1. (a) Assuming the material is a perfect dielectric,

   i. (2 points) Impedance is given by

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_o}{\epsilon_r \epsilon_o}} = \frac{\eta_o}{\sqrt{12}} = 108.83\,[\Omega]$$

   and the velocity

$$v = \frac{1}{\sqrt{\mu\epsilon}} = \frac{c}{\sqrt{12}} = 8.66 \times 10^7\,[\frac{\text{m}}{\text{s}}].$$

   ii. (2 points) Wavelength in the medium can be calculated by

$$\lambda = \frac{v}{f} = 144.3\,[\text{nm}].$$

   To determine the color of the light, we determine the free space wavelength of the light

$$\lambda_o = \frac{c}{f} = 500\,[\text{nm}].$$

   Hence it is seen that the free space wavelength lies in the visible region with the color green. The wavevector can be derived from the wavelength in the medium,

$$\beta = \frac{2\pi}{\lambda} = 4.35 \times 10^7\,[\text{m}^{-1}].$$

   iii. (2 points) The electric field and magnetic field can be written in terms of known parameters

$$\mathbf{E} = E_o \cos(\omega t + \beta y)\hat{x} = E_o \cos(12\pi \times 10^{14} t + 4.35 \times 10^7 y)\hat{x}\,[\tfrac{\text{V}}{\text{m}}]$$
$$\mathbf{H} = \tfrac{E_o}{\eta} \cos(\omega t + \beta y)\hat{z} = \tfrac{E_o}{108.83} \cos(12\pi \times 10^{14} t + 4.35 \times 10^7 y)\hat{z}\,[\tfrac{\text{A}}{\text{m}}].$$

   iv. (2 points) The phasor forms of electric field and magnetic field

$$\tilde{\mathbf{E}} = E_o \exp(j\beta y)\hat{x} = E_o \exp(j4.35 \times 10^7 y)\hat{x}\,[\tfrac{\text{V}}{\text{m}}]$$
$$\tilde{\mathbf{H}} = \tfrac{E_o}{\eta} \exp(j\beta y)\hat{z} = \tfrac{E_o}{108.83} \exp(j4.35 \times 10^7 y)\hat{z}\,[\tfrac{\text{A}}{\text{m}}].$$

   v. (2 points) The complex Poynting vector is given by

$$\tilde{\mathbf{S}} = \tilde{\mathbf{E}} \times \tilde{\mathbf{H}}^* = -\frac{E_o^2}{\eta}\hat{y} = -\frac{E_o^2}{108.83}\hat{y}\,[\text{W/m}^2].$$

   Taking the time average of the poynting vector

$$\left\langle \tilde{\mathbf{S}} \right\rangle = \frac{1}{2} Re\{\tilde{\mathbf{E}} \times \tilde{\mathbf{H}}^*\} = -\frac{E_o^2}{2\eta}\hat{y} = -\frac{E_o^2}{217.66}\hat{y}\,[\text{W/m}^2].$$

(b) Assuming the perfect dielectric assumption,

i. (2 points) Permittivity can be found from the velocity

$$\epsilon = \frac{c^2}{v^2}\epsilon_o = 4\epsilon_o$$

Hence, the relative permittivity is $\epsilon_r = 4$. The impedance can be derived as

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = \frac{\eta_o}{2} = 60\pi\ [\Omega]$$

ii. (2 points) Radial frequency is given by

$$\omega = 2\pi\frac{v}{\lambda} = 9.42 \times 10^{14}\ [\frac{\text{rad}}{\text{s}}]$$

Time period is the inverse of frequency

$$T = \frac{1}{f} = 6.67 \times 10^{-15}\ [s].$$

The free space wavelength determines the color of the wave

$$\lambda_o = \sqrt{\epsilon_r}\,\lambda = 2\ [\mu m].$$

Therefore, the wave lies in the infrared region and is not visible to the human eye.

