ECE 329	Introduction to Electromagnetic Fields	Spring 13

University of Illinois — **Gilbert, Kim, Schutt-Aine, Waldrop**

Exam 3

Thursday, Apr. 18, 2013 — 7:00-8:15 PM

Name:	
Section:	9 AM 12 Noon 1 PM 2 PM

Please clearly PRINT your name in CAPITAL LETTERS and CIRCLE YOUR SECTION in the above boxes.

This is a closed book exam and calculators are not allowed. Please show all your work and make sure to include your reasoning for each answer. **All answers should include units wherever appropriate**.

Problem 1 (25 points)	
Problem 2 (25 points)	
Problem 3 (25 points)	
Problem 4 (25 points)	
TOTAL (100 points)	

1. A TEM wave is propagating through a good conductor, with time-domain electric and magnetic fields given by:

$$\mathbf{E}(y,t) = 100\sqrt{2}e^{10\pi y}\sin(2\pi \times 10^8 t + \beta y + \phi)\hat{x} \text{ V/m}$$

$$\mathbf{H}(y,t) = 50e^{10\pi y}\sin(2\pi \times 10^8 t + \beta y + \frac{\pi}{3})\hat{h} \text{ A/m}$$

a) (5 pts) What is the complex propagation constant γ?

b) (5 pts) What is the magnitude and direction of the phase velocity $\mathbf{v_p}$ of the wave?

c) (5 pts) What is $\hat{h}$, the direction of the magnetic intensity field $\mathbf{H}$?

d) (5 pts) What is the complex impedance η of the material?

e) (5 pts) What is the phase ϕ of the electric field?

2. (25 points) A plane wave field $\mathbf{E_i} = E_0 \cos(\omega t - \beta_1 y)\hat{x} - 2E_0 \sin(\omega t - \beta_1 y)\hat{z}$ V/m is propagating in vacuum in the region $y < 0$ at a frequency $\omega = 2\pi \times 10^8$ rad/s. The wave is incident upon the $y = 0$ plane, which is the boundary of a perfect dielectric material with permittivity $9\epsilon_0$ and permeability μ_0 in the region $y > 0$.

   a) (5 points) Determine the magnetic wavefield $\mathbf{H}_i$ corresponding to the $\mathbf{E}_i$ field given above in the region $y < 0$.

   b) (5 points) Determine the reflection and transmission coefficients, Γ and τ, at the boundary.

   c) (10 points) Determine the phasor expressions for the incident, reflected and transmitted electric fields, $\tilde{\mathbf{E}}_\mathbf{i}$, $\tilde{\mathbf{E}}_\mathbf{r}$, and $\tilde{\mathbf{E}}_\mathbf{t}$. Write your expression in terms of the reflection and transmission coefficient, Γ and τ, instead of the actual numerical values found in part (b) above. ***Be sure to include explicit values for the wavenumber*** $\boldsymbol{\beta}$***.***

   d) (5 points) Determine the polarizations of the incident and reflected waves.

3. **PART ONE**: A plane TEM wave is confined to propagate in a perfect dielectric material (μ, ϵ) between two perfectly conducting plates located in the $x = 0$ and $x = d$ planes, respectively, which each have width W and length L. Neglecting frinding fields, the guided TEM wave satisfies Maxwell's equations, with Ampere's and Faraday's Law written in simplified form as:

$$\frac{\partial H_y}{\partial z} = -\epsilon \frac{\partial E_x}{\partial t}$$

$$\frac{\partial E_x}{\partial z} = -\mu \frac{\partial H_y}{\partial t}$$

a) (2 pts) What is the polarization of this guided TEM wave?

   i. right-handed circular

   ii. left-handed circular

   iii. linear along $\hat{z}$

   iv. linear along $\hat{x}$

   v. this wave is not polarized

b) (4 pts) Defining the voltage drop between the plates as V, what is $\dfrac{\partial V}{\partial z}$ in terms of the current I along the plates, the dielectric material properties (μ and/or ϵ), and the plate dimensions?

