ECE 329 **Introduction to Electromagnetic Fields** **Spring 13**

University of Illinois **Gilbert, Kim, Schutt-Aine, Waldrop**

Exam 2

Thursday, Mar. 14, 2013 — 7:00-8:15 PM

Name:				
Section:	9 AM	12 Noon	1 PM	2 PM

Please clearly PRINT your name in CAPITAL LETTERS and CIRCLE YOUR SECTION in the above boxes.

This is a closed book exam and calculators are not allowed. Please show all your work and make sure to include your reasoning for each answer. **All answers should include units wherever appropriate**.

Problem 1 (25 points)	
Problem 2 (25 points)	
Problem 3 (25 points)	
Problem 4 (25 points)	
TOTAL (100 points)	

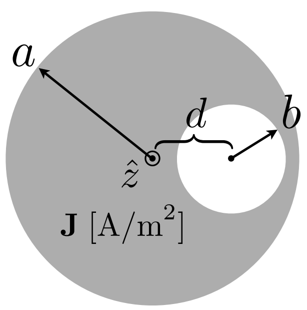
1. Consider a uniform volumetric current $\mathbf{J} = J_0\hat{z}$ A/m^2 flowing along an inifinitely long cylinder of radius a centered on the $\hat{z}$-axis.

   a) (5 pts) Use Ampere's Law in integral form to find the vector magnetic field $\mathbf{H}(r)$ *outside* the cylinder as a function of radius $r > a$.

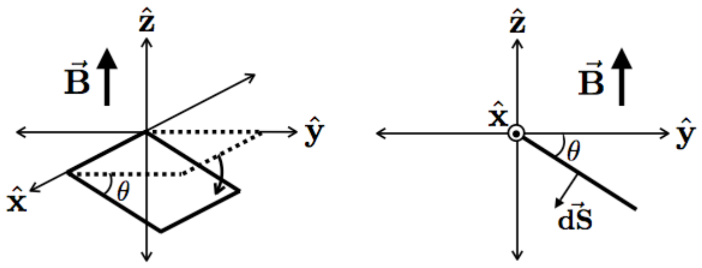
   b) (6 pts) Use Ampere's Law in integral form to find the vector magnetic field $\mathbf{H}(r)$ *inside* the cylinder as a function of radius $r < a$.

   c) (4 pts) What is the magnitude and direction of the magnetic field $\mathbf{H}$ along the center of the cylinder (i.e., at $r = 0$)?

   d) (10 pts) Now consider that the current distribution is not uniform across the cross-section of the cylinder, such that it instead contains an infinitely long cylindrical hole of radius b whose center is parallel to the $\hat{z}$-axis but offset by a distance d (see diagram). What is the magnitude of the magnetic field $|\mathbf{H}|$ along the center axis of the hole? **Hint**: Use superposition of the above results.



2. A square loop of wire of some finite resistance R and 4 cm^2 surface area is located within a region of constant magnetic field $\mathbf{B} = 4\hat{z}$ Wb/m^2 as illustrated in the following diagrams (perspective and side views are shown).



a) (5 pts) What is the magnetic flux Φ through the loop when the orientation angle of the loop is $\theta = 180°$? In your flux calculation make use of $\mathbf{dS}$ orientation shown in the diagram on the right.

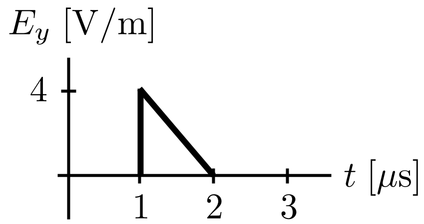
b) (5 pts) What is the flux Φ as a function of angle θ (using the same sign convention as in part a)?

c) (5 pts) What is the direction and magnitude of induced current flow when the loop is held stationary at $\theta = 0°$?

d) (5 pts) Now consider that the loop is no longer stationary and is instead rotating, such that angle θ is time varying at a rate of $\frac{d\theta}{dt} = 2\pi$ rad/s. What is the emf $\mathcal{E}$ around the rotating loop at the instant when $\theta = 135°$?

e) (5 pts) What is the direction and magnitude of induced current flow around the loop at the same instant? You may draw a picture to explain your answer. Be sure to justify your answer.

