University of Illinois **Chen, Shao, Waldrop**

Exam 3

Monday, April 22, 2024 — 7:00-8:15 PM

Please clearly PRINT your name in CAPITAL LETTERS and circle your section in the boxes below.

Name:	
Section:	12 PM 1 PM 2 PM

This is a closed-book and closed-notes exam and calculators are not allowed. You may bring a single 4"x6" index card of handwritten notes. Please show all your work and make sure to include your reasoning for each answer. All answers should include units wherever appropriate. You may use the back of the exam as scratch paper, but no credit will be given for work done on scratch pages.

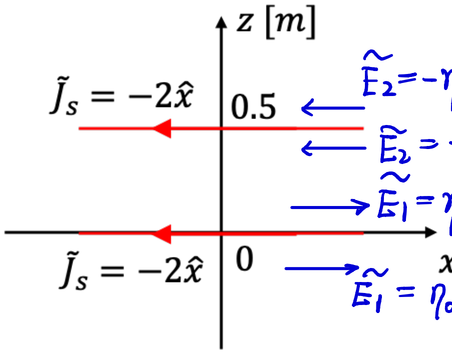
Problem 1 (18 points)	
Problem 2 (30 points)	
Problem 3 (18 points)	
Problem 4 (34 points)	
TOTAL (100 points)	

	Condition	β	α	$\lvert\eta\rvert$	τ	$\lambda = \frac{2\pi}{\beta}$	$\delta = \frac{1}{\alpha}$
Perfect dielectric	$\sigma = 0$	$\omega\sqrt{\epsilon\mu}$	0	$\sqrt{\frac{\mu}{\epsilon}}$	0	$\frac{2\pi}{\omega\sqrt{\epsilon\mu}}$	∞
Imperfect dielectric	$\frac{\sigma}{\omega\epsilon} \ll 1$	$\sim \omega\sqrt{\epsilon\mu}$	$\beta\frac{1}{2}\frac{\sigma}{\omega\epsilon} = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}$	$\sim \sqrt{\frac{\mu}{\epsilon}}$	$\sim \frac{\sigma}{2\omega\epsilon}$	$\sim \frac{2\pi}{\omega\sqrt{\epsilon\mu}}$	$\frac{2}{\sigma}\sqrt{\frac{\epsilon}{\mu}}$
Good conductor	$\frac{\sigma}{\omega\epsilon} \gg 1$	$\sim \sqrt{\pi f\mu\sigma}$	$\sim \sqrt{\pi f\mu\sigma}$	$\sqrt{\frac{\omega\mu}{\sigma}}$	$45°$	$\sim \frac{2\pi}{\sqrt{\pi f\mu\sigma}}$	$\sim \frac{1}{\sqrt{\pi f\mu\sigma}}$
Perfect conductor	$\sigma = \infty$	∞	∞	0	-	0	0

1. (18 points) Two infinite sheet of current lies on $z = 0$ and $z = 0.5$ m respectively. The sheets have identical surface current density as $\widetilde{\mathbf{J}}_s = -2\hat{x}$ $[A/m]$. The entire space is free space having ε_o, μ_o and η_o. Assume $f = 300\ MHz$. Simplify your answer using the relationships $e^{j\pi} = -1$, $\frac{e^{j\theta}+e^{-j\theta}}{2} = cos(\theta)$, and $\frac{e^{j\theta}-e^{-j\theta}}{2j} = sin(\theta)$. Note that 2 points will be given for simplifying your answers.

In free space $\lambda = \frac{c}{f} = \frac{3\times10^8}{300\times10^6} = 1\ m.$

$\beta = \frac{2\pi}{\lambda} = 2\pi.$



$\widetilde{E}_2 = -\eta_o e^{-j\beta z}\hat{x}$

$\widetilde{E}_2 = -\eta_o e^{j\beta z}\hat{x}$

$\widetilde{E}_1 = \eta_o e^{-j\beta z}\hat{x}$

$\widetilde{E}_1 = \eta_o e^{j\beta z}\hat{x}$

a) (9 points) Write the general expressions for electric field phasors in the entire space. Simplify your answer.