iii. (2 points) Following the same approach, the electric field and magnetic field can be written as

$$\mathbf{E} = \eta H_o\cos(\omega t - \beta x)\hat{y} = 60\pi H_o\cos(9.42\times 10^{14}t - 2\pi\times 10^6 x)\hat{y}\ [\tfrac{\text{V}}{\text{m}}].$$
$$\mathbf{H} = H_o\cos(\omega t - \beta x)\hat{z} = H_o\cos(9.42\times 10^{14}t - 2\pi\times 10^6 x)\hat{z}\ [\tfrac{\text{A}}{\text{m}}].$$

iv. (2 points) The phasor forms

$$\tilde{\mathbf{E}} = \eta H_o\exp(-j\beta x)\hat{y} = 60\pi H_o\exp(-j2\pi\times 10^6 x)\hat{y}\ [\tfrac{\text{V}}{\text{m}}].$$
$$\tilde{\mathbf{H}} = H_o\exp(-j\beta x)\hat{z} = H_o\exp(-j2\pi\times 10^6 x)\hat{z}\ [\tfrac{\text{A}}{\text{m}}].$$

v. (2 points) The complex poynting vector is given by

$$\tilde{\mathbf{S}} = \tilde{\mathbf{E}}\times\tilde{\mathbf{H}}^* = \eta H_o^2\hat{x} = 60\pi H_o^2\hat{x}\ [\frac{\text{W}}{\text{m}^2}].$$

Taking the time average of the poynting vector

$$\left\langle\tilde{\mathbf{S}}\right\rangle = \frac{1}{2}Re\{\tilde{\mathbf{E}}\times\tilde{\mathbf{H}}^*\} = \frac{\eta H_o^2}{2}\hat{x} = 30\pi H_o^2\hat{x}\ [\frac{\text{W}}{\text{m}^2}].$$

(c) i. (2 points) As the wave travels at the speed of light

$$\mu = \mu_o$$

and the impedance is given by

$$\eta = \eta_o$$

ii. (2 points) Wavelength can be calculated by

$$\lambda = \frac{v}{f} = 0.1\ [m].$$

Since the wave travels at the speed of light, the above wavelength represents the free space wavelength. This wave lies in the radio wave region and is not visible. Consequently, the wavevector is

$$\beta = \frac{2\pi}{\lambda} = 20\pi\ [m^{-1}].$$

iii. (2 points) Since the electric field is right hand circular polarized, the form is different from the last two parts,

$$\begin{aligned}\mathbf{E} &= E_o[\cos(\omega t - \beta z)\hat{x} + \sin(\omega t - \beta z)\hat{y}] \\ &= E_o[\cos(6\pi \times 10^9 t - 20\pi z)\hat{x} + \sin(6\pi \times 10^9 t - 20\pi z)\hat{y}]\ [\tfrac{\mathrm{V}}{\mathrm{m}}]. \\ \mathbf{H} &= \tfrac{E_o}{\eta}[\cos(\omega t - \beta z)\hat{y} - \sin(\omega t - \beta z)\hat{x}] \\ &= \tfrac{E_o}{120\pi}[\cos(6\pi \times 10^9 t - 20\pi z)\hat{y} - \sin(6\pi \times 10^9 t - 20\pi z)\hat{x}]\ [\tfrac{\mathrm{A}}{\mathrm{m}}].\end{aligned}$$

iv. (2 points) The phasor forms of electric field and magnetic field

$$\tilde{\mathbf{E}} = E_o \exp(-j\beta z)(\hat{x} - j\hat{y}) = E_o \exp(-j20\pi z)(\hat{x} - j\hat{y})\ [\tfrac{\mathrm{V}}{\mathrm{m}}].$$
$$\tilde{\mathbf{H}} = \tfrac{E_o}{\eta} \exp(-j\beta z)(\hat{y} + j\hat{x}) = \tfrac{E_o}{120\pi} \exp(-j20\pi z)(\hat{y} + j\hat{x})\ [\tfrac{\mathrm{A}}{\mathrm{m}}].$$

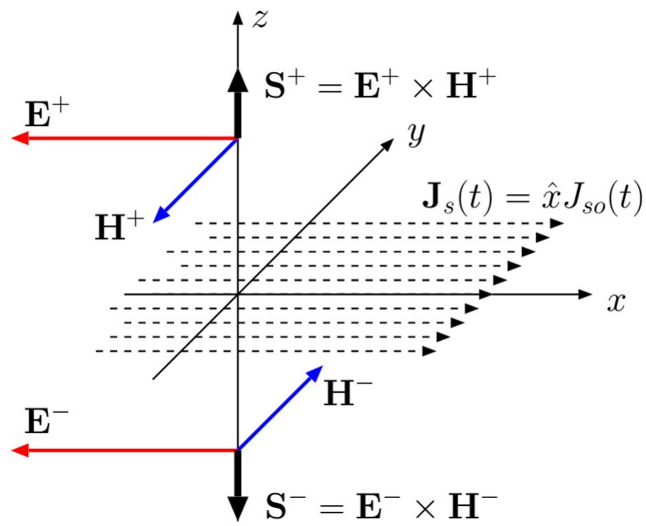
v. (2 points) The complex poynting vector is given by