c) (3 pts) If the width of the plates is increased, which parameters associated with the guided TEM wave would change? (choose all that apply)

   i. the surface current density $\mathbf{J_s}$ on the top plate

   ii. the capacitance of the system per unit length, $\mathcal{C}$

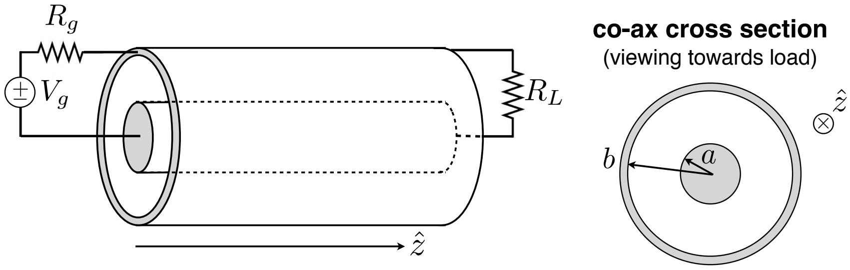
   iii. the intrinsic impedance of the dielectric material, η

   iv. the characteristic impedance of the system, Z_0

   v. the voltage drop from the bottom to the top plate at a given distance z

   vi. the propagation speed of the wave

PART TWO: Consider a guided TEM wave propagating in a vacuum between two perfectly conducting cylindrical shells having length $L = 1.5$ cm along the $\hat{z}$ direction and radii a and b, respectively, where $L \gg b$ (see diagram below). The TEM wave is generated by voltage and current variations introduced at one end of the line from a voltage source $V_g = 10u(t)$ V having internal Thevenin resistance $R_g = Z_0 = 50\ \Omega$. A resistive load $R_L = Z_0$ is connected to the other end of the line.



a) (3 pts) How long will it take the transmission line to reach steady state equilibrium?

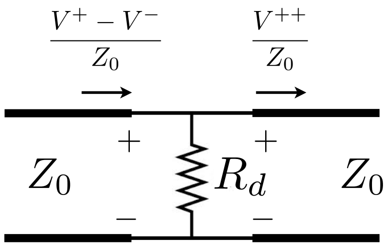
b) (4 pts) Draw vectors in the region between the cylinders in the figure on the right above to indicate the directions of the electric and magnetic fields once steady state is reached. Note that $\hat{z}$, the direction towards the load, is pointing into the page.

c) (2 pts) **TRUE** or **FALSE**: Once it reaches steady state equilibrium, the surface current on the inner cylinder is flowing in the $+\hat{z}$ direction.

d) (2 pts) **TRUE** or **FALSE**: Once it reaches steady state equilibrium, the current densities on the inner and outer conductor surfaces have different absolute magnitudes $|J_s|$.

e) (5 pts) Once it reaches steady state equilibrium, what is the power P_L delivered to the load R_L?

4. A transmission line, which has a length 600 m and is characterized by $Z_0 = 400\ \Omega$ and $v_p = c$, has a defect halfway along the line (i.e., at 300 m) that can be modeled as a "shunt" resistance $R_d = 200\ \Omega$ as shown in the figure below.

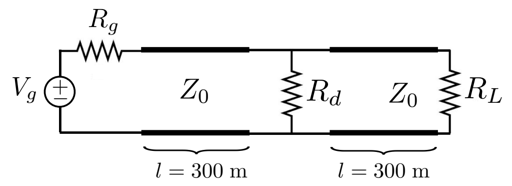


Incident voltage waveforms, denoted V^+, will be reflected at the junction such that $\frac{V^-}{V^+} \equiv \Gamma_d = \frac{Z_{eq} - Z_0}{Z_{eq} + Z_0}$, where the reflection coefficient Γ_d is defined in terms of an equivalent impedance Z_{eq} associated with the parallel combination of R_d and the rest of the transmission line.

a) (4 pts) What is Z_{eq} and Γ_d in this case? (Note that you do not need to derive Z_{eq} explicitly.)

