1. (a) (3 points) The direct approach is to find the voltage and current at the load. Knowing the voltage and impedance at the load, we can find:

$$\begin{aligned} V_L &= j15\,[\mathrm{V}] \\ I_L &= \frac{V_L}{R_L} = \frac{j15}{150} = j0.1\,[\mathrm{A}] \\ P_L &= \frac{1}{2} Re\{V_L I_L^*\} = 0.75\,[\mathrm{W}] \end{aligned}$$

(b) (5 points) In this circuit there are two elements that dissipate power: Z_g and R_L. The shortcut is to realize time-average power indicates steady-state, and thus TL doesn't dissipate any power, thus the system can be think of as a simple circuit of two impedances Z_g and Z_{in}, where $Z_{in} = \frac{1}{2} \times 75 = 37.5\Omega$ is the input impedance of the TL looking from the load, connected in series. Z_{in} can be found easily either by formulas or using Smith Chart with noting that we have a $\frac{3\lambda}{4}$-long TL. Thus, the power dissipation is proportional to impedance.

$$P_g = \frac{0.75}{37.5} \times 37.5 = 0.75\,[\mathrm{W}]$$

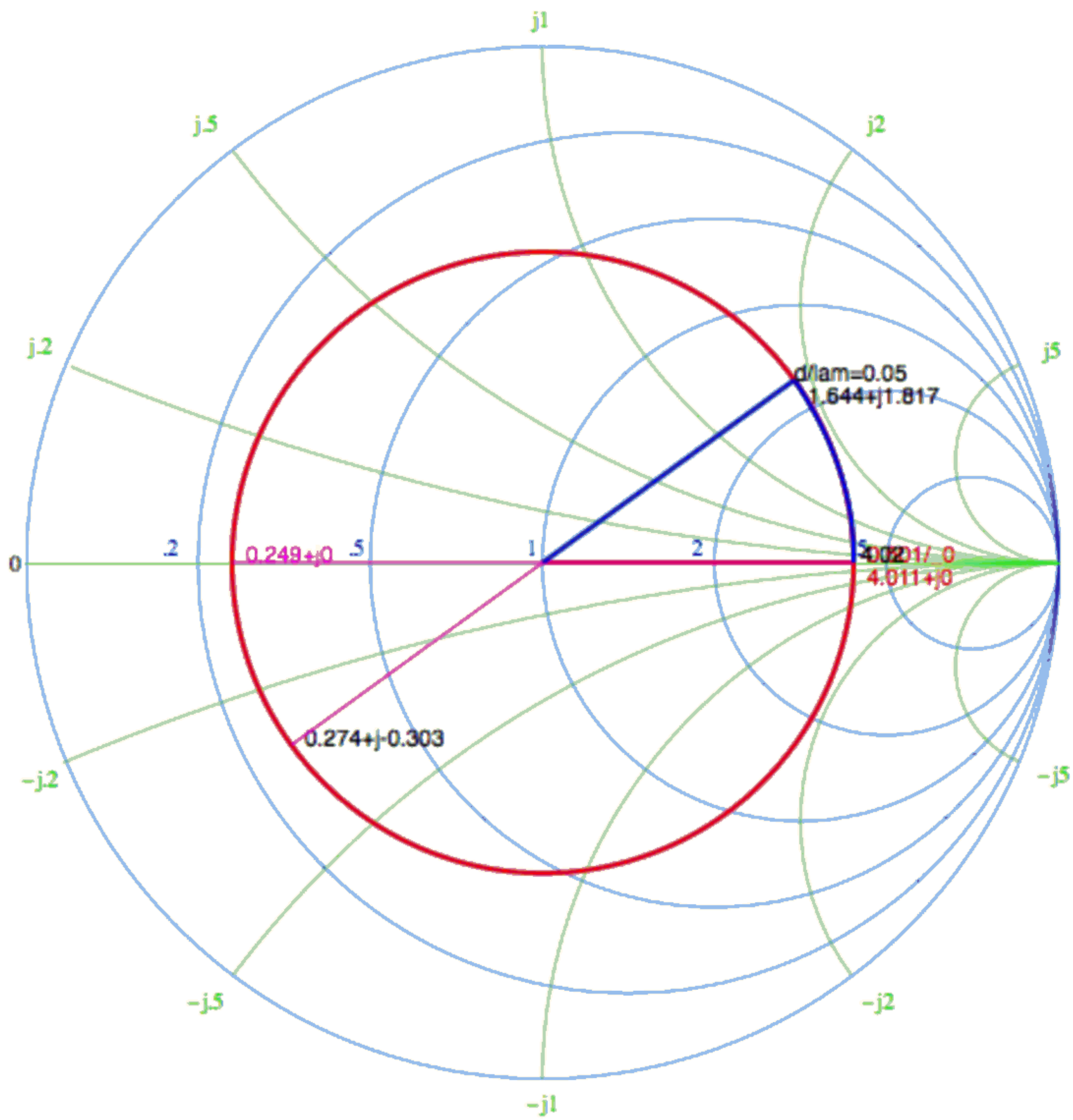
and then the total power delivered by the generator is $P_{gen} = P_L + P_g = 1.5\,[\mathrm{W}]$
An alternative interpretation is to regard the voltage-source-plus-internal-impedance as the geneartor. In this case, as long as you realize TL does not dissipate power in steady-state, the power dissipated at load is the same as the power delivered by geneator. Then $P_{gen} = 0.75\,[\mathrm{W}]$.
Note: Another approach would be to split the line into a quarter-wave and a half-wave transfomer then apply their transformation successively we end at the same result.