3. A plane TEM wave is generated by a surface current $\mathbf{J_s}(t)$ on the $x = 0$ plane and propagates away from the source in a vacuum ($v = c \approx 3 \times 10^8$ m/s and $\eta = \eta_0 \approx 120\pi$ Ω). The electric field is observed to vary with time at $x = 300$ m according to $E_y(300 \text{ m}, t) = 4u(t - t_0)\triangle(\frac{t - t_0}{\tau})$ V/m, as depicted in the figure below:

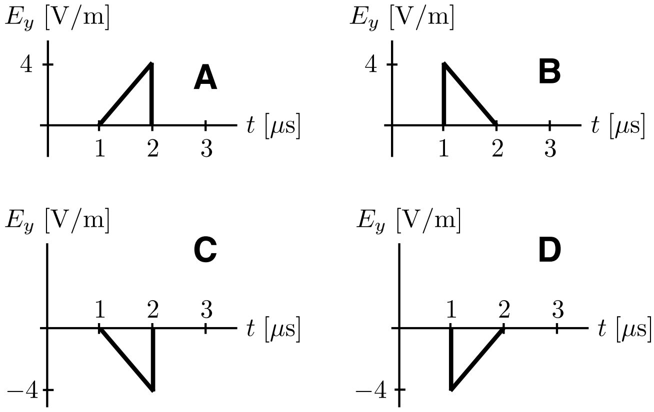


a) (2 pts) What are the numerical values of the time constants t_0 and τ used in the expression above to describe the electric field variation at $x = 300$ m?

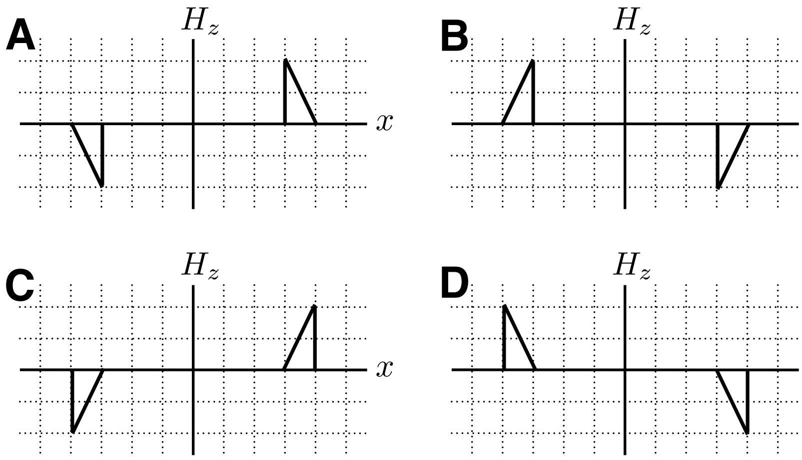
b) (4 pts) Write the expression for the surface current density $\mathbf{J_s}(t)$ located at $x = 0$ that generates the TEM wave.

c) (6 pts) Write the expression for associated vector wavefield $\mathbf{H}(x, t)$ explicitly in terms of all space and time variables x and t.

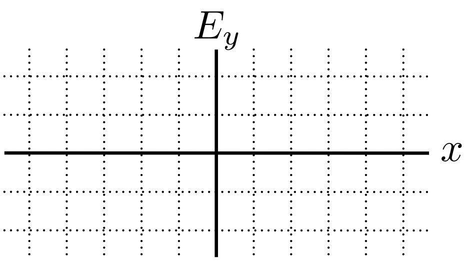
d) (3 pts) Which of the following figures depicts $E_y(x,t)$ vs time t at position $x = -300$ m?



e) (5 pts) Identify which of the following figures depicts $H_z(x,t)$ vs position y at time $t = 3$ μs and add appropriate numerical values along each axis in the figure you choose.



f) (5 pts) Plot $E_y(x,t)$ vs position x at time $t = 0.5$ μs. Be sure to label the values on the E_y and x axes.