Current sheet at $z=0$ $\quad \widetilde{E}_1 = \eta_o \frac{J_{so}}{2} e^{\mp j\beta z}\hat{x} = \eta_o e^{\mp j\beta z}\hat{x} \quad z \gtrless 0$

Current sheet at $z=0.5$

$z \gtrless 0.5 \quad \widetilde{E}_2 = \eta_o e^{\mp j\beta(z-0.5)}\hat{x} = \eta_o e^{\mp j\beta z}\cdot e^{\pm j\beta\cdot 0.5}\hat{x} = \eta_o e^{\mp j\beta z}e^{\pm j\pi}\hat{x} = -\eta_o e^{\mp j\beta z}\hat{x}$

Your Answer (include direction and appropriate units):

i) $\widetilde{\mathbf{E}}(z < 0) =$ $0 \quad V/m$

ii) $\widetilde{\mathbf{E}}(0 < z < 0.5) =$ $-240j\sin(2\pi z)\hat{x} \quad V/m$

iii) $\widetilde{\mathbf{E}}(z > 0.5) =$ $0 \quad V/m$

for $z<0$. $\widetilde{E}_{tot} = \eta_o e^{+j\beta z} - \eta_o e^{+j\beta z} = 0$

for $0<z<0.5$ $\widetilde{E}_{tot} = \eta_o e^{-j\beta z}\hat{x} - \eta_o e^{j\beta z}\hat{x}$

$= -2\eta_o j\sin(\beta z)\hat{x} \quad V/m$

for $z>0.5$ $\widetilde{E}_{tot} = \eta_o e^{-j\beta z}\hat{x} - \eta_o e^{-j\beta z}\hat{x}$

$= 0$

b) (9 points) Write the general expressions for magnetic field phasors in the entire space. Simplify your answer.

$\widetilde{H}_1 = \pm\frac{J_{so}}{2} e^{\mp j\beta z}\hat{y} = e^{\mp j\beta z}\hat{y} \quad z \gtrless 0$

$\widetilde{H}_2 = \pm\frac{J_{so}}{2} e^{\mp j\beta(z-0.5)}\hat{y} = \mp e^{\mp j\beta z}\hat{y} \quad z \gtrless 0.5$

Your Answer (include direction and appropriate units):

i) $\widetilde{\mathbf{H}}(z < 0) =$ $0 \quad A/m$

ii) $\widetilde{\mathbf{H}}(0 < z < 0.5) =$ $2\cos(2\pi z)\hat{y} \quad A/m.$

iii) $\widetilde{\mathbf{H}}(z > 0.5) =$ $0 \quad A/m$

for $z<0$ $\quad \widetilde{H}_{tot} = 0$

for $0<z<0.5$ $\quad \widetilde{H}_{tot} = e^{-j\beta z}\hat{y} + e^{j\beta z}\hat{y}$

$= 2\cos(\beta z)\hat{y} \quad A/m$

for $z>0.5$ $\quad \widetilde{H}_{tot} = 0$

$\vec{E} = E_z \hat{z}$

2. (30 points) A cosinusoidal TEM wave, linearly polarized in the $\hat{z}$ direction, is generated by a current sheet on the $x = 0$ plane and propagates in the $+\hat{x}$ direction through a good conducting material (having a large conductivity σ relative to $\omega\epsilon$). The penetration depth of the wave is $\delta = 1$ mm, and the electric field at time $t = 0$ and distance $x = 0$ is given by $\mathbf{E}(0, 0) = 10\hat{z}$ V/m.

$\alpha = \beta$

$= \frac{1}{\alpha}$

a) (5 points) What is the propagation constant γ of this wave?

$\gamma = \alpha + j\beta = \alpha(1+j) = \frac{1}{\delta}(1+j)$

Your Answer (include units when appropriate):

$\gamma = 10^3(1+j)$ $[m^{-1}]$

$\frac{1}{1\,mm} = \frac{1}{10^{-3}\,m} = 10^3\,[m^{-1}]$

b) (5 points) Write an expression for the amplitude of the oscillating electric field at $\underbrace{x = 2\text{ mm}}_{=2\delta}$. You do NOT need to evaluate it numerically.

