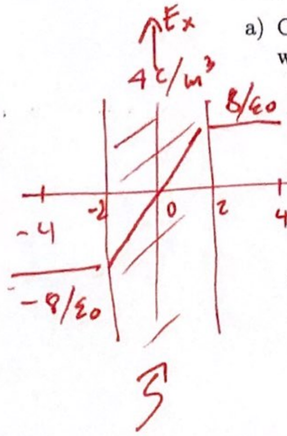
1. Two parts of this question are independent:



$\rho_s = 16\ \frac{C}{m^2}$

a) Consider an infinite slab of width $W = 4$ m occupying the region $|x| < 2$ m. The charge density within the slab is $\rho = 4$ C/m^3 and it is zero outside the slab.

i. (4 pts) Determine the electric field vector $\mathbf{E}(x, 0, 0)$ for $x = 4$ m and $x = -4$ m.

$$\bar{E}(x=-4m) = -\frac{8}{\varepsilon_0}\hat{x}\ V/m$$

$$\bar{E}(x=+4m) = \frac{8}{\varepsilon_0}\hat{x}\ V/m$$

ii. (4 pts) Determine the electric field vector $\mathbf{E}(x, 0, 0)$ for $-2 < x < 2$ m within the slab.

$$E(x) = \hat{x}\underbrace{\frac{4}{\varepsilon_0}x}_{E_x}\ V/m \qquad \text{s.t. } \bar{E}(x=2m) = \hat{x}\frac{8}{\varepsilon_0}\ V/m$$

iii. (2 pts) What is $\mathbf{E} \cdot d\mathbf{l}$ within the slab?

$$\bar{E}\cdot d\bar{l} = E_x dx + \cancelto{0}{E_y} dy + \cancelto{0}{E_z} dz = \frac{4x}{\varepsilon_0}dx$$

iv. (5 pts) Determine the electrostatic potential function $V(x, y, z)$ within the slab if $V(0, 0, 0) = 0$ V? **Hint/sanity check:** make sure V only depends on x and $\mathbf{E}$ field vectors point in the direction of decreasing V everywhere in the region.

$$V(X,y,z) = \int_X^0 \bar{E}\cdot d\bar{l} = \int_X^0 \frac{4x}{\varepsilon_0}dx = \frac{2x^2}{\varepsilon_0}\Big|_X^0 = 0 - \frac{2X^2}{\varepsilon_0} \Rightarrow V(x,y,z) = -\frac{2x^2}{\varepsilon_0}V$$

b) Consider the electric field vector

$$\mathbf{E} = \underbrace{\hat{x}\ \sin y}_{\text{div free}} + \underbrace{\hat{y}\ \cos y}_{\text{curl free}}\ \frac{\text{V}}{\text{m}}.$$

i. (4 pts) Determine $\nabla \cdot \mathbf{E}$ and $\nabla \times \mathbf{E}$.

$$\nabla\cdot\bar{E} = \cancel{\frac{\partial E_x}{\partial x}} + \frac{\partial E_y}{\partial y} + \cancel{\frac{\partial E_z}{\partial z}} = \frac{\partial}{\partial y}\cos(y) = -\sin(y)$$

$$\nabla\times\bar{E} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \sin y & \cos y & 0 \end{vmatrix} = \hat{x}\cdot 0 - \hat{y}\cdot 0 + \hat{z}\left(0 - \frac{\partial}{\partial y}\sin(y)\right) = -\hat{z}\cos(y)$$

ii. (3 pts) Which component of $\mathbf{E}$ is electrostatic? Explain.