4. Consider a monochromatic TEM plane wave of frequency $f = 1$ GHz that is traveling along the $-\hat{z}$ direction at a propagation speed $v_p = c/3$ through a homogeneous, non-conducting medium characterized by $\epsilon_r = 3$. At $t = 0$ and $z = 0$, the electric wavefield has equal amplitude *positive* E_x and E_y components, magnitude $|\mathbf{E}| = 2$ V/m, and zero phase angle.

a) (3 pts) What are the relative permeability μ_r and intrinsic impedance η of the medium?

b) (3 pts) Write the expression for the vector wavefield $\mathbf{E}(z, t)$ propagating in the $-\hat{z}$ direction in terms of angular frequency ω and wavenumber β.

c) (2 pts) What are the numerical values for ω and β?

d) (4 pts) Write the expression for the associated *time-domain* wavefield $\mathbf{H}$ and *phasor* wavefield $\tilde{\mathbf{H}}$ propagating in the $-\hat{z}$ direction.

e) (3 pts) Choose which set of simplified curl equations describes the TEM wave propagation for this geometry.

i. $-\frac{\partial E_x}{\partial z}\hat{x} + \frac{\partial E_y}{\partial z}\hat{y} = -\mu\frac{\partial H_y}{\partial t}\hat{x} - \mu\frac{\partial H_x}{\partial t}\hat{y}$ and $-\frac{\partial H_x}{\partial z}\hat{x} + \frac{\partial H_y}{\partial z}\hat{y} = \epsilon\frac{\partial E_y}{\partial t}\hat{x} + \epsilon\frac{\partial E_x}{\partial t}\hat{y}$

ii. $-\frac{\partial E_z}{\partial x}\hat{x} + \frac{\partial E_z}{\partial y}\hat{y} = -\mu\frac{\partial H_z}{\partial t}\hat{x} - \mu\frac{\partial H_z}{\partial t}\hat{y}$ and $-\frac{\partial H_z}{\partial x}\hat{x} + \frac{\partial H_z}{\partial y}\hat{y} = \epsilon\frac{\partial E_z}{\partial t}\hat{x} + \epsilon\frac{\partial E_z}{\partial t}\hat{y}$

iii. $-\frac{\partial E_y}{\partial z}\hat{x} + \frac{\partial E_x}{\partial z}\hat{y} = -\mu\frac{\partial H_x}{\partial t}\hat{x} - \mu\frac{\partial H_y}{\partial t}\hat{y}$ and $-\frac{\partial H_y}{\partial z}\hat{x} + \frac{\partial H_x}{\partial z}\hat{y} = \epsilon\frac{\partial E_x}{\partial t}\hat{x} + \epsilon\frac{\partial E_y}{\partial t}\hat{y}$

iv. $-\frac{\partial E_y}{\partial z}\hat{x} + \frac{\partial E_x}{\partial z}\hat{y} = -\mu\frac{\partial H_y}{\partial t}\hat{x} - \mu\frac{\partial H_x}{\partial t}\hat{y}$ and $-\frac{\partial H_y}{\partial z}\hat{x} + \frac{\partial H_x}{\partial z}\hat{y} = \epsilon\frac{\partial E_y}{\partial t}\hat{x} + \epsilon\frac{\partial E_x}{\partial t}\hat{y}$

f) (2 pts) **TRUE** or **FALSE**: The time-averaged power transmitted by the wave described above is constant and uniform everywhere $z < 0$.

g) (2 pts) **TRUE** or **FALSE**: The wave described above is characterized by a linear dispersion relation.

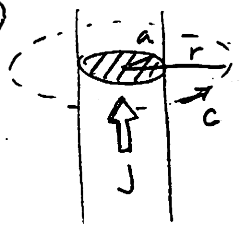
h) (2 pts) **TRUE** or **FALSE**: For general EM waves, the Poynting vector can be oriented in a different direction as the propagation velocity.

i) (2 pts) **TRUE** or **FALSE**: In general, Maxwell's equations dictate that all transverse electromagnetic waves must be planar.

j) (2 pts) **TRUE** or **FALSE**: In general, some solutions to Maxwell's equations do not satisfy the wave equation: $\nabla^2\mathbf{E} = \mu\epsilon\frac{\partial^2\mathbf{E}}{\partial t^2}$

1. Cylindrical symmetry: expect $\vec{H} = H_\phi \hat{\phi}$ and $H_\phi(r) = \text{const.}$

ⓐ

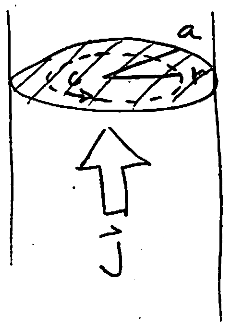


$$\oint_c \vec{H} \cdot d\vec{l} = H_\phi \int_c d\phi = H_\phi (2\pi r) = I_{encl}$$

$$I_{encl} = \int_s \vec{J} \cdot d\vec{s} = J_o \int ds = J_o (\pi a^2)$$

so $\vec{H}(r) = \hat{\phi} \dfrac{J_o \pi a^2}{2\pi r}$ [A/m].