Your Answer (include units when appropriate):

$E_z(0,0)\,e^{-\alpha(2\delta)} = 10\,e^{-\alpha(2\cdot\frac{1}{\alpha})} = 10\,e^{-2}$ $[V/m]$

c) (5 points) Which of the following statements is true regarding the power density $\mathbf{S} = \mathbf{E} \times \mathbf{H}$ (and its time averaged value $\langle \mathbf{S} \rangle$) that is transported by this wave? Circle **all** correct answers.

~~i.~~ $-\nabla \cdot \mathbf{S}$ equals the time rate of change of the energy density stored in the wavefield — neglects dissipation

ii. $\nabla \cdot \langle \mathbf{S} \rangle$ equals the time-averaged rate of energy dissipation in the medium — equal but opposite

iii. $\langle \mathbf{S} \rangle$ decreases as the wave propagates along $+\hat{x}$ — $\langle \vec{S} \rangle \propto e^{-2\alpha x}$

~~iv.~~ $\langle \mathbf{S} \rangle$ would increase if the wave were to propagate along $-\hat{x}$ — $\langle \vec{S} \rangle \propto e^{+2\alpha x}$ with $x<0$

~~v.~~ $|\langle \mathbf{S} \rangle| = \dfrac{|\mathbf{E}|^2}{2\eta_0}$ W/m^2 at $x = 0$ — $\eta \neq \eta_0$ otherwise correct

Poynting Theorem: $\nabla \cdot \vec{S} + \frac{\partial}{\partial t}\left(\frac{1}{2}\vec{D}\cdot\vec{E} + \frac{1}{2}\vec{B}\cdot\vec{H}\right) + \vec{J}\cdot\vec{E} = 0$

time-avg: $\nabla \cdot \langle \vec{S} \rangle + 0 + \langle \vec{J}\cdot\vec{E} \rangle = 0$

$\langle \vec{S} \rangle = \frac{1}{2}\mathrm{Re}\{\tilde{E} \times \tilde{H}^*\} = \frac{1}{2}\frac{|E|^2 e^{-2\alpha x}}{\eta}$

$\eta = \sqrt{\frac{\omega\mu}{\sigma}}\,e^{j\pi/4} \neq \eta_0$ for good conductor

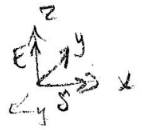
d) (5 points) If the above TEM wave were superposed with a sinusoidal $\hat{y}$ component, such that the resulting wave is left-handed circularly polarized, which of the following expressions corresponds to the phasor representation of the associated ***magnetic field*** (where $H_0 = |\mathbf{H}(0,0)|$)? Circle **one** correct answer.

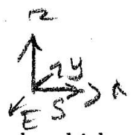
i. $\tilde{\mathbf{H}} = H_0\, e^{-\alpha x} e^{-j\beta x}\, (\hat{z} + j\hat{y})$ A/m
ii. $\tilde{\mathbf{H}} = H_0\, e^{\alpha x} e^{j\beta x}\, (\hat{z} - j\hat{y})$ A/m
iii. $\tilde{\mathbf{H}} = H_0\, e^{-\alpha x} e^{-j\beta x}\, (-\hat{y} - j\hat{z})$ A/m
iv. $\tilde{\mathbf{H}} = H_0\, e^{\alpha x} e^{j\beta x}\, (\hat{y} + j\hat{z})$ A/m
v. $\tilde{\mathbf{H}} = H_0\, e^{-\alpha x} e^{-j\beta x}\, (\hat{y} - j\hat{z})$ A/m
vi. $\tilde{\mathbf{H}} = H_0\, e^{\alpha x} e^{-j\beta x}\, (-\hat{y} - j\hat{z})$ A/m

prop: $+\hat{x}$
E: $+\hat{z}$

$$\tilde{E} = E_0 e^{-\gamma x} (\hat{z} - j\hat{y})$$
$$E_0 e^{-\alpha x} e^{-j\beta x} (\hat{z} - j\hat{y})$$
$$\tilde{H} = H_0 e^{-\alpha x} e^{-j\beta x} (-\hat{y} - j\hat{z}) \ \frac{A}{m}$$

Ahead
$\hat{z} \times \hat{y} = -\hat{x}$
Behind
(Not $+\hat{x}$, so left hand)





e) (10 points) If the frequency of the propagating wave were increased, which of the following parameters would **stay the same**? Circle **all** correct answers.