2. (a) (5 points) First of all, ECE 329 website provides a SC applet, which is good enough in most cases, but we also encourage you to look for online interactive SC tools on your own. Of course you can also use printed SC.

Normalized impedance is:

$$z_L = \frac{Z_L}{Z_0} = 4$$

we look for $4 + j0$ point on the S.C., and then rotate the unit circle clockwise (towards the generator) by 0.45λ (Remember rotating a full circle corresponds to half wavelength, so 0.45λ should correspond to 324 degrees). The result is shown below:



from which we find:
(i) $\Gamma_L = 0.6\angle 0^0$
(ii) $\Gamma(l) = 0.6\angle 36^0$
(iii) $Z(l) = 25 \times (1.64 + j1.81) = 41 + j45.25\,[\Omega]$

(b) (2 points) At the generator end (source plane, source port, input port, etc), the generator sees nothing but two lumped elements with impedance $50\,\Omega$and $41 + j45.25\,\Omega$. A simple voltage dividing rule gives:

$$V(l) = \frac{10 \times (41 + j45.25)}{50 + 41 + j45.25} = 5.600 + j2.188 = 6.01\angle 21.34^0\,[\mathrm{V}]$$

(c) (2 points) At $d = l$, which is the input port, $\beta d = \frac{2\pi}{\lambda} \times 0.45\lambda = 0.9\pi$, so:

$$\begin{aligned} V(l) &= V^+(e^{j\beta l} + \Gamma_L e^{-j\beta l}) \\ \rightarrow 5.600 + j2.188\,V &= V^+(e^{j0.9\pi} + 0.6e^{-j0.9\pi}) \end{aligned}$$

this equation can be solved directly by Wolfram Alpha to obtain:

