ECE 329 Fields and Waves I
Homework 3

Instructors: Chen, Goddard, Shao

Due February 9, 2023, 11:59 PM

Homework Policy:

- Write your name and NetID on top of every page. This habit will help you in exams in the event of having loose page(s).
- Tag all the questions in Gradescope. Failure to do so results in a 5 points deduction.
- Cheating results in ZERO and 50% reduction in HW average on first offense. A 100% reduction in HW average on second offense.
- Please show detailed process for each problem instead of just an answer. No partial credits would be given otherwise. All answers should include units wherever appropriate.
- No late HW is accepted.
- Regrade requests are available one week following grade release.

You are allowed to work with anyone else, but the work you submit should only belong to you. Note that if you have knowledge of a violation of the Honor Code, then you are obligated to report it. By submitting this homework, you are agreeing to the Honor Code: "I have neither given nor received unauthorized aid on this homework, nor have I concealed any violations of the Honor Code."

Question	Points	Score
1	5	
2	5	
3	10	
4	10	
5	5	
6	5	
7	10	
8	10	
Total:	60	

1. (5 points) Is $\mathbf{E} = z\hat{x} - y\hat{y} + x\hat{z}$ V/m a possible electrostatic field in free space? Explain.

2. (5 points) In free space, find the electric field $\mathbf{E}$ and static charge density $\rho(x, y, z)$ if the scalar potential is $V = \sin(x)e^{-y}z^2$ V. Briefly explain why $\mathbf{E}$ is a conservative field.

3. (10 points) Given $\mathbf{E} = 1\hat{x}+\sin(z)\hat{y}+y\cos(z)\hat{z}\ \frac{\text{V}}{\text{m}}$, determine the potential $V(1, 2, 3)$ if $V(0, 0, 0) =$ 3 V.

4. Given the two fields $\mathbf{E} = y\hat{x} \pm x\hat{y}$ V/m,
   (a) (6 points) Determine the circulation $\oint_C \mathbf{E}\cdot\mathbf{dl}$ for a triangular path C traversing in order its vertices at $(x, y, z) = (1, -1, 0), (-1, -1, 0)$, and $(1, 1, 0)$ m. **Hint:** $\mathbf{dl} \equiv dx\hat{x} + dy\hat{y} + dz\hat{z}$ always. On the slant edge of C, $x = y$ and $z = 0$ so $\mathbf{dl} = (\hat{x} + \hat{y})dx$ and $\mathbf{E} = x\hat{x} \pm x\hat{y}$.
   (b) (4 points) Verify your calculations by using Stokes' theorem to find $\iint_S (\nabla \times \mathbf{E}) \cdot \mathbf{dS}$. **Hint:** Think of the area of the region bounded by C.

5. (5 points) Between a pair of infinitesimally thin sheets located on the $z = -2$ and $z = 2$ m surfaces, a constant electric field is observed to be $\mathbf{E} = -4\hat{z}$ V/m. What is the voltage drop (or rise) V_p from the $z = 2$ m plane to the $z = -2$ m plane?

6. Consider a static sheet of charge $\rho(x, y, z) = \rho_0\delta(z)$ C/m^3.
   (a) (3 points) Write down the expression for $\mathbf{D}$ as a vector above and below the plate.
   (b) (2 points) Label $z > 0$ as region 1 and $z < 0$ as region 2 and $\hat{a}_n = \hat{z}$ which points from region 2 into region 1. Using your result from part (a), show that $D_{1n} - D_{2n} \equiv \hat{a}_n \cdot (\mathbf{D}_1 - \mathbf{D}_2) = \rho_0$.

7. Consider a static charge distribution given by $\rho_0 = 3\delta(y)+\rho_0'\delta(y-2)$ C/m^3 in free space. Along the $\hat{y}$ axis (i.e., for $x = z = 0$), the displacement field is known to be $\mathbf{D} = 2\hat{y} - 2\hat{z}$ C/m^2 for $0 < y < 2$ m and $D_y = 1$ C/m^2 for $y > 2$ m. Note that this field is a superposition of the field generated by ρ_0 together with a constant background field generated by a far away sheet charge source with surface charge density ρ_s.
   (a) (2 points) Determine the unknown charge density ρ_0' (be sure to include units in your answer). **Hint:** Use the boundary conditions from the previous problem.
   (b) (2 points) Determine $\mathbf{D}(0, y, 0)$ for the region $y > 2$ m.
   (c) (2 points) Determine $\mathbf{D}(0, y, 0)$ for the region $y < 0$ m.
   (d) (4 points) Determine ρ_s and write two possible expressions for the volumetric charge density $\rho(x, y, z)$ of the far away sheet charge if its distance from the $\hat{y}$ axis is known to be 30 m.

8. (10 points) Three important vector identities which are true for any scalar field $f(x, y, z)$ and vector field $\mathbf{A}(x, y, z)$ are:

$$\nabla \times (\nabla f) = \mathbf{0}$$

$$\nabla \cdot (\nabla \times \mathbf{A}) = 0$$

$$\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$$

where

$$\nabla^2 \mathbf{A} \equiv (\tfrac{\partial^2}{\partial x^2} + \tfrac{\partial^2}{\partial y^2} + \tfrac{\partial^2}{\partial z^2})\mathbf{A}$$

is the Laplacian of $\mathbf{A}$ and $\nabla(\nabla \cdot \mathbf{A})$ is the gradient of the divergence of $\mathbf{A}$.

Verify the identities for $f = x + yz$, $\mathbf{A} = (x + z)y\hat{x} + (x - z)\hat{z}$ by calculating each side of each identity and showing them to be the same.

There is no bonus question this week.