ⓑ



$\oint \vec{H} \cdot d\vec{l} = H_\phi (2\pi r) = I_{encl}$ as before

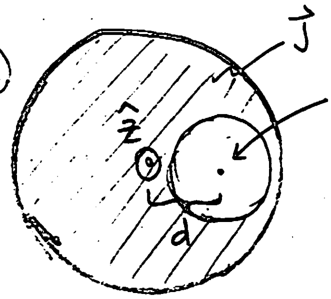
$I_{encl} = J_o \int ds = J_o (\pi r^2) < J_o (\pi a^2)$!

so $\vec{H}(\vec{r}) = \hat{\phi} \dfrac{J_o \pi r^2}{2\pi r} = \hat{\phi} J_o \dfrac{r}{2}$ [A/m]

ⓒ @ $r = 0$, $\vec{H} = 0$.

since current is symmetrically distributed around $\hat{z}$ axis.

ⓓ



"hole" = $J_o\hat{z} - J_o\hat{z}$

field here is superposition of two $\vec{H}$ fields:

① generated by $J_o\hat{z}$ cylinder of radius a

$\vec{H}(r=d) = \hat{\phi} J_o \dfrac{d}{2}$ [A/m]

② generated by $-J_o\hat{z}$ cylinder of radius b

$\vec{H}(r=0) = 0$. (since along center axis)

so $\vec{H}$ along axis of hole is the same as if the hole were not there: $\vec{H} = \hat{\phi} J_o \dfrac{d}{2}$ [A/m]

EOPS.

2. (a) For $\theta = 180°$, $\vec{ds}$ is along $+\hat{z}$.

$$\Phi = \int_S \vec{B}\cdot\vec{ds} = \int B_z \hat{z}\cdot\hat{z}\,ds = B_z \underbrace{\int ds}_{=4\,cm^2 = 4(10^{-2})^2 m^2} = 4(4\cdot 10^{-4}) = 1.6\times 10^{-3}\ [Wb]$$

(b) $\vec{ds}$ points along $-\hat{z}$ for $\theta = 0$. → cosine
along $-\hat{y}$ for $\theta = \pi/2$ → sine.

so $\vec{ds} = (-\hat{z}\cos\theta - \hat{y}\sin\theta)\,ds$

$$\Phi = \int_S B_z\hat{z}\cdot\vec{ds} = \int_S B_z\cos\theta\underbrace{(-1)}_{\text{from } \hat{z}\cdot-\hat{z}}\,ds = -1.6\times 10^{-3}\cos\theta\ [Wb]$$

(c) @ $\theta = 0°$, $\Phi = -1.6\times 10^{-3}$ [Wb] but if stationary, $\frac{d\Phi}{dt} = 0$.
so $\mathcal{E} = 0$, no induced current flow.

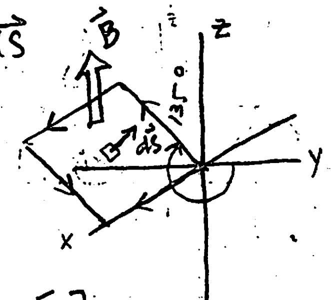
(d) $$\mathcal{E} = -\frac{d\Phi}{dt} = -\frac{d}{dt}\left(-1.6\times 10^{-3}\cos\theta\right) = 1.6\times 10^{-3}\frac{d}{dt}(\cos\theta)$$
$$= -1.6\cdot 10^{-3}\sin\theta\,\frac{d\theta}{dt} \quad \left(\frac{d\theta}{dt} = 2\pi\right)$$

at $\theta = 135°$, $\mathcal{E} = -3.2\pi\cdot 10^{-3}\underbrace{\sin(135°)}_{-\sqrt{2}/2}$ [V]

$$\mathcal{E} = +1.6\pi\sqrt{2}\cdot 10^{-3}\ [V]$$

(e) $\mathcal{E} > 0$ so expect current to flow in the same direction as the contour associated with $\vec{ds}$

