

ECE 329 Fields and Waves I

Homework 9

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Due March 30, 2023, 11:59 PM

Homework Policy:

- Write your name and NetID on top of every page. This habit will help you in exams in the event of having loose page(s).
- Tag all the questions in Gradescope. Failure to do so results in a 5 points deduction.
- Cheating results in ZERO and 50% reduction in HW average on first offense. A 100% reduction in HW average on second offense.
- Please show detailed process for each problem instead of just an answer. No partial credits would be given otherwise. All answers should include units wherever appropriate.
- No late HW is accepted.
- Regrade requests are available one week following grade release.

You are allowed to work with anyone else, but the work you submit should only belong to you. Note that if you have knowledge of a violation of the Honor Code, then you are obligated to report it. By submitting this homework, you are agreeing to the Honor Code: “I have neither given nor received unauthorized aid on this homework, nor have I concealed any violations of the Honor Code.”

Question	Points	Score
1	30	
2	10	
3	10	
4	10	
5	15	
6	10	
Total:	85	

1. For each of the three electromagnetic waves described in (a), (b), and (c), find the requested parameter:
- (a) $\mu = \mu_o$ and $\varepsilon = 12\varepsilon_o$, electric field linearly polarized in the \hat{x} direction, poynting vector in the $-\hat{y}$ direction, electric field amplitude E_o , radial frequency $\omega_o = 12\pi \times 10^{14}$ rad/s:
- (2 points) Impedance of the medium and the speed of wave propagation.
 - (2 points) Wavelength (λ) and Wavevector (β) for the wave. Is this wave visible to the human eye? If so, what color would we associate with this wave? If not, what wavelength range does it fall into?
 - (2 points) Write the Electric and Magnetic fields as cosinusoidal monochromatic waves $\mathbf{E}(\mathbf{r}, t)$, $\mathbf{H}(\mathbf{r}, t)$
 - (2 points) Write the Electric and Magnetic fields in phasor notation \tilde{E} , \tilde{H} .
 - (2 points) Give the complex Poynting Vector \tilde{S} and time averaged Poynting Vector $\langle \mathbf{S} \rangle$.
- (b) $\mu = \mu_o$ and $v = c/2$, magnetic field linearly polarized in the \hat{z} direction, electric field in the \hat{y} direction, magnetic field amplitude H_o , wavelength in the medium $\lambda = 1\mu\text{m}$:
- (2 points) Impedance of the medium and the material permittivity.
 - (2 points) Radial frequency (ω) and Wave-period (T) for the wave. Is this wave visible to the human eye? If so, what color would we associate with this wave? If not, what wavelength range does it fall into?
 - (2 points) Write the Electric and Magnetic fields as cosinusoidal monochromatic waves $\mathbf{E}(\mathbf{r}, t)$, $\mathbf{H}(\mathbf{r}, t)$
 - (2 points) Write the Electric and Magnetic fields in phasor notation \tilde{E} , \tilde{H} .
 - (2 points) Give the complex Poynting Vector \tilde{S} and time averaged Poynting Vector $\langle \mathbf{S} \rangle$.
- (c) $\varepsilon = \varepsilon_o$ and $v = c$, right-hand circularly polarized, propagating in the $+\hat{z}$ direction, electric field amplitude E_o , frequency $f = 3\text{GHz}$:
- (2 points) Impedance of the medium and the material permeability.
 - (2 points) Wavelength (λ) and wavevector (β) for the wave. Is this wave visible to the human eye? If so, what color would we associate with this wave? If not, what wavelength range does it fall into?
 - (2 points) Write the Electric and Magnetic fields as cosinusoidal monochromatic waves $\mathbf{E}(\mathbf{r}, t)$, $\mathbf{H}(\mathbf{r}, t)$
 - (2 points) Write the Electric and Magnetic fields in phasor notation \tilde{E} , \tilde{H} .
 - (2 points) Give the complex Poynting Vector \tilde{S} and time averaged Poynting Vector $\langle \mathbf{S} \rangle$.

2. Consider an infinite surface current density

$$\mathbf{J}_s = \hat{x} J_{so} \sin(\omega t)$$

flowing on $z = 0$ surface, where $J_{so} > 0$ is real-valued amplitude of the monochromatic surface current measured in A/m units. It is found that \mathbf{J}_s injects field energy into propagating transverse electromagnetic (TEM) waves away from the $z = 0$ plane at an average rate of 4 W/m^2 — that is, the magnitude of the average Poynting vector $\langle \mathbf{S} \rangle$ is 2 W/m^2 for the waves excited above and below the surface current.

- (3 points) Denoting the TEM waves excited by \mathbf{J}_s (above and below the $z = 0$ plane) as $\mathbf{E} = -\hat{x} E_o \sin(\omega t \mp \beta z)$ V/m and $\mathbf{H} = \mp \hat{y} H_o \sin(\omega t \mp \beta z)$ A/m, where wavenumber $\beta = \frac{\omega}{c}$, determine the numerical values of wave amplitudes E_o and H_o in V/m and A/m units (assuming wave propagation in free space).
- (3 points) Determine the value of J_{so} in A/m units.
- (4 points) Write down explicitly the phasors of \mathbf{E} and \mathbf{H} fields of part (a) (for regions above and below $z = 0$ plane) using the numerical values of E_o and H_o determined in that part.