$$\tilde{\mathbf{S}} = \tilde{\mathbf{E}} \times \tilde{\mathbf{H}}^* = \frac{2E_o^2}{\eta}\hat{z} = \frac{E_o^2}{60\pi}\hat{z}\,[\frac{\mathrm{W}}{\mathrm{m}^2}],$$

and the time average poynting vector by

$$\left\langle \tilde{\mathbf{S}} \right\rangle = \frac{1}{2} Re\{\tilde{\mathbf{E}} \times \tilde{\mathbf{H}}^*\} = \frac{E_o^2}{\eta}\hat{z} = \frac{E_o^2}{120\pi}\hat{z}\,[\frac{\mathrm{W}}{\mathrm{m}^2}].$$

2. The pulse of sheet current $\mathbf{J}_s(t) = \hat{x}J_{so}\sin(\omega t)$ A/m will produce magnetic and electric fields as shown in the following figure.



(a) (3 points) The average Poynting vector is $\langle S\rangle_{z>0} = \frac{1}{2}E_oH_o\hat{z}$ for the wave propagating in $+z$ direction while it is $\langle S\rangle_{z<0} = -\frac{1}{2}E_oH_o\hat{z}$ for the wave propagating in $-z$ direction. Since the average power density provided by the current sheet is 4 $[\frac{\mathrm{W}}{\mathrm{m}^2}]$, we can write

$$\langle S\rangle_{z>0} + \langle S\rangle_{z<0} = E_oH_o = \frac{E_o^2}{\eta_o} = 4\,[\frac{\mathrm{W}}{\mathrm{m}^2}]$$

Knowing that $\eta_0 = 120\pi\Omega$, we get

$$E_o = \sqrt{4\eta_o} \approx 38.83\,[\mathrm{V/m}],$$

and

$$H_o = \frac{E_o}{\eta_o} = 0.103\,[\mathrm{A/m}].$$

(b) (3 points) The amount of instantaneous electromagnetic density power injected by the current surface is

$$-\mathbf{J}(t)\cdot\mathbf{E}(0,t) = J_{so}E_0\sin^2(\omega t)$$

from which we obtain the average power density

$$\langle -\mathbf{J}(t)\cdot\mathbf{E}(0,t)\rangle = \frac{1}{2}J_{so}E_0 = 4\,[\frac{\mathrm{W}}{\mathrm{m}^2}],$$

which implies that

$$J_{so} = \frac{8}{E_o} \approx 0.206\,[\mathrm{A/m}].$$

(c) (4 points) The phasors $\tilde{\mathbf{E}}$ of part (a) is given by

$$\tilde{\mathbf{E}} = jE_o e^{\mp j\beta z}\hat{x}\,[\mathrm{V/m}]$$

And the phasor $\tilde{\mathbf{H}}$ is

$$\tilde{\mathbf{H}} = \pm jE_o e^{\mp j\beta z}\hat{y}\,[\mathrm{A/m}]$$

Explicitly

$$\begin{cases} \tilde{\mathbf{E}} = 38.83je^{-j\beta z}\hat{x}\,[\mathrm{V/m}] \\ \tilde{\mathbf{H}} = 0.103je^{-j\beta z}\hat{y}\,[\mathrm{A/m}] \end{cases} \quad z > 0$$

and

$$\begin{cases} \tilde{\mathbf{E}} = 38.83je^{j\beta z}\hat{x}\,[\mathrm{V/m}] \\ \tilde{\mathbf{H}} = -0.103je^{j\beta z}\hat{y}\,[\mathrm{A/m}] \end{cases} \quad z < 0$$

3. Recall

$$\gamma\eta = j\omega\mu \text{ and } \frac{\gamma}{\eta} = \sigma + j\omega\epsilon$$

as well as

$$\mu = \frac{\gamma\eta}{j\omega},\ \sigma = \text{Re}\{\frac{\gamma}{\eta}\},\ \epsilon = \frac{1}{\omega}\text{Im}\{\frac{\gamma}{\eta}\}$$