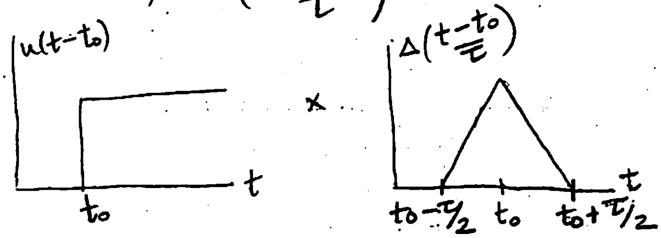
Lenz Law: need to generate $\vec{B}$ with $+\hat{z}$ component to oppose the decrease in flux at this time



magnitude of current $I = \frac{\mathcal{E}}{R}$ [A].

3. $E_y(300\,m, t) = 4u(t-t_0)\,\Delta\left(\frac{t-t_0}{\tau}\right)$

looks like:



ⓐ $t_0 = 1\,\mu s$ → makes sense: wave is observed after traveling 300 m at $v = c = 300\,m/\mu s$.

$\tau = 2\,\mu s$ since ½ width of Δ function is 1 µs.

ⓑ $t_0 = 300m/c = x_0/c$ @ $x=0$, there will be no delay.

$\vec{E}(x,t) = -\frac{\eta_0}{2}\vec{J}_s(t \mp x/c)$ so $\vec{J}_s(t) = -\frac{2}{\eta_0}\vec{E}(x=0,t)$.

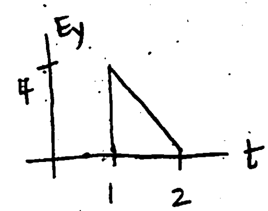
so $\vec{J}_s(t) = -\hat{y}\frac{8}{\eta_0}u(t)\,\Delta\left(\frac{t}{\tau}\right)$ [A/m]

ⓒ $\frac{|E|}{|H|} = \eta_0$ and $\hat{e}\times\hat{h} = \hat{v} = \pm\hat{x}$ in this case

so $\hat{h} = \pm\hat{z}$ since $\hat{e} = +\hat{y}$.

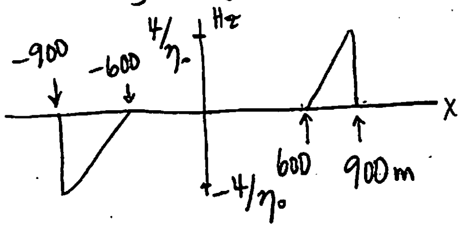
$\vec{H}(x,t) = \pm\hat{z}\frac{4}{\eta_0}u(t \mp x/c)\,\Delta\left(\frac{t \mp x/c}{\tau}\right)$ [A/m] for $x \gtrless 0$

ⓓ $E_y(-300, t) = E_y(+300, t)$ so:



ⓔ $H_z(x>0, t) > 0$ (eliminates 2 choices)

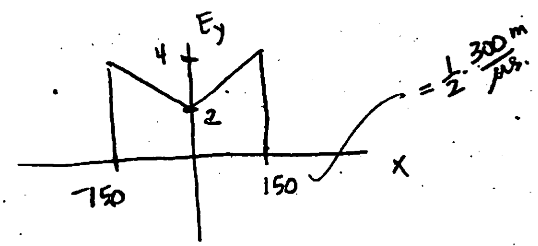
and leading edge — travels farthest — happened first at the source.



in 3µs, wavefront traveled

$3\mu s \times \frac{300m}{\mu s} = 900\,m$.

ⓕ



@ $t = \frac{1}{2}\mu s$, only ½ the waveform has even occurred at the source!