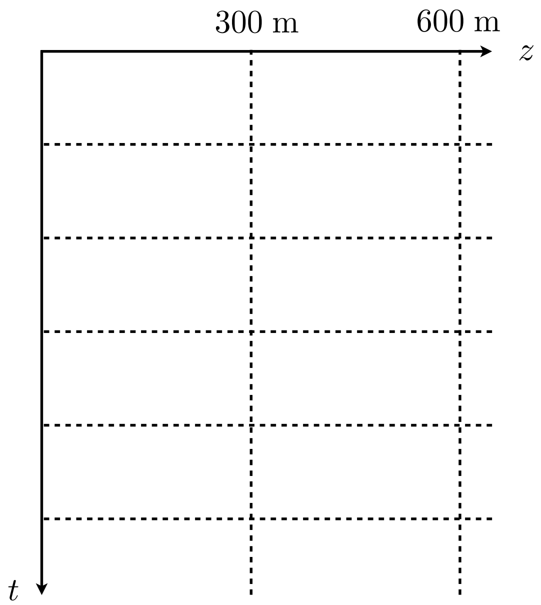
b) (4 pts) Use lumped circuit rules at the junction to show that the transmission coefficient through the defect, denoted τ_d, is equal to $\frac{V^{++}}{V^+} \equiv \tau_d = 1 + \Gamma_d$.

c) Now consider (for parts c through e) that this transmission line, including its defect, is connected at time $t = 0$ to a DC step voltage source of magnitude $V_g = 100$ V and internal resistance $R_g = 400\ \Omega$ and a load resistor $R_L = 1.2\ \text{k}\Omega$ (see diagram).



(6 pts) What is the injection coefficient τ_g, the load reflection coefficient Γ_L, and the source reflection coefficient Γ_g associated with this transmission line network?

d) (8 pts) On the axes below, draw a bounce diagram for the ***voltage waveforms*** propagating on both segments of the line for the first 5 μs after the switch is closed. Indicate voltage magnitudes in terms of τ_g, Γ_g, Γ_L, Γ_d, and τ_d. Be sure to mark all relevant values on the time axis.



e) (3 pts) What voltage would be measured on the line at $z = 150$ m at time $t = 4\ \mu$s? (choose one):

i. $V(150\ \text{m}, 4\ \mu\text{s}) = 0$

ii. $V(150\ \text{m}, 4\ \mu\text{s}) = V_g \tau_g \tau_d^2 \Gamma_L$

iii. $V(150\ \text{m}, 4\ \mu\text{s}) = V_g \tau_g (1 + \Gamma_d + \tau_d^2 \Gamma_L)$

iv. $V(150\ \text{m}, 4\ \mu\text{s}) = V_g \tau_g (1 + \Gamma_d + \Gamma_g \Gamma_d + \tau_d^2 \Gamma_g \Gamma_d^2)$

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1. A TEM wave is propagating through a good conductor, with time-domain electric and magnetic fields given by:

$$\mathbf{E}(y,t) = 100\sqrt{2}e^{10\pi y}\sin(2\pi \times 10^8 t + \beta y + \phi)\hat{x} \text{ V/m}$$

α

$$\mathbf{H}(y,t) = 50e^{10\pi y}\sin(2\pi \times 10^8 t + \beta y + \frac{\pi}{3})\hat{h} \text{ A/m}$$

a) (5 pts) What is the complex propagation constant γ?

$\gamma = \alpha + j\beta$. $\alpha = 10\pi$. good conductor, so $\alpha \simeq \beta$.

$\boxed{\gamma = 10\pi(1+j)}$

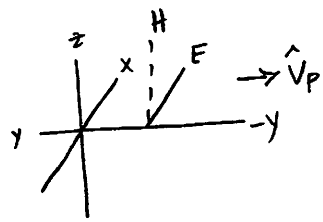
b) (5 pts) What is the magnitude and direction of the phase velocity $\mathbf{v_p}$ of the wave?

$\hat{v}_p = -\hat{y}$

$v_p = \frac{\omega}{\beta} = \frac{2\pi \cdot 10^8}{10\pi} = \frac{1}{5}\cdot 10^8 = 2\cdot 10^7 \text{ m/s}$

c) (5 pts) What is $\hat{h}$, the direction of the magnetic intensity field $\mathbf{H}$?

$\hat{e} = \hat{x}$, $\hat{v}_p = -\hat{y}$. $\hat{e} \times \hat{h} = \hat{v}_p \Rightarrow \boxed{\hat{h} = \hat{z}}$



d) (5 pts) What is the complex impedance η of the material?