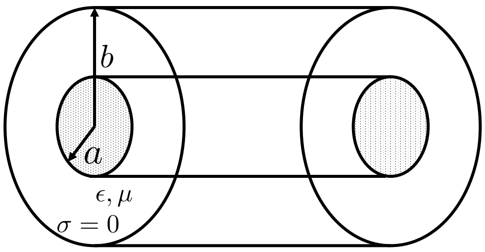
i. The amplitude of $\mathbf{H}(0,0)$
ii. The amplitude of $\mathbf{E}(0,0)$
iii. The penetration depth
iv. The speed of propagation
v. The amount of average power transported across the $x = \frac{1}{10}$ m plane
vi. The amount of average power absorbed between the $x = \frac{1}{10}$ m plane and the $x = \frac{1}{5}$ m plane
vii. The phase shift between $\mathbf{E}$ and $\mathbf{H}$
viii. The polarization of the TEM wave
ix. The intrinsic impedance of the material X
x. The loss tangent X

i. Amplitude of H(0,0) → $|H(0,0)| = \frac{J_{so}}{2} e^{-\gamma x}|_{x=0} = \frac{J_{so}}{2}$ → same amplitude
ii. Amplitude of E(0,0). → $|E(0,0)| = \eta \frac{J_{so}}{2}$. Because η is related to frequency, so different
iii. Penetration depth. → $\delta = \frac{1}{\alpha}$, and α is related to frequency → different
iv. speed of propagation. → $v_p = \frac{\omega}{\beta}$, again related to frequency → different
v. average power transported across the plane. → related to $e^{-2\alpha x}$, α is related to frequency, so different
vi. average power. → same reason as above → different
vii. phase shift between E and H. τ is related to frequency → different
viii. polarization. → depends on E direction, which is the opposite direction as current sheet, so same
ix. Eta → η is related to frequency → different
x. loss tangent $\frac{\sigma}{\omega\epsilon}$ → is related to frequency → different

3. (18 points) Consider a coaxial cable transmission line as illustrated. Recall from Lecture 10 of course notes that the per-unit-length capacitance of this transmission line is $\mathcal{C} = \frac{2\pi}{\ln \frac{b}{a}}\epsilon$ [F/m]. Given this information, find the per-unit-length industance $\mathcal{L}$, the characteristic impedance Z_0, and the propagation velocity of voltage and current waves v_p of the line. You should express your answers in terms of the variables a, b, ϵ, μ.

Per-unit length capacitance,

$$C = \frac{2\pi}{\ln b/a} \epsilon \; [F/m]$$



$$L = \frac{\ln b/a}{2\pi} \mu$$ ⟶ Inductance per unit length.

$$Z_0 = \sqrt{\frac{L}{C}} = \frac{1}{GF} \sqrt{\frac{\mu}{\epsilon}}$$ ⟶ Characteristic impedance.