$$V^+ = 3.94\angle - 154^0\,[\mathrm{V}]$$

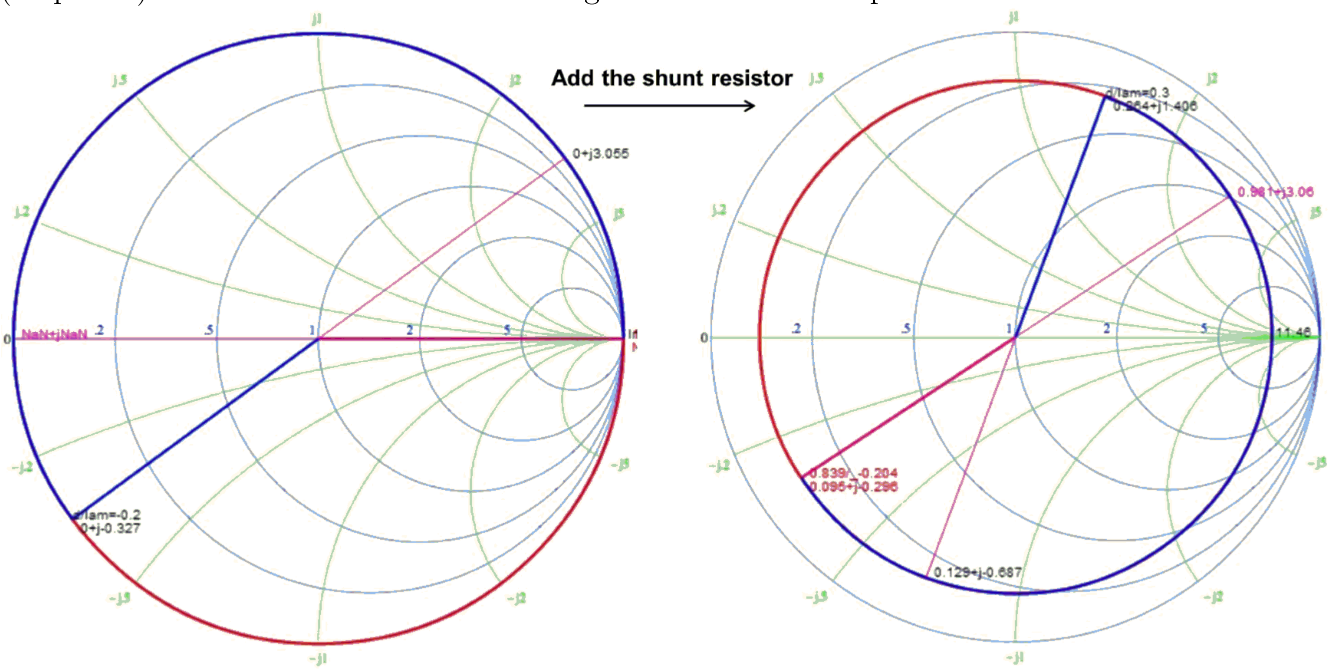
(d) (2 points) Always refer to $V(d) = V^+(e^{j\beta d} + \Gamma_L e^{-j\beta d})$ for voltage at arbitrary point along the TL. At $d = 0$:

$$\begin{aligned} V(0) &= V^+(1 + \Gamma_L) \\ &= 3.94\angle - 154^0 \times 1.6 \\ &= 6.304\angle - 154^0\,[\mathrm{V}] \end{aligned}$$

(e) (1 point) Ohm's law applies:

$$I(0) = \frac{V(0)}{Z_L} = 0.06\angle - 154^0\,[\mathrm{A}]$$

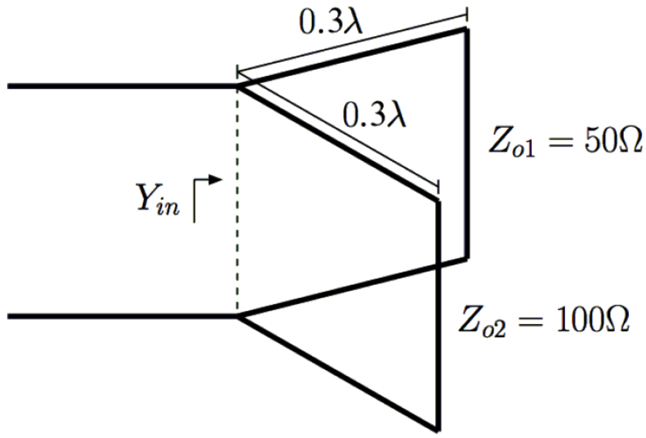
3. (10 points) Please think of movement along TL as a transform process.