$\frac{E}{H} = \eta = |\eta| e^{j\tau}$. $|\eta| = \frac{|E|}{|H|} = \frac{100\sqrt{2}}{50} = 2\sqrt{2}$

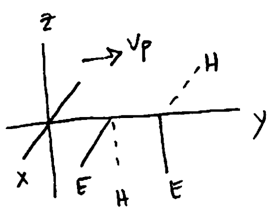
$\boxed{\eta = 2\sqrt{2}e^{j\pi/4}}$ $\tau = \angle\eta = \pi/4$ for good conductors

e) (5 pts) What is the phase ϕ of the electric field?

$\frac{\angle\tilde{E}}{\angle\tilde{H}} = \frac{e^{j\phi}}{e^{j\pi/3}} = e^{j\pi/4}$ so $\phi - \frac{\pi}{3} = \frac{\pi}{4} \Rightarrow \boxed{\phi = \frac{7\pi}{12}}$

2. (25 points) A plane wave field $\mathbf{E_i} = E_0 \cos(\omega t - \beta_1 y)\hat{x} - 2E_0 \sin(\omega t - \beta_1 y)\hat{z}$ V/m is propagating in vacuum in the region $y < 0$ at a frequency $\omega = 2\pi \times 10^8$ rad/s. The wave is incident upon the $y = 0$ plane, which is the boundary of a perfect dielectric material with permittivity $9\epsilon_0$ and permeability μ_0 in the region $y > 0$.

a) (5 points) Determine the magnetic wavefield $\mathbf{H_i}$ corresponding to the $\mathbf{E_i}$ field given above in the region $y < 0$.



$$\vec{H}_i = \frac{E_0}{\eta_1}\cos(\omega t - \beta_1 y)(-\hat{z}) + \frac{2E_0}{\eta_1}\sin(\omega t - \beta_1 y)(-\hat{x}) \quad \text{A/m}$$

$$\text{where } \eta_1 = \eta_0, \quad \beta_1 = \frac{\omega}{c} = \frac{2\pi \cdot 10^8}{3 \cdot 10^8} = \frac{2\pi}{3} \ \frac{\text{rad}}{\text{m}}$$

b) (5 points) Determine the reflection and transmission coefficients, Γ and τ, at the boundary.

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \quad \text{where } \eta_2 = \sqrt{\frac{\mu_0}{9\varepsilon_0}} = \frac{1}{3}\eta_0.$$

$$\Gamma = \frac{\frac{1}{3} - 1}{\frac{1}{3} + 1} = \frac{-2/3}{4/3} = -\frac{1}{2}.$$

$$\tau = 1 + \Gamma = 1 - \frac{1}{2} = \frac{1}{2}.$$

c) (10 points) Determine the phasor expressions for the incident, reflected and transmitted electric fields, $\mathbf{\tilde{E}_i}$, $\mathbf{\tilde{E}_r}$, and $\mathbf{\tilde{E}_t}$. Write your expression in terms of the reflection and transmission coefficient, Γ and τ, instead of the actual numerical values found in part (b) above. ***Be sure to include explicit values for the wavenumber β.***

$$\tilde{E}_i = E_0 e^{-j\beta_1 y}\hat{x} - 2E_0(-j)e^{-j\beta_1 y}\hat{z} = E_0 e^{-j\beta_1 y}(\hat{x} + 2j\hat{z}) \quad \text{V/m}$$

$$\text{where } \beta_1 = 2\pi/3 \text{ as above.}$$

$$\tilde{E}_r = \Gamma E_0 e^{+j\beta_1 y}(\hat{x} + 2j\hat{z})$$

$$\tilde{E}_t = \tau E_0 e^{-j\beta_2 y}(\hat{x} + 2j\hat{z}) \quad \text{where } \beta_2 = \frac{\omega}{v_{p2}} = \frac{\omega}{c}\cdot 3 = 3\beta_1 = 2\pi \ //$$

$$\left(v_{p2} = \frac{1}{\sqrt{\mu_0 9\varepsilon_0}} = \frac{c}{3}\right)$$

d) (5 points) Determine the polarizations of the incident and reflected waves.

incident: $\hat{x} \times (-\hat{z}) = +\hat{y} = +\hat{v}_p \quad \therefore$ right handed.

reflected: $\hat{x} \times (-\hat{z}) = +\hat{y} = -\hat{v}_p \quad \therefore$ left handed

both waves have $|E_x| \neq |E_z|$ so both are elliptical, not circular.

