilon} = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}$ | $\sim \sqrt{\frac{\mu}{\epsilon}}$ | $\sim \frac{\sigma}{2\omega\epsilon}$ | $\sim \frac{2\pi}{\omega\sqrt{\epsilon\mu}}$ | $\frac{2}{\sigma}\sqrt{\frac{\epsilon}{\mu}}$ |
| Good conductor | $\frac{\sigma}{\omega\epsilon} \gg 1$ | $\sim \sqrt{\pi f\mu\sigma}$ | $\sim \sqrt{\pi f\mu\sigma}$ | $\sqrt{\frac{\omega\mu}{\sigma}}$ | $45°$ | $\sim \frac{2\pi}{\sqrt{\pi f\mu\sigma}}$ | $\sim \frac{1}{\sqrt{\pi f\mu\sigma}}$ |
| Perfect conductor | $\sigma = \infty$ | $\infty$ | $\infty$ | $0$ | - | $0$ | $0$ |

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \qquad \tau = \frac{2\eta_2}{\eta_2 + \eta_1} = 1 + \Gamma$$

$$v = \frac{\omega}{\beta} = \lambda f$$

$$\nabla^2 \tilde{E} = (j\omega\mu)(\sigma + j\omega\epsilon)\tilde{E}$$

***TLs***:

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\tau_g = \frac{Z_0}{R_g + Z_0}$$

$$\Gamma_g = \frac{R_g - Z_0}{R_g + Z_0}$$

$$\Gamma_{jk} = \frac{Z_k - Z_j}{Z_k + Z_j}$$

$$\tau_{jk} = 1 + \Gamma_{jk}$$

Half-wave:

$$V_{in} = -V_{out}$$

$$I_{in} = -I_{out}$$

$$Z_{in} = Z_{out}$$

Quarter-wave:

$$V_{in} = jI_{out}Z_0$$

$$I_{in} = \frac{jV_{out}}{Z_0}$$

$$Z_{in} = \frac{Z_0^2}{Z_{out}}$$

# Units

Charge $Q$: C
Current $I$: A
Electric field strength $\vec{E}$: N/C or V/m
Electric flux density $\vec{D}$: C/m$^2$
Polarization field $\vec{P}$: C/m$^2$
Electric potential $V$: V
Capacitance $C$: F
Magnetic flux density $\vec{B}$: T or Wb/m$^2$
Magnetic field strength $\vec{H}$: A/m
Magnetic flux $\Psi$: Wb
Electromotive force $\varepsilon$: V
Inductance $L$: H

Electric permittivity $\epsilon$: F/m
Magnetic permeability $\mu$: H/m
Conductivity $\sigma$: Si/m

Charge density $\rho$: C/m$^3$
Surface charge density $\rho_S$: C/m$^2$
Current density $\vec{J}$: A/m$^2$

Intrinsic impedance $\eta$: Ohm
Wave number $\beta$: rad/m

Characteristic impedance $Z$: Ohm

# Office Hours

Any questions?