Using the Smith chart and working in terms of impedances we find:

- Impedance at the load $Z_L = \infty$,
- Normalized input impedance at the load $z_L = \frac{Z_L}{Z_0} = \infty$, now rotate from point z_L towards the generator (clockwise) by $d = 0.2\lambda$ until we reach A (at shunt connection)
- Normalized input impedance A at shunt connection is $z_a = -j0.327$.
- Normalized input admittance A at shunt connection is $y_a = 1/z_a = j3.055$.
- The normalized shunt resistor is $z_R = R/Z_o = 50/50 = 1$, and thus $y_R = 1$.
- Then, normalized input admittance B at shunt connection is $y_b = y_a + y_R = 1 + j3.055$.
- Normalized input impedance B at shunt connection is given by $z_b = \frac{1}{1+y_b} = 0.097 - j0.296$, now rotate from point z_b towards the generator (clockwise) by $d = 0.2\lambda$ until we reach the generator.
- Normalized impedance at the generator will be $z(l) = 0.264 + j1.408$.
- Impedance at the generator is $Z(l) = 50 \cdot z(l) = 13.2 + j70.4\,[\Omega]$.
- Normalized admittance at the generator is $y(l) = 1/z(l) = 0.129 - j0.687$.
- Finally, admittance at the generator $Y(l) = \frac{1}{50} \cdot y(l) = 0.0026 - j0.014\,[\text{S}]$

4. (5 points) The two short-terminated T.L. stubs connected in parallel is shown in the following figure. Using the Smith chart we find:



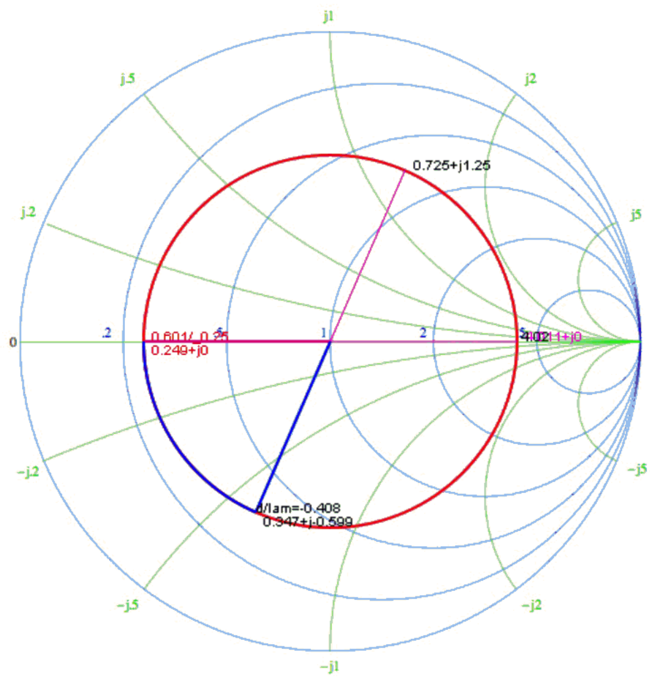
- Normalized impedance at the load $z(0) = 0$
- Normalized admittance at the load $y(0) = \infty$
- Normalized admittance at $z = l$ (connection point) $y(l) = j0.326$
- Admittance of the first stub $Y_1 = \frac{1}{Z_{01}} y(l) = j6.52\times10^{-3}$ [S]
- Admittance of the second stub $Y_2 = \frac{1}{Z_{02}} y(l) = j3.26\times10^{-3}$ [S]
- Admittance of the combined network $Y_{in} = Y_1 + Y_2 = j9.78 \times 10^{-3}$ [S].

5. (a) (2 points) According to the definition,

$$VSWR = \frac{8.4}{2.1} = 4$$

(b) (2 points) You should remember the standing wave pattern along a TL has a period of half wavelength. Then d can be reduced by as many half-wavelength as possible. We will have: $d_{min} = 0.092\lambda$.