4. $f = 1\,\text{GHz} = 1\cdot 10^9\ \text{Hz}$ $\qquad \mu = ?$

$\hat{v}_p = -\hat{z}, \quad v_p = c/3$ $\qquad \varepsilon = 3\varepsilon_0$

$\qquad \sigma = 0$

$\vec{E}(0,0) = E_x\hat{x} + E_y\hat{y}$

and $E_x = E_y$, both >0, $|E| = \sqrt{2E_x^2} = 2$

(a) $v_p = \dfrac{1}{\sqrt{\mu\varepsilon}} = \dfrac{1}{\sqrt{\mu\cdot 3\varepsilon_0}} = \dfrac{c}{3} = \dfrac{1}{3\sqrt{\mu_0\varepsilon_0}} \Rightarrow \mu = 3\mu_0$

(b) $\vec{E}(z,t) = E_x\cos(\omega t + \beta z)\hat{x} + E_y\cos(\omega t + \beta z)\hat{y}$

where $E_x = \sqrt{2} = E_y$.

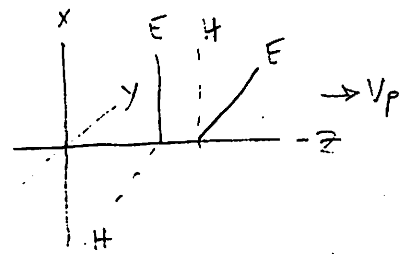
$\vec{E}(z,t) = \sqrt{2}\cos(\omega t + \beta z)(\hat{x} + \hat{y})\ [V/m]$

→ cos not sin since it evaluates to $\sqrt{2}$ @ $(0,0)$

(c) $\omega = 2\pi f = 2\pi\cdot 10^9\ [rad/m]$

$\beta = \dfrac{\omega}{v_p} = \dfrac{2\pi\cdot 10^9}{(3\cdot 10^8)/3} = 20\pi\ [rad/m]$

(d) $\vec{H}(z,t) = \dfrac{\sqrt{2}}{\eta}\cos(\omega t + \beta z)(\hat{x} - \hat{y})\ [A/m]$



$\tilde{H}(z) = \dfrac{\sqrt{2}}{\eta} e^{j\beta z}(\hat{x} - \hat{y})\ [A/m]$

where $\eta = \sqrt{\dfrac{\mu}{\varepsilon}} = \sqrt{\dfrac{3\mu_0}{3\varepsilon_0}} = \sqrt{\dfrac{\mu_0}{\varepsilon_0}} = \eta_0 = 120\pi\ \Omega$

(e) $\vec{E} = E_x(z,t)\hat{x} + E_y(z,t)\hat{y}$ so $\vec{H} = H_x(z,t)\hat{x} + H_y(z,t)\hat{y}$

partial derivatives $\partial/\partial x$ and $\partial/\partial y \to 0$ (divergence free fields)

and $\dfrac{\partial E_x}{\partial z}\hat{x}$ or $\dfrac{\partial H_y}{\partial t}\hat{x} \to 0$ (not correct form of curl or time deriv)

so: $-\dfrac{\partial E_y}{\partial z}\hat{x} + \dfrac{\partial E_x}{\partial z}\hat{y} = -\mu\dfrac{\partial H_x}{\partial t}\hat{x} - \mu\dfrac{\partial H_y}{\partial t}\hat{y}$ ✓

and $-\dfrac{\partial H_y}{\partial z}\hat{x} + \dfrac{\partial H_x}{\partial z}\hat{y} = \varepsilon\dfrac{\partial E_x}{\partial t}\hat{x} + \varepsilon\dfrac{\partial E_y}{\partial t}\hat{y}$ ✓

(f) time averaged power transmitted

$$\langle \vec{S} \rangle = \frac{1}{2} \mathrm{Re}\{ \tilde{E} \times \tilde{H}^* \} = \frac{1}{2}\left(\frac{2|E_x|^2}{\eta} \right) \neq f(x,y,z,t)$$

yes : uniform and constant

(g) linear dispersion relation $\frac{\beta}{\omega}$ = constant
true for this wave $= \sqrt{\mu\varepsilon}$

(h) Poynting vector $\vec{S}$ always $\parallel$ to $\hat{v}_p$

(i) not all TEM waves are planar. there can be spherical wavefronts etc.
($\vec{E} \times \vec{H} = \hat{v}_p$)

(j) wave equ $\nabla^2 \vec{E} = \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}$ assumes: $\sigma = 0$, material is homogeneous & source free.

waves prop. in conductor (for example) do NOT satisfy this wave equ
but do satisfy Maxwell's equ.

TOPS 35500