3. **PART ONE**: A plane TEM wave is confined to propagate in a perfect dielectric material (μ, ϵ) between two perfectly conducting plates located in the $x = 0$ and $x = d$ planes, respectively, which each have width W and length L. Neglecting frinding fields, the guided TEM wave satisfies Maxwell's equations, with Ampere's and Faraday's Law written in simplified form as:

$$\frac{\partial H_y}{\partial z} = -\epsilon \frac{\partial E_x}{\partial t}$$

$$\frac{\partial E_x}{\partial z} = -\mu \frac{\partial H_y}{\partial t}$$

a) (2 pts) What is the polarization of this guided TEM wave?

i. right-handed circular

ii. left-handed circular

iii. linear along $\hat{z}$

iv. linear along $\hat{x}$

v. this wave is not polarized

$\frac{\partial E_x}{\partial z} \Rightarrow \vec{E} = E_x(z,t)\hat{x}$

b) (4 pts) Defining the voltage drop between the plates as V, what is $\frac{\partial V}{\partial z}$ in terms of the current I along the plates, the dielectric material properties (μ and/or ϵ), and the plate dimensions?

$[V/m]\ E_x = V/d.$ (via $\int_0^d \vec{E}\cdot d\vec{l} = V$)

$[A/m]\ H_y = I/W$ (via $\int_0^W \vec{J_s}\cdot dl = I$ and $J_{sz} = H_y$)

so $\frac{\partial E_x}{\partial z} = -\mu \frac{\partial H_y}{\partial t}$

$\downarrow \qquad \downarrow$

$\frac{\partial V}{\partial z}\cdot\left(\frac{1}{d}\right) = -\mu \frac{\partial I}{\partial t}\left(\frac{1}{W}\right)$

so $\boxed{\frac{\partial V}{\partial z} = -\mu\left(\frac{d}{W}\right)\frac{\partial I}{\partial t}}$

c) (3 pts) If the width of the plates is increased, which parameters associated with the guided TEM wave would change? (choose all that apply)

~~i.~~ the surface current density $\mathbf{J_s}$ on the top plate $= H_y$

ii. the capacitance of the system per unit length, C $= \varepsilon(W/d)$

~~iii.~~ the intrinsic impedance of the dielectric material, η $\longrightarrow = \sqrt{\mu/\varepsilon}$

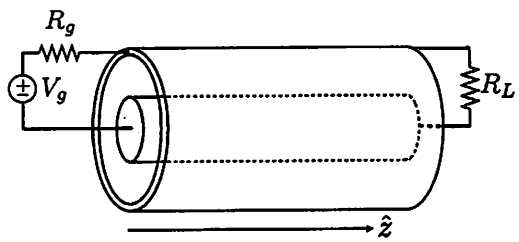
iv. the characteristic impedance of the system, Z_0 $= \sqrt{\mu/\varepsilon}\,(d/W)$

~~v.~~ the voltage drop from the bottom to the top plate at a given distance z $= E_x d$

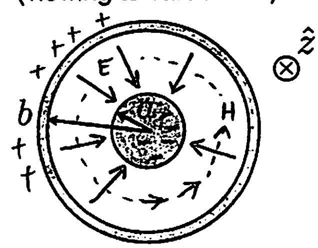
~~vi.~~ the propagation speed of the wave $= \frac{1}{\sqrt{\mu\varepsilon}}$

$V_p = c$

PART TWO: Consider a guided TEM wave propagating in a vacuum between two perfectly conducting cylindrical shells having length $L = 1.5$ cm along the $\hat{z}$ direction and radii a and b, respectively, where $L \gg b$ (see diagram below). The TEM wave is generated by voltage and current variations introduced at one end of the line from a voltage source $V_g = 10u(t)$ V having internal Thevenin resistance $R_g = Z_0 = 50\ \Omega$. A resistive load $R_L = Z_0$ is connected to the other end of the line.



co-ax cross section
(viewing towards load)



a) (3 pts) How long will it take the transmission line to reach steady state equilibrium?

matched load $\rightarrow$ no reflections $\rightarrow$ equilibrium once wave reaches load.