These relations are valid for the given plane wave propagating in a non-magnetic material ($\mu = \mu_0$) with

$$\mathbf{H} = \hat{x}25e^{-z}\cos\left(8\pi\times10^6t - \sqrt{3}z - \frac{\pi}{3}\right)\ [\text{A/m}].$$

(a) (2 points) By comparing with the given expression and the general expression

$$\mathbf{H} = \hat{x}H_0e^{-\alpha z}\cos(\omega t - \beta z - \phi)\ [\text{A/m}],$$

we find that $\alpha = 1$ [Np/m], $\beta = \sqrt{3}$ [rad/m], $\gamma = \alpha + j\beta = 1 + j\sqrt{3}$ [1/m].
From $\gamma\eta = j\omega\mu$, we have

$$\eta = \frac{j\omega\mu}{\gamma} = \frac{j\times8\pi\times10^6\times4\pi\times10^{-7}}{1+j\sqrt{3}} = \frac{4\pi^2}{5}(\sqrt{3}+j)\ [\Omega].$$

(b) (2 points)

$$\frac{\gamma}{\eta} = \frac{1+j\sqrt{3}}{\frac{4\pi^2}{5}(\sqrt{3}+j)} = 0.110 + j0.0633\ [\text{S/m}].$$

$$\begin{cases} \epsilon &= \frac{1}{\omega}Im(\frac{\gamma}{\eta}) = \frac{1}{8\pi\times10^6}\times0.0633 = 2.52\times10^{-9}\ [\text{F/m}] \\ \sigma &= Re(\frac{\gamma}{\eta}) = 0.110\ [\text{S/m}] \end{cases}$$

(c) (1 point)

$$\tilde{\mathbf{H}} = \hat{x}25e^{-(1+j\sqrt{3})z-j\frac{\pi}{3}}\ [\text{A/m}].$$

(d) (1 point)

$$\mathbf{E} = \eta\mathbf{H}\times\hat{v},$$

$$\tilde{\mathbf{E}} = \eta\tilde{\mathbf{H}}\times(\hat{z}) = \left(\frac{8\pi^2}{5}e^{j\frac{\pi}{6}}\right)\cdot(-\hat{y})\cdot25e^{-(1+j\sqrt{3})z-j\frac{\pi}{3}} = -\hat{y}40\pi^2e^{-(1+j\sqrt{3})z-j\frac{\pi}{6}}\ [\text{V/m}].$$

(e) (2 points)

$$\begin{aligned} <\mathbf{E}\times\mathbf{H}> &= \tfrac{1}{2}Re(\tilde{\mathbf{E}}\times\tilde{\mathbf{H}}^*) \\ &= \tfrac{1}{2}Re\left(\hat{z}1000\pi^2e^{-2z+j\frac{\pi}{6}}\right) \\ &= \hat{z}500\pi^2e^{-2z}\left(\tfrac{\sqrt{3}}{2}\right) \\ &= \hat{z}250\sqrt{3}\pi^2e^{-2z}\ [\text{W/m}^2]. \end{aligned}$$

(f) (2 points) Notice that the wave travels along z direction, the cross-sectional area of given the cubic volumn is $1m^2$. The power loss when the wave travels from $z = 0$ to $z = 1$ is

$$\triangle|<\mathbf{E}\times\mathbf{H}>|\cdot1 = 250\sqrt{3}\pi^2\left(1 - e^{-2\times1}\right) \approx 3.6953\,[\text{kW}].$$

4. We observe that

$$\frac{\sigma}{\omega\epsilon} = \frac{4}{2\pi \times 90 \times 10^3 \times 81 \times 8.85 \times 10^{-12}} \gg 1,$$

which means ocean water can be treated as a good conductor at 90 kHz.

