ECE 329 Fields and Waves I
Homework 5

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Due February 23, 2023, 11:59 PM

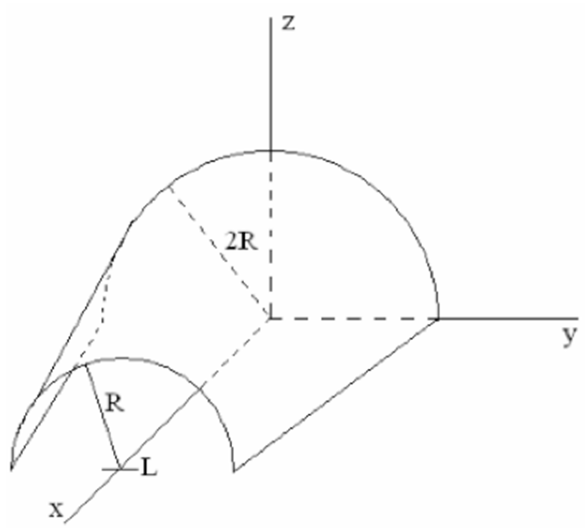
Homework Policy:

- Write your name and NetID on top of every page. This habit will help you in exams in the event of having loose page(s).
- Tag all the questions in Gradescope. Failure to do so results in a 5 points deduction.
- Cheating results in ZERO and 50% reduction in HW average on first offense. A 100% reduction in HW average on second offense.
- Please show detailed process for each problem instead of just an answer. No partial credits would be given otherwise. All answers should include units wherever appropriate.
- No late HW is accepted.
- Regrade requests are available one week following grade release.

You are allowed to work with anyone else, but the work you submit should only belong to you. Note that if you have knowledge of a violation of the Honor Code, then you are obligated to report it. By submitting this homework, you are agreeing to the Honor Code: "I have neither given nor received unauthorized aid on this homework, nor have I concealed any violations of the Honor Code."

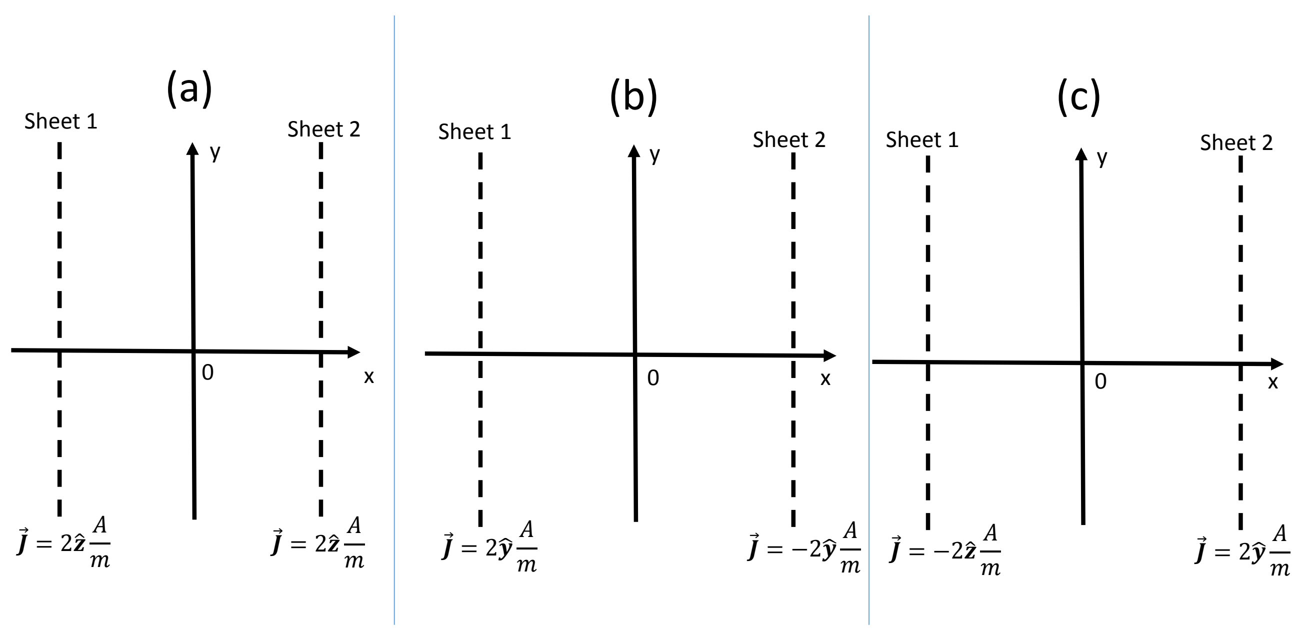
Question	Points	Score
1	10	
2	15	
3	15	
4	10	
Total:	50	