Propagation velocity, $$v = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu\epsilon}}$$

$$\frac{\ln b/a}{2\pi} \cdot \sqrt{\frac{\mu}{\epsilon}}$$

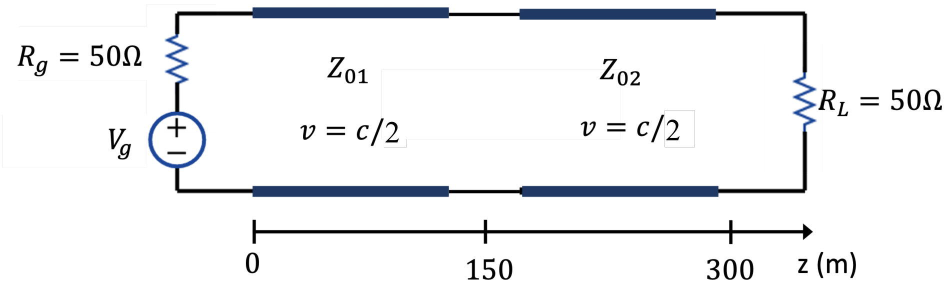
Your Answer (include units when appropriate):

$\mathcal{L} =$ $\frac{\ln b/a}{2\pi} \mu$ [H/m]

$Z_0 =$ $\frac{\ln b/a}{2\pi} \sqrt{\frac{\mu}{\epsilon}}$ [Ω]

$v_p =$ $\frac{1}{\sqrt{\mu\epsilon}}$ [m/s]

4. (34 points) Two transmission lines with the same length $l/2$ are connected to a Thevenin equivalent source at $z = 0$ with $R_g = 50\Omega$ and $v_g(t)$. The load resister is $Z_L = 50\Omega$ located at $z = l = 300\,\text{m}$. The characteristic impedance of the transmission line are Z_{01} and Z_{02}, respectively. The velocity of wave on both lines is $v_p = \frac{c}{2}$, where c is the speed of light in a vacuum.



a) (12 points) Given $Z_{01} = 150\ \Omega$ and $Z_{02} = 50\ \Omega$, generator voltage is $V_g(t) = 80u(t)\ V$, where $u(t)$ is unit step function. Complete the voltage bounce diagram below for $t \in [0, 4]\mu s$. Be sure to indicate the injection coefficient τ_g, generator and load reflection coefficieints Γ_g and Γ_L, and label all values clearly.

$$\tau_g = \frac{Z_{01}}{R_g + Z_{01}} = \frac{150}{50+150} = \frac{3}{4}$$

$$V^+ = \tau_g V_g = \frac{3}{4} \cdot 80 = 60\,V$$

Forward wave

$$\Gamma_{12} = \frac{Z_{02} - Z_{01}}{Z_{02} + Z_{01}} = \frac{50-150}{50+150} = -\frac{1}{2}$$

$$\tau_{12} = 1 + \Gamma_{12} = \frac{1}{2}$$

$$\Gamma_L = \frac{R_L - Z_{02}}{R_L + Z_{02}} = \frac{50-50}{50+50} = 0$$

Backward wave

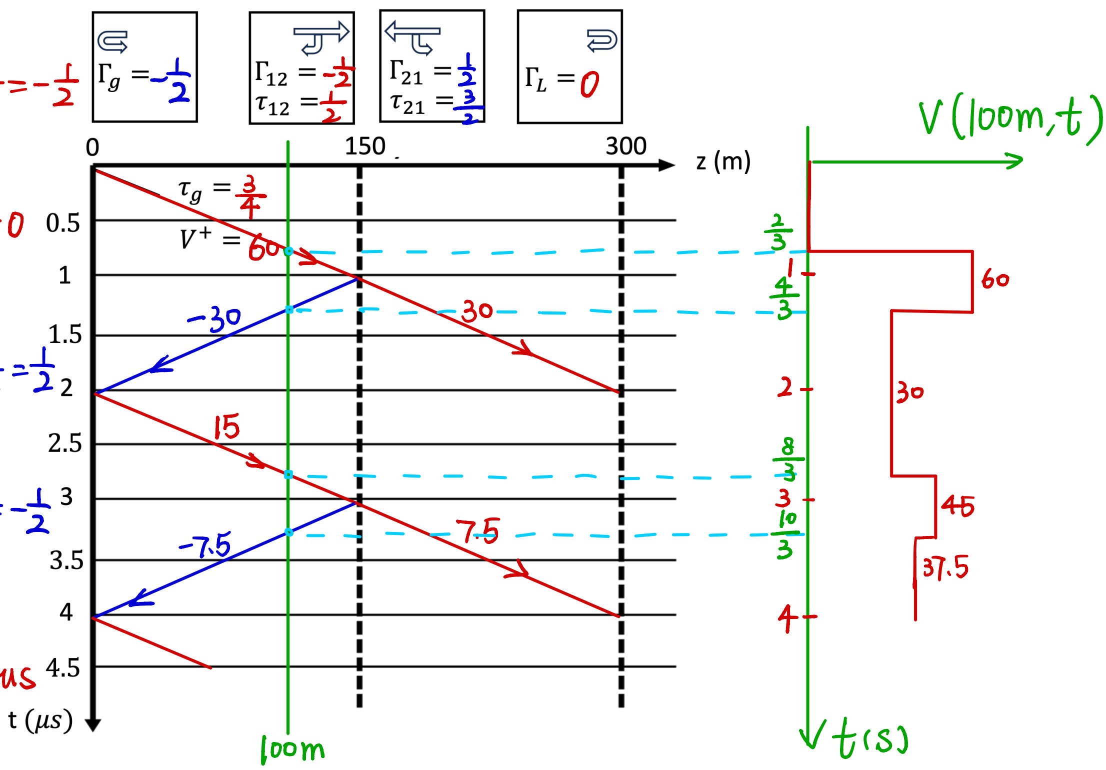
$$\Gamma_{21} = \frac{Z_{01} - Z_{02}}{Z_{01} + Z_{02}} = \frac{150-50}{150+50} = \frac{1}{2}$$