(a) (2 points) In a good conductor

$$\alpha[\text{Np/m}] = \beta[\text{rad/m}] \approx \sqrt{\pi f \mu\sigma} = \sqrt{\pi \times 90 \times 10^3 \times 4\pi \times 10^{-7} \times 4} = 1.1922$$

$$|\eta| = \sqrt{\frac{\omega\mu}{\sigma}} = 0.4215$$

because it is a good conductor, $\tau = 45^o$

$$\eta = 0.243\angle 45^o \ [\Omega].$$

(b) (2 points) Assume the distance between the submarine and the ship is d, we want

$$\begin{aligned} e^{-\alpha d} &\geqslant 1\% = 0.01 \\ -\alpha d &\geqslant ln(0.01) \\ d &\leqslant \tfrac{-ln(0.01)}{\alpha} \approx 3.8627\,[\text{m}] \end{aligned}$$

(c) (1 point) Based on β the in Part a), the wavelength is

$$\lambda = \frac{2\pi}{\beta} \approx 5.27\,[\text{m}].$$

(d) (5 points) At $f = 900Hz$, since

$$\frac{\sigma}{\omega\epsilon} = \frac{4}{2\pi \times 300 \times 81 \times 8.85 \times 10^{-12}} = 2.96 \times 10^6 \gg 1,$$

we can still treat ocean water as a good conductor at $900Hz$. By using the general formula:

$$\alpha[\text{Np/m}] = \beta[\text{rad/m}] \approx \sqrt{\pi f \mu\sigma} = \sqrt{\pi \times 9 \times 10^2 \times 4\pi \times 10^{-7} \times 4} = 0.119,$$

$$|\eta| = \sqrt{\frac{\omega\mu}{\sigma}} = 0.0421,$$

$$\eta = 0.0421\angle 45^o \ [\Omega],$$

$$d \leqslant \frac{-ln(0.01)}{\alpha} \approx 38.63 \ [\text{m}],$$

$$\lambda = \frac{2\pi}{\beta} = 57.71\,[\text{m}].$$

5. Since, the wave frequency for the current source is equal to the wave frequency of the resulting TEM waves, the wave frequency $\omega = 6\pi \times 10^8$[rad/s]

   (a) (1 point) The wave propagation velocity will be perpendicular to the current sheet i.e. in $\pm\hat{x}$ direction and is given by

$$|\mathbf{v_p}| = \sqrt{\frac{1}{\mu\epsilon}} = \sqrt{\frac{1}{\frac{9}{4}\mu_o\epsilon_o}} = \frac{2}{3}c\,[\text{m/s}],$$

   where c is the speed of light.

   (b) (2 points) For wave number β and wavelength λ, we can write

$$\beta = \frac{\omega}{|\mathbf{v_p}|} = \frac{6\pi \times 10^8}{\frac{2}{3} \times 3 \times 10^8} = 3\pi\,[\text{m}^{-1}]$$

   Then we get

$$\lambda = \frac{2\pi}{\beta} = \frac{2}{3}\,[\text{m}]$$

   (c) (1 point) Intrinsic impedance η for pefect dielectrics is given by

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_o}{\frac{9}{4}\epsilon_o}} = \frac{2}{3}\eta_o\,[\Omega]$$

   (d) (4 points) The wavefields can be expressed as

$$\begin{aligned} \mathbf{E} &= -\eta\cos\left(\omega t \mp \beta\left(x-2\right)\right)\hat{y}\,[\tfrac{\text{V}}{\text{m}}] \\ \tilde{\mathbf{E}} &= -\eta e^{\mp j\beta(x-2)}\hat{y}\,[\tfrac{\text{V}}{\text{m}}] \\ \mathbf{H} &= \mp\cos\left(\omega t \mp \beta\left(x-2\right)\right)\hat{z}\,[\tfrac{\text{A}}{\text{m}}] \\ \tilde{\mathbf{H}} &= \mp e^{\mp j\beta(x-2)}\hat{z}\,[\tfrac{\text{A}}{\text{m}}] \end{aligned} \qquad x \gtrless 2$$

   (e) (3 points) Let's assume a general form for the current density $\mathbf{J_s}(t) = A\cos(\omega t)\hat{y}\,[\frac{\text{A}}{\text{m}}]$., the electric field and the magnetic field in the region $x > 2$ is given by

$$\begin{aligned} \mathbf{E} &= -\tfrac{A\eta}{2}\cos\left(\omega t - \beta\left(x - x_0\right)\right)\hat{y} \\ \mathbf{H} &= -\tfrac{A}{2}\cos\left(\omega t - \beta\left(x - x_0\right)\right)\hat{z} \end{aligned}$$

   Since $\mathbf{J} = 0$, the reduced Poynting's theorem can be written as

$$\nabla\cdot(\mathbf{E}\times\mathbf{H}) + \frac{\partial}{\partial t}\left(\frac{1}{2}\epsilon\mathbf{E}\cdot\mathbf{E} + \frac{1}{2}\mu\mathbf{H}\cdot\mathbf{H}\right) = 0$$