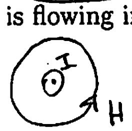
$$T = \frac{L}{V_p} = \frac{1.5 \cdot 10^{-2}}{3 \cdot 10^{8}} = \frac{1}{2} \cdot 10^{-10} = 5 \cdot 10^{-11}\ \text{s} = 500\ \text{ns}$$

b) (4 pts) Draw vectors in the region between the cylinders in the figure on the right above to indicate the directions of the electric and magnetic fields once steady state is reached. Note that $\hat{z}$, the direction towards the load, is pointing into the page.

$\vec{E}$ along $-\hat{r}$, $\vec{E} \times \vec{H} = \hat{z}$ so $\vec{H}$ along $-\hat{\phi}$

c) (2 pts) **TRUE or FALSE**: Once it reaches steady state equilibrium, the surface current on the inner cylinder is flowing in the $+\hat{z}$ direction.

for $\hat{z} \otimes$



current is along $-\hat{z}$ via Ampere's Law

d) (2 pts) **TRUE or FALSE**: Once it reaches steady state equilibrium, the current densities on the inner and outer conductor surfaces have different absolute magnitudes $|J_s|$.

$I_A = J_{sa} \cdot 2\pi a = I_b = J_{sb} \cdot 2\pi b$ so $J_{sa} \neq J_{sb}$.

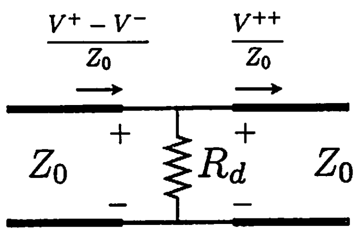
e) (5 pts) Once it reaches steady state equilibrium, what is the power P_L delivered to the load R_L?

$$P_L = V_L I_L = V_L \left(\frac{V_L}{R_L}\right) = \frac{5^2}{50} = \frac{1}{2}\ \text{W}$$

where $V_L = V_{ss} = V_g \tau_g$ (v^+ only)

$$= V_g \cdot \frac{Z_0}{R_g + Z_0} = 10 \cdot \frac{50}{100} = 5$$

4. A transmission line, which has a length 600 m and is characterized by $Z_0 = 400\ \Omega$ and $v_p = c$, has a defect halfway along the line (i.e., at 300 m) that can be modeled as a "shunt" resistance $R_d = 200\ \Omega$ as shown in the figure below.



Incident voltage waveforms, denoted V^+, will be reflected at the junction such that $\frac{V^-}{V^+} \equiv \Gamma_d = \frac{Z_{eq} - Z_0}{Z_{eq} + Z_0}$, where the reflection coefficient Γ_d is defined in terms of an equivalent impedance Z_{eq} associated with the parallel combination of R_d and the rest of the transmission line.

a) (4 pts) What is Z_{eq} and Γ_d in this case? (Note that you do not need to derive Z_{eq} explicitly.)

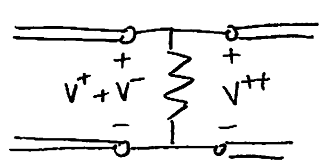
$$\frac{1}{Z_{eq}} = \frac{1}{R_d} + \frac{1}{Z_o} \quad \text{so} \quad Z_{eq} = \frac{R_d Z_o}{R_d + Z_o} = \frac{200 \cdot 400}{600} = \frac{400}{3}\ \Omega$$

$$\Gamma_d = \frac{Z_{eq} - Z_o}{Z_{eq} + Z_o} = \frac{400/3 - 400}{400/3 + 400} = \frac{1/3 - 1}{1/3 + 1} = \frac{-2/3}{4/3} = -\frac{1}{2}.$$

b) (4 pts) Use lumped circuit rules at the junction to show that the transmission coefficient through the defect, denoted τ_d, is equal to $\frac{V^{++}}{V^+} \equiv \tau_d = 1 + \Gamma_d$.