1. (10 points) Gauss' Law for $\mathbf{B}$ states that $\oint_S \mathbf{B} \cdot \mathbf{dS} = 0$ over any closed surface S enclosing a volume V. Given that $\mathbf{B} = B_0(-3\hat{x} + 2\hat{y} - 1\hat{z})$ Wb/m^2, determine the magnetic flux through the partial cone surface shown in the following figure:



2. (15 points) An infinite current sheet with a uniform current density $\mathbf{J} = J_s\hat{y}\ \frac{\text{A}}{\text{m}}$ produces magneto-static fields $\mathbf{B}$ with $\frac{\mu_o J_s}{2}$ magnitude on both sides of the sheet and with opposing directions in consistency with the right-hand-rule and the Biot-Savart law.

   Determine the **magnetic field intensity** $\mathbf{H} = \frac{\mathbf{B}}{\mu_o}$ at origin O in the following diagrams due to a pair of current sheets with specified $\mathbf{J}$ vectors.



3. Consider a hollowed out cylinder centered on the z-axis with inner radius a and outer radius b such that it is described by $a < r < b$. The hollow cylinder conducts a uniform current density of $\mathbf{J} = -J_0\hat{z}$ A/m^2 in the region $a < r < b$. Outside this region, that is for $r > b$ and $r < a$, the charge and current densities are zero.
   (a) (5 points) Using the integral form of Ampere's law find $\mathbf{B}$ everywhere. (You can simplify your answer using $I = J_0 \cdot A$)
   (b) (2 points) For $r > b$ what does the magnetic field $\mathbf{B}$ produced by a hollow cylinder look like? (Hint: See Lecture 12 online, this is the magnetic analog to a hollow sphere of charge).
   (c) (5 points) Now assume the hollow cylinder conducts a non-uniform current density of $\mathbf{J} = -\frac{J_0}{r}\hat{z}$. Find $\mathbf{B}$ everywhere.
   (d) (3 points) Prove that Gauss's Law for the magnetic field found in part (c) is satisfied. In cylindrical coordinates:
$$\nabla \cdot \mathbf{F} = \frac{1}{r}\frac{\partial (rF_r)}{\partial r} + \frac{1}{r}\frac{\partial F_\theta}{\partial \theta} + \frac{\partial F_z}{\partial z}.$$

4. Consider an infinite slab (extending in y and x directions) of a finite width $W = 4$ m described by $-3 < z < 1$. The slab is electrically neutral but it conducts a uniform current density of $\mathbf{J} = 2\hat{y}$ A/m^2 (meaning that it contains equal densities of positive and negative charge carriers moving in opposite directions parallel to $\hat{y}$). Outside the slab, that is for $z > 1$ and $z < -3$, the charge and current densities are zero.
   (a) (2 points) Using the right-hand-rule and Biot-Savart law, discuss why the current slab should generate equal and opposite directed magnetic fields in $\pm\hat{x}$ directions in front of and behind the plane of symmetry of the slab.
   (b) (1 point) Based on part (a), what is $\mathbf{B}$ on the $z = -1$ m plane? Briefly explain the reasoning behind your answer.
   (c) (3 points) Next, make use of the integral form of Ampere's law and the deductions of parts (a) and (b), to find $B_x(z)$ in the regions outside the slab. Hint: make use of a shifted coordinate system with its origin at the center of the slab.
   (d) (2 points) Use Ampere's law to find $B_x(z)$ at a distance z within the current slab.
   (e) (2 points) Plot B_x as a function of z over the region $-5 < z < 3$. Be sure to label all relevant values of B_x and z.

There is no Bonus Problem this week.