3. In general conducting media, plane TEM wave parameters γ and η satisfy

$$\gamma\eta = j\omega\mu \quad \text{and} \quad \frac{\gamma}{\eta} = \sigma + j\omega\epsilon,$$

as well as

$$\mu = \frac{\gamma\eta}{j\omega}, \quad \sigma = \text{Re}\left\{\frac{\gamma}{\eta}\right\}, \quad \epsilon = \frac{1}{\omega} \text{Im}\left\{\frac{\gamma}{\eta}\right\}.$$

Using these relations, for a plane wave propagating in a non-magnetic material ($\mu = \mu_o$) with

$$\mathbf{H} = \hat{x} 25e^{-z} \cos(8\pi \cdot 10^6 t - \sqrt{3}z - \frac{\pi}{3}) \frac{\text{A}}{\text{m}},$$

Determine:

- (2 points) The propagation constant γ and intrinsic impedance η ,
- (2 points) The permittivity ϵ and conductivity σ ,
- (1 point) Phasor magnetic field $\tilde{\mathbf{H}}$,
- (1 point) The corresponding phasor electric field $\tilde{\mathbf{E}}$,
- (2 points) Time-averaged Poynting vector $\langle \mathbf{E} \times \mathbf{H} \rangle$,
- (2 points) The time averaged power dissipated in cubic volume bounded by the planes $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$, all in meters.

4. A submarine submerged in the ocean ($\sigma = 4 \text{ S/m}$, $\epsilon_r = 81$, $\mu_r = 1$) wants to receive an electromagnetic signal from a ship located at the surface and transmitting at $f = \frac{\omega}{2\pi} = 90 \text{ kHz}$.
- (2 points) Calculate the numerical values of the propagation constant $\gamma = \alpha + j\beta$ and intrinsic impedance η . Use the approximations appropriate for the associated loss tangent $\frac{\sigma}{\omega\epsilon}$.
 - (2 points) How close by must the submarine be located in order to receive at least 1% of the signal amplitude at the surface?
 - (1 point) What is the wavelength $\lambda = \frac{2\pi}{\beta} = \frac{v_p}{f}$ of the EM signal at this operating frequency?
 - (5 points) Repeat parts (a-c) for ship transmission at 900 Hz.
5. An infinite plane current sheet of uniform, time-varying density $\mathbf{J}_s(t) = 2 \cos(6\pi \times 10^8 t) \hat{y} \text{ A/m}$ exists at the $x = 2 \text{ m}$ plane within a perfect dielectric medium having an electric permittivity of $\epsilon = \frac{9}{4}\epsilon_0$ and a magnetic permeability of $\mu = \mu_0$. Answer the following questions (using appropriate units) about the plane TEM waves which will propagate away from this surface current source.
- (1 point) What is the magnitude and direction of the wave propagation velocity \mathbf{v}_p of the TEM wave through the dielectric medium?
 - (2 points) What is the wave number β and wavelength λ of the TEM wave?
 - (1 point) What is the intrinsic impedance η of the dielectric medium?
 - (4 points) Write the expressions for wavefields \mathbf{E} and $\tilde{\mathbf{E}}$ (the phasor of \mathbf{E}) and \mathbf{H} and $\tilde{\mathbf{H}}$ (the phasor of $\tilde{\mathbf{H}}$) in terms of ω , β , and η for each of the regions $x > 2$ and $x < 2$ on either side of the current sheet source.
 - (3 points) Verify that the TEM wave satisfies the Poynting theorem, $\nabla \cdot (\mathbf{E} \times \mathbf{H}) + \frac{\partial}{\partial t} (\frac{1}{2}\epsilon \mathbf{E} \cdot \mathbf{E} + \frac{1}{2}\mu \mathbf{H} \cdot \mathbf{H}) + \mathbf{J} \cdot \mathbf{E} = 0$, in the region $x > 2$. **Hint:** you should be able to prove this for a TEM wave propagating at an arbitrary frequency ω in an arbitrary perfect dielectric in terms of μ and ϵ in order to avoid explicit use of constants found in parts (a-c).
 - (2 points) What is the time-averaged power transported by the TEM wave across a square surface on the $x = 4 \text{ m}$ plane having area $A = 2 \text{ m}^2$?
 - (2 points) Verify that the *time-averaged* Poynting theorem is satisfied at $x = 2$ (i.e., at the location of the current sheet).
6. **Bonus Problem** An infinitely conducting metallic region resides starting on the $z = z_0$ plane and extends into the positive z direction. For each of the following waves, identify 1) the polarization direction, 2) the propagation direction, and 3) $\hat{k} \cdot \mathbf{E}$. Using your knowledge of boundary conditions, describe (in words and diagrams) what happens to the following electric fields when they reach the boundary coming from from $z < z_0$. **Hint:** Consider transverse and normal components of the electric field. If the field has more than one vector component, you may treat each independently. The component of field propagating in the \hat{x} -direction does not interact with the surface. You will learn more about *oblique incidence* in ECE 350.
- (5 points) $\tilde{\mathbf{E}} = \hat{y} E_0 e^{-j \frac{k}{\sqrt{2}}(x+z)} \text{ V/m}$
 - (5 points) $\tilde{\mathbf{E}} = \frac{(\hat{x}-\hat{z})}{\sqrt{2}} E_0 e^{-j \frac{k}{\sqrt{2}}(x+z)} \text{ V/m}$