at junction, KVL ⇒ voltage is continuous

so

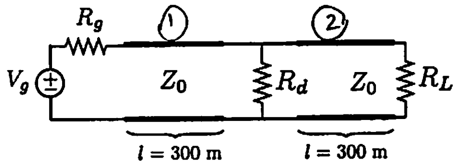


$$V^+ + V^- = V^{++} \qquad \frac{V^-}{V^+} = \Gamma_L$$

$$V^+ + \Gamma_L V^+ = \tau_g V^+ \qquad \frac{V^{++}}{V^+} = \tau_g$$

$$1 + \Gamma_L = \tau_g \quad //$$

c) Now consider (for parts c through e) that this transmission line, including its defect, is connected at time $t = 0$ to a DC step voltage source of magnitude $V_g = 100$ V and internal resistance $R_g = 400\ \Omega$ and a load resistor $R_L = 1.2\ \mathrm{k}\Omega$ (see diagram).



(6 pts) What is the injection coefficient τ_g, the load reflection coefficient Γ_L, and the source reflection coefficient Γ_g associated with this transmission line network?

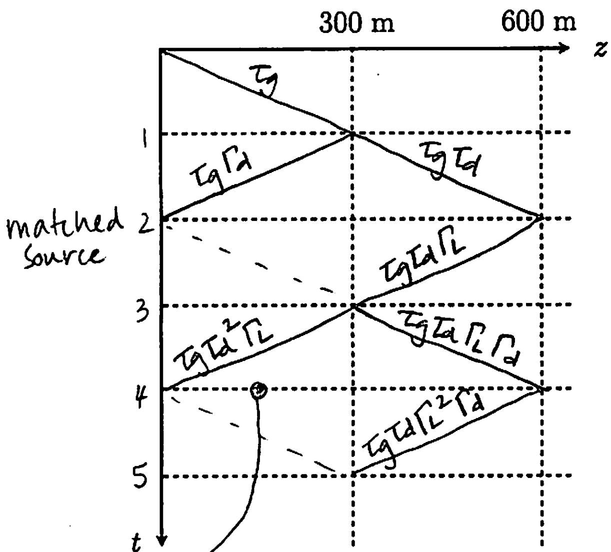
$$\tau_g = \frac{Z_0}{R_g + Z_0} = \frac{400}{800} \qquad \boxed{\tau_g = \frac{1}{2}}$$

$$\Gamma_L = \frac{R_L - Z_0}{R_L + Z_0} = \frac{1200 - 400}{1600} \qquad \boxed{\Gamma_L = \frac{1}{2}}$$

$$\Gamma_g = \frac{R_g - Z_0}{R_g + Z_0} \qquad \boxed{\Gamma_g = 0}$$

since $R_g = Z_0$ matched source

d) (8 pts) On the axes below, draw a bounce diagram for the ***voltage waveforms*** propagating on both segments of the line for the first 5 μs after the switch is closed. Indicate voltage magnitudes in terms of τ_g, Γ_g, Γ_L, Γ_d, and τ_d. Be sure to mark all relevant values on the time axis.



reflection from 2 onto 1

$$\Gamma_{21} = \frac{Z_{eq} - Z_0}{Z_{eq} + Z_0}$$

both segments have the same impedance, so Γ_d, τ_d are valid for either direction of incidence.

e) (3 pts) What voltage would be measured on the line at $\underbrace{z = 150\text{ m}}_{\text{middle of line 1}}$ at time $\underbrace{t = 4\ \mu\text{s}}_{\text{3 wavefronts have passed by in this time}}$? (choose one):

i. $V(150\text{ m}, 4\ \mu\text{s}) = 0$

ii. $V(150\text{ m}, 4\ \mu\text{s}) = V_g\tau_g\tau_d^2\Gamma_L$

iii. $V(150\text{ m}, 4\ \mu\text{s}) = V_g\tau_g(1 + \Gamma_d + \tau_d^2\Gamma_L)$

iv. $V(150\text{ m}, 4\ \mu\text{s}) = V_g\tau_g(1 + \Gamma_d + \Gamma_g\Gamma_d + \tau_d^2\Gamma_g\Gamma_d^2)$

Since $f(t) = V_g u(t)$, all magnitudes superpose.