$$\tau_{21} = 1 + \Gamma_{21} = \frac{3}{2}$$

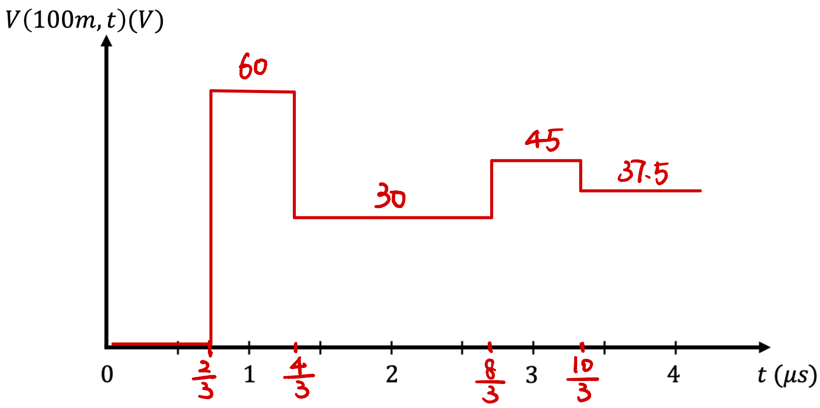
$$\Gamma_g = \frac{R_g - Z_{01}}{R_g + Z_{01}} = \frac{50-150}{50+150} = -\frac{1}{2}$$

Travel time for $l/2$

$$T = \frac{l/2}{v_p} = \frac{150}{\frac{1}{2} \times 3 \times 10^8} = 1\,\mu s$$

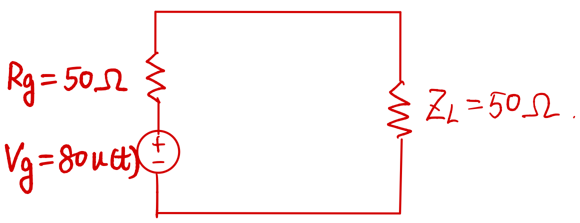


b) (14 points) Use the bounce diagram in part (a) to find the voltage at $z = 100\,\text{m}$ for $t \in [0, 4]\mu s$. Be sure to mark all magnitudes.



c) (7 points) As $t \to \infty$, what is the DC steady-state current at $z = l/2$?

$$I_{SS} = \frac{V_g}{R_g + Z_L} = \frac{80}{50+50} = 0.8\,A$$



Your Answer (include units when appropriate):

$I_{SS}(z = l/2) =$ 0.8 A.

(Scratch Page – No credit given for work done on this page.)