Electrostatic component is the curl free part, $\hat{y}\cos(y)$ V/m

iii. (3 pts) What is the static charge density ρ implied by $\mathbf{E}$ at the location $\mathbf{r} = \hat{y}\frac{\pi}{2}$ m?

$$\rho = \nabla\cdot(\varepsilon_0\bar{E}) = -\varepsilon_0\sin(y) \Rightarrow \text{at } \bar{r} = \hat{y}\frac{\pi}{2}m,\ \rho = -\varepsilon_0\underbrace{\sin\left(\frac{\pi}{2}\right)}_{1} = -\varepsilon_0$$

2. Two parts of the following question are independent:

   a) (10 pts) Given that $\mathbf{E} = 2x\,\hat{x} + 2\sin(z)\,\hat{z}$ V/m, determine the electrostatic potential $V(x, y, z)$ if $V(0, 0, 0) = 1$ V.

$$-dV = \bar{E}\cdot d\bar{l} = 2x\,dx + 2\sin(z)\,dz = d(x^2 - 2\cos(z))$$

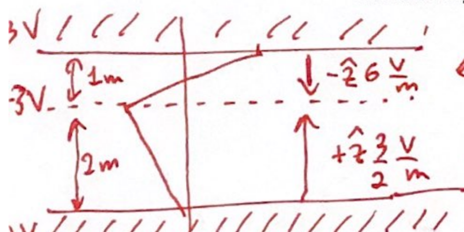
$$\Rightarrow V(x,y,z) = -x^2 + 2\cos(z) + C$$

To find C, $V(0,0,0) = -0 + 2 + C = 1 \Rightarrow \underline{\underline{C = -1}}$ ✓

$$\Leftrightarrow V(x,y,z) = -x^2 + 2\cos(z) - 1 \;//$$

   b) Conducting plates placed on $z = 0$ and $z = 3$ m surfaces (of infinite extent) are separated by vacuum and maintained at potentials of $V(0) = 0$ V and $V(3) = 3$ V. Also, the potential of the $z = 2$ m surface that supports some surface charge density ρ_{s2} is $V(2) = -3$ V.

   i. (5 pts) What are the electric field vectors $\mathbf{E}_1$ and $\mathbf{E}_2$ below and above the $z = 2$ m surface? Include in your answer an appropriate diagram where $\mathbf{E}_1$ and $\mathbf{E}_2$ vectors are shown explicitly.



an $\bar{E}_2 \Rightarrow \bar{E}_2 = -\hat{z}6\frac{V}{m} // \Rightarrow \bar{D}_2 = -\hat{z}6\epsilon_0 \frac{C}{m^2} //$

an $\bar{E}_1 \Rightarrow \bar{E}_1 = +\hat{z}\frac{3}{2}\frac{V}{m} // \Rightarrow \bar{D}_1 = +\hat{z}\frac{3}{2}\epsilon_0 \frac{C}{m^2} //$

   ii. (5 pts) What are the surface charge densities ρ_{s0} and ρ_{s3} at $z = 0$ and $z = 3$ m, respectively? Justify your answers.

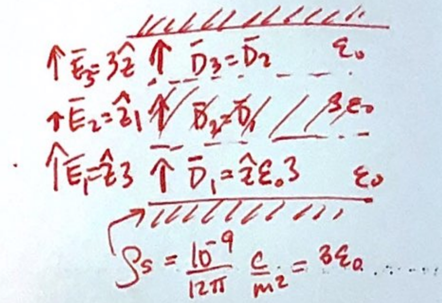
$$\rho_{s0} = \frac{3}{2}\underbrace{\epsilon_0}_{\frac{10^{-9}}{36\pi}}\frac{C}{m^2} //$$

$$\rho_{s3} = 6\epsilon_0 \frac{C}{m^2} //.$$

   iii. (5 pts) What is the surface charge density ρ_{s2} at $z = 2$ m? Justify your answer.

$$\rho_{s2} = \hat{z}\cdot(\bar{D}_2 - \bar{D}_1) = \hat{z}\cdot\left(-\hat{z}6\epsilon_0 - \hat{z}\frac{3}{2}\epsilon_0\right) = -6\epsilon_0 - \frac{3}{2}\epsilon_0 = \underline{\underline{-7.5\epsilon_0 \frac{C}{m^2}}}$$

3. Consider conducting plates placed on $z = 0$ and $z = 3$ m surfaces (of infinite extent) holding $\rho_{s0} = \frac{10^{-9}}{12\pi}$ C/m^2 and $\rho_{s3} = -\frac{10^{-9}}{12\pi}$ C/m^2 (i.e. of $\pm 3\epsilon_o$ magnitude) of surface charge densities, respectively. The region between the plates is free space except for an infinite dielectric slab of 1 m width occupying the region $1 < z < 2$ m with a permittivity of $\epsilon = 3\epsilon_o$, where $\epsilon_o = \frac{10^{-9}}{36\pi}$ F/m is the permittivity of free space. There are no free charges in $0 < z < 3$ m region. Determine (indicating the proper units):



a) (3 pts) Displacement **D** in $0 < z < 1$ m region. Explain.