   Evaluating individual components of the equation

$$\begin{aligned} \nabla\cdot(\mathbf{E}\times\mathbf{H}) &= \nabla\cdot\left[-\tfrac{A\eta}{2}\cos\left(\omega t - \beta\left(x-x_0\right)\right)\hat{y} \times -\tfrac{A}{2}\cos\left(\omega t - \beta\left(x-x_0\right)\right)\hat{z}\right] \\ &= \nabla\cdot\left[\tfrac{A^2\eta}{4}\cos^2\left(\omega t - \beta\left(x-x_0\right)\right)\hat{x}\right] \\ &= \tfrac{A^2\eta\beta}{4}\sin\left(2\omega t - 2\beta\left(x-x_0\right)\right) \end{aligned}$$

$$\begin{aligned}
\frac{\partial}{\partial t}\left(\frac{1}{2}\epsilon\mathbf{E}\cdot\mathbf{E}+\frac{1}{2}\mu\mathbf{H}\cdot\mathbf{H}\right) &= \frac{\partial}{\partial t}\left[\frac{1}{2}\epsilon\frac{A^2\eta^2}{4}\cos^2\left(\omega t-\beta\left(x-x_0\right)\right)+\frac{1}{2}\mu\frac{A^2}{4}\cos^2\left(\omega t-\beta\left(x-x_0\right)\right)\right] \\
&= \frac{\partial}{\partial t}\left[\frac{A^2}{4}\left(\epsilon\eta^2+\mu\right)\cos^2\left(\omega t-\beta\left(x-x_0\right)\right)\right] \\
&= -\frac{A^2\omega}{8}\left(\epsilon\eta^2+\mu\right)\sin\left(2\omega t-2\beta\left(x-x_0\right)\right) \\
&= -\frac{A^2\eta\beta}{4}\sin\left(2\omega t-2\beta\left(x-x_0\right)\right)
\end{aligned}$$

It can be seen that adding both component terms leads to zero. Hence, the Poynting's theorem is satisfied in the region $x > 2$

(f) (2 points) The time-average power is given by

$$\langle P\rangle = \langle\mathbf{S}\rangle\cdot(A\hat{n})$$

Substituting values from part d gives

$$\begin{aligned}
\langle P\rangle &= \langle\eta\cos^2\left(\omega t-\beta\left(x-x_0\right)\right)\hat{x}\rangle\cdot 2\hat{x} \\
&= \left(\frac{\eta}{2}\hat{x}\right)\cdot 2\hat{x} \\
& \quad \eta\,[\mathrm{W}].
\end{aligned}$$

(g) (2 points) The time-average Poynting theorem is

$$\oint_A \langle\mathbf{S}\rangle\cdot d\mathbf{A} = -\int_V \langle\mathbf{J}\cdot\mathbf{E}\rangle\, dV$$

In words, the total time-averaged power generated by the current sheet is equal to the time-averaged power transported by the waves on both sides of the sheet. The time-averaged power generated by the current sheet is given by

$$\begin{aligned}
\langle\mathbf{J}\cdot\mathbf{E}(2,t)\rangle &= \langle -2\cos(\omega t)\times\eta\cos(\omega t)\rangle \\
&= \langle -2\eta\cos^2(\omega t)\rangle \\
&= -\eta\left[\frac{\mathrm{W}}{\mathrm{m}^3}\right],
\end{aligned}$$

which is twice of the time-average of the Poynting vector in part (f), on one side of the sheet.

6. (a) (5 points) For the wave $\tilde{\mathbf{E}} = \hat{y}E_0 e^{-j\frac{k}{\sqrt{2}}(x+z)}$, the electric field is in $\hat{y}$ direction whereas the wave propagates along $\hat{k} = \frac{\hat{x}+\hat{z}}{\sqrt{2}}$ direction, the field is hence called transverse to the wave, i.e. $\hat{k}\cdot\mathbf{E} = 0$.

Hence, we can decompose the wave into 2 propagation phenomena. One is (E_y, H_z) propagating along $\hat{x}$ direction. The other is (E_y, H_{-x}) propagating along $\hat{z}$ direction. Notice that we only have one component of **E** field (i.e. E_y) but two different components of **H** field.

Since the first wave component propagates along $\hat{x}$ direction, it doesn't interact with the surface, hence, remains propagating along $\hat{x}$ direction. (But it doesn't mean the field directions are to be the same, we will see that in a minute)

The second wave component (i.e. the pair (E_y, H_{-x}) - denoted as $\mathbf{E^{inc}}$ and $\mathbf{H_2^{inc}}$ on the figure) tries to penetrate through the boundary and gets partially transmitted as well as reflected. Boudary condition for **E** field at the boundary surface gives

$$E_y^{inc} + E_y^{refl} = E_y^{trans}$$

Roughly speaking, we know that electric field inside a conducting object should be zero, hence $E_y^{trans} = 0$. Thus, $E_y^{inc} = -E_y^{refl}$. That is the reflected electric field is in the opposite direction compared to the incident one, while the reflected magnetic field is in the same direction as before. It confirms that there exists the reflected wave propagating in $-\hat{z}$ direction, hence, termed *reflected*. In the figure, this fact is illustrated as $\mathbf{H_2^{inc}}$ and $\mathbf{H_2^{refl}}$ are both in $-\hat{x}$ direction while $\mathbf{E^{inc}}$ and $\mathbf{E^{refl}}$ are in opposite direction.

Notice that the first wave component doesn't change its propagating direction, namely $\hat{x}$. However, due to the fact that **E** field reverses its direction, so does the associated **H** field component, namely, $\mathbf{H_1^{refl}}$ in the figure. This reversed direction phenomenon of $\mathbf{H_1}$ from $\mathbf{H_1^{inc}}$ to $\mathbf{H_1^{refl}}$ is consistent with the normal boundary condition:

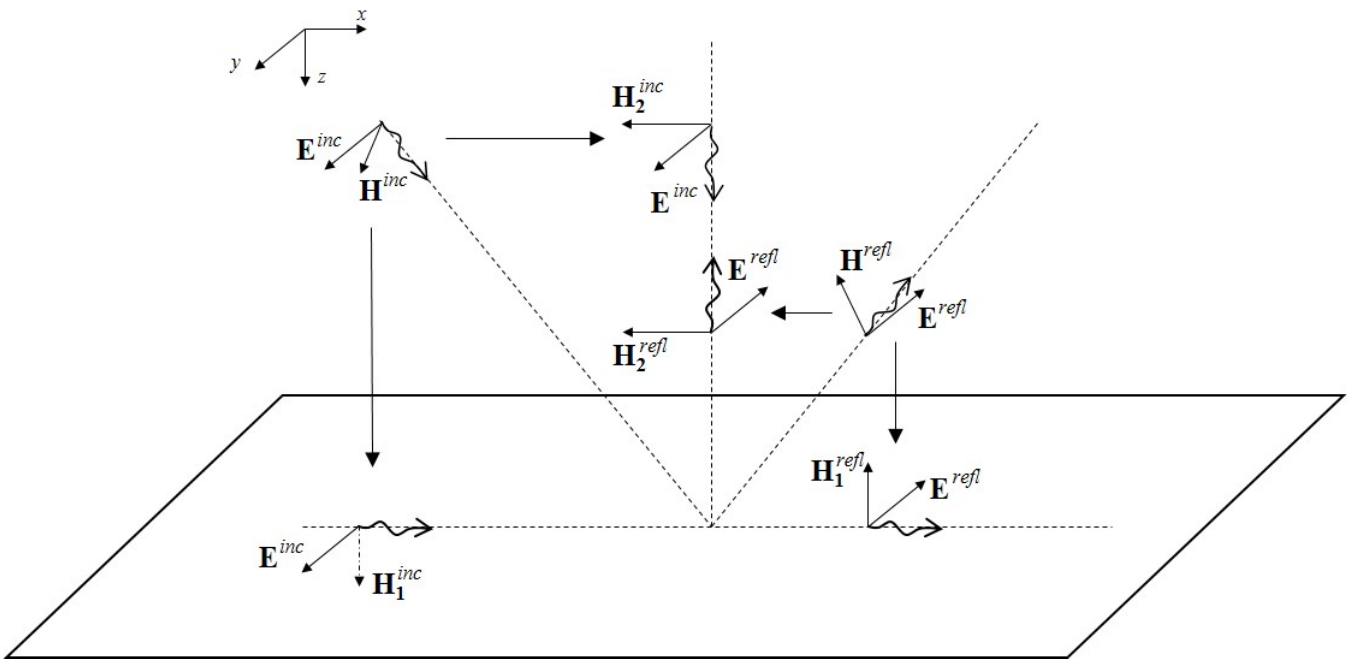
$$H_1^{inc} + H_1^{refl} = H_1^{trans} = 0$$

Again, the very good conductor is assumed. Thus, not only no electric field but also no magnetic field inside the conductor as well.