$\bar{D} = \hat{z}3\varepsilon_o = \hat{z}\frac{10^{-9}}{12\pi}\frac{C}{m^2}$ (matching $\bar{D}$ to ρ_{s0})

b) (3 pts) Displacement **D** in $1 < z < 2$ m region. Explain.

$\bar{D} = \hat{z}3\varepsilon_o$ again (no free surface charges at $z = 1$ m surface)

c) (3 pts) Electric field vector **E** in $1 < z < 2$ m region. Explain.

$\bar{E} = \frac{\bar{D}}{\varepsilon} = \frac{\hat{z}3\varepsilon_o}{3\varepsilon_o} = \hat{z}1\frac{V}{m}$

d) (3 pts) Polarization field vector **P** in $1 < z < 2$ m region. Explain.

$\bar{P} = \bar{D} - \varepsilon_o\bar{E} = \hat{z}3\varepsilon_o - \hat{z}\varepsilon_o = \hat{z}2\varepsilon_o = \hat{z}\frac{10^{-9}}{18\pi}\frac{C}{m^2}$

e) (3 pts) Electric field vector **E** in $2 < z < 3$ m region. Explain.

$\bar{E} = \frac{\bar{D}}{\varepsilon} = \frac{\hat{z}3\varepsilon_o}{\varepsilon_o} = \hat{z}3\frac{V}{m}$

f) (3 pts) Polarization field vector **P** in $2 < z < 3$ m region. Explain.

$\bar{P} = 0$ here because no polarizable material.

g) (3 pts) Potential $V(z = 3\,\text{m})$ if $V(z = 0) = 0$ V. Explain.

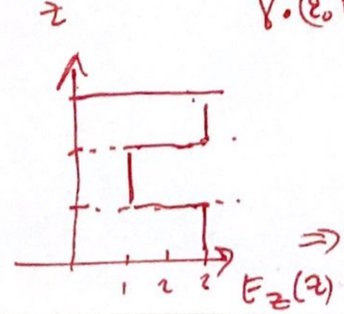
$$V(3) = \int_3^0 \bar{E}\cdot d\bar{l} = \int_3^0 E_z dz = \underbrace{\int_3^2 E_z dz}_{-3} + \underbrace{\int_2^1 E_z dz}_{-1} + \underbrace{\int_1^0 E_z dz}_{-3} = -7\ V$$

h) (4 pts) Compare the variations of functions $\nabla\cdot\epsilon\mathbf{E}$ and $\nabla\cdot\epsilon_o\mathbf{E}$ with variable z throughout the region $0 < z < 3$ m — which of these exhibit a simpler variation and how would you interpret the scalar function $\nabla\cdot\epsilon_o\mathbf{E}$? Discuss!

$\nabla\cdot(\underbrace{\varepsilon\bar{E}}_{\bar{D}}) = \nabla\cdot\bar{D} = 0$ ← Because $\bar{D}$ is independent of z ← simpler

$\nabla\cdot(\varepsilon_o\bar{E}) = \varepsilon_o\nabla\cdot\bar{E} = \varepsilon_o\frac{\partial}{\partial z}E_z = 2\varepsilon_o(\delta(z-2) - \delta(z-1))\frac{C}{m^3}$ ← indicates eqv. bound charge at $z = 1$ & $z = 2$ m surfaces.

Does not show up in $\nabla\cdot\bar{D}$ because bound charge is hidden within $\bar{D}$!!



$\Rightarrow \frac{\partial E_z}{\partial z} = -2\delta(z-1) + 2\delta(z-2)$