Future lectures will give you a more rigorous proof on this phenomenon.

(b) (5 points) This case is the complementary part of part (a). This time, we have 2 components of **E** field and 1 components of **H** field, $\hat{H} = \hat{k}\times\hat{E} = \frac{\hat{x}+\hat{z}}{\sqrt{2}} \times \frac{\hat{x}-\hat{z}}{\sqrt{2}} = \hat{y}$. It is, hence, called the parallel polarization wave as the **E** field is parallel to the wave propagation direction, i.e. $\hat{k} = \frac{\hat{x}+\hat{z}}{\sqrt{2}}$, and $\hat{k}\cdot\mathbf{E} = 0$.

The same phenomenon as above, the wave gets reflected, direction of **H** field remains $\hat{y}$ but that of **E** field changes to $\frac{-\hat{x}-\hat{z}}{\sqrt{2}}$.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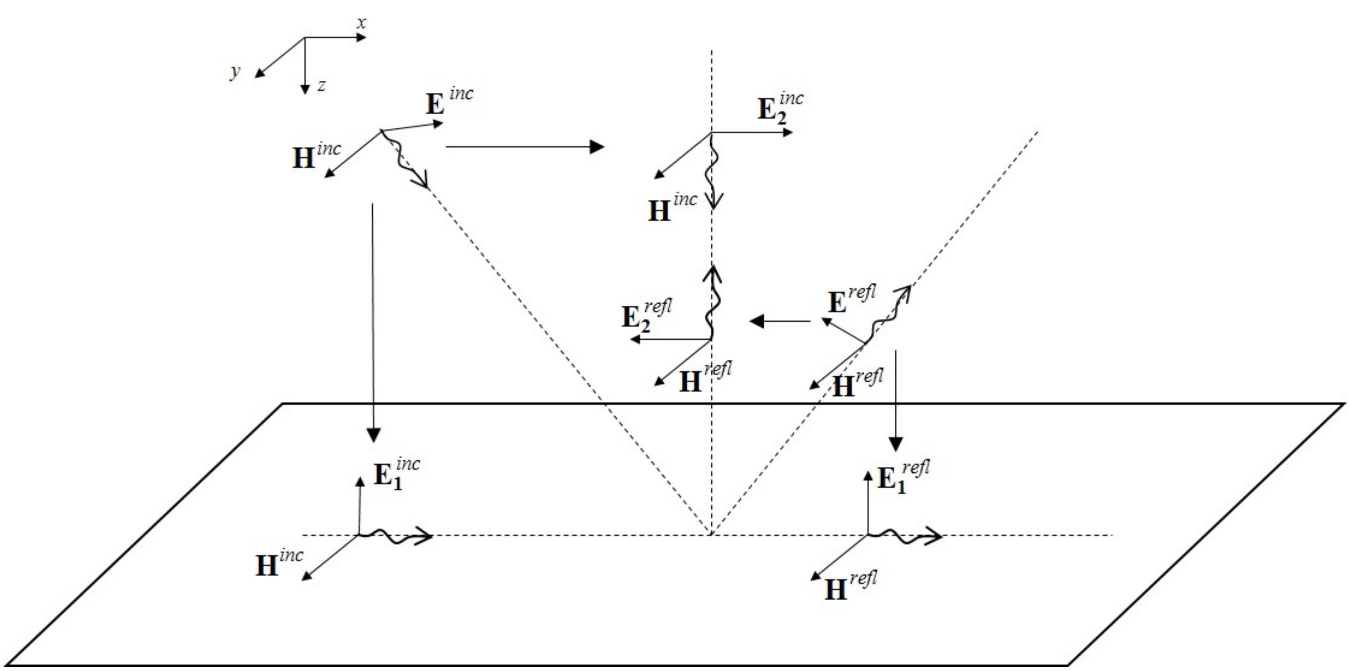
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