(c) (3 points) First, we should determine the constant Γ circle. We can label two points $z(d_{max}) = VSWR + j0$ and $z(d_{min}) = \frac{1}{VSWR} + j0$ as A and B, respectively. By counter-clockwise rotating from the point B to the load by a distance of $d_{min} = 0.092\lambda$, we will reach another point that we label as C. Then we can read from the Smith Chart $\Gamma_L = 0.60\angle\theta_L$, where $\theta_L = -180 + \frac{0.092\lambda}{0.5\lambda} \times 360 = -114$, i..e. $\Gamma_L = 0.60\angle{-114}^\circ$



(d) (2 points) From the SC, we can read $z_L = 0.35 - j0.6$, so $Z_L = 50 \times (0.35 - j0.6) = 17.5 - j30\,[\Omega]$.

(e) (4 points) Another way to interpret VSWR is to consider in terms of constructive/destructive interaction between forward/backward traveling waves. You will realize $|V|_{max} = |V^+| + |V^-|$, $|V|_{min} = |V^+| - |V^-|$. So, $|V^+| = 5.25\,[\text{V}]$, $|V^-| = 3.15\,[\text{V}]$.

(f) (2 points) Instead of calculating the power at the load, we can calculate the power at some point of pure real impedance like: d_{max}. The normalized impedance at d_{max} is $z(d_{max}) = VSWR = 4$, then $Z(d_{max}) = Z_o \cdot z(d_{max}) = 200\,[\Omega]$. Therefore

$$P_{delivered} = \frac{1}{2} Re V I^* = \frac{1}{2} Re\left\{\frac{|V|^2}{Z^*}\right\} = \frac{|V_{max}|^2}{2Z\left(d_{max}\right)} = \frac{8.4^2}{2 \times 200} = 0.1764\,[\text{W}].$$

Since the line is lossless, $P_{delivered}$ is exactly the same power delivered to the load.

6. This problem is very similar to Example 2 of Lecture 37.

(a) We can find the distance d_1 by using the Smith chart.

- First, we find the normalized load impedance $z_L = \frac{Z_L}{Z_0}$ on SC,
- Then, we should draw a circle whose radius will be equal to the distance from the center of SC to z_L
- The point $z_L^{'}$ where the circle intersects the real axis (i.e. where the impedance is purely real),
- Starting at z_L, measure the angle you rotate towards the generator (i.e. clockwise) until you reach $z_L^{'}$,
- This angle will give you the distance d_1 in l-units,
- Finally, the impedance at this point will $Z(d_1) = Z_o \cdot z_L^{'}$.

i. (2 points) Given that $Z_L = 100\,\Omega$, then $z_L = 2$. Since z_L is already on real axis, we do not have to find $z_L^{'}$. Thus, we have $d_1 = 0\lambda$ and $Z(d_1) = Z_L = 100\,\Omega$. The characteristic impedance of the quarter-wave transformer is

$$Z_{qo} = \sqrt{Z_o{\cdot}Z(d_1)} = \sqrt{50{\cdot}100} = 70.71\,[\Omega]$$

ii. (2 points) Given that $Z_L = 100 + j100\,\Omega$, then $z_L = 2 + j2$. Now, we will draw a circle passing through z_L and this circle will intersect the real axis at $z_L^{'} = 4.26$. The angle rotated from z_L to $z_L^{'}$ gives us the distance as $d_1 = 0.0413\lambda$ and $Z(d_1) = Z_o \cdot z_L^{'} = 50 \cdot 4.26 = 213\,[\Omega]$. The characteristic impedance of the quarter-wave transformer is

$$Z_{qo} = \sqrt{Z_o{\cdot}Z(d_1)} = \sqrt{50{\cdot}213} = 103.19\,[\Omega]$$

(b) For $Z_L = 100 + j100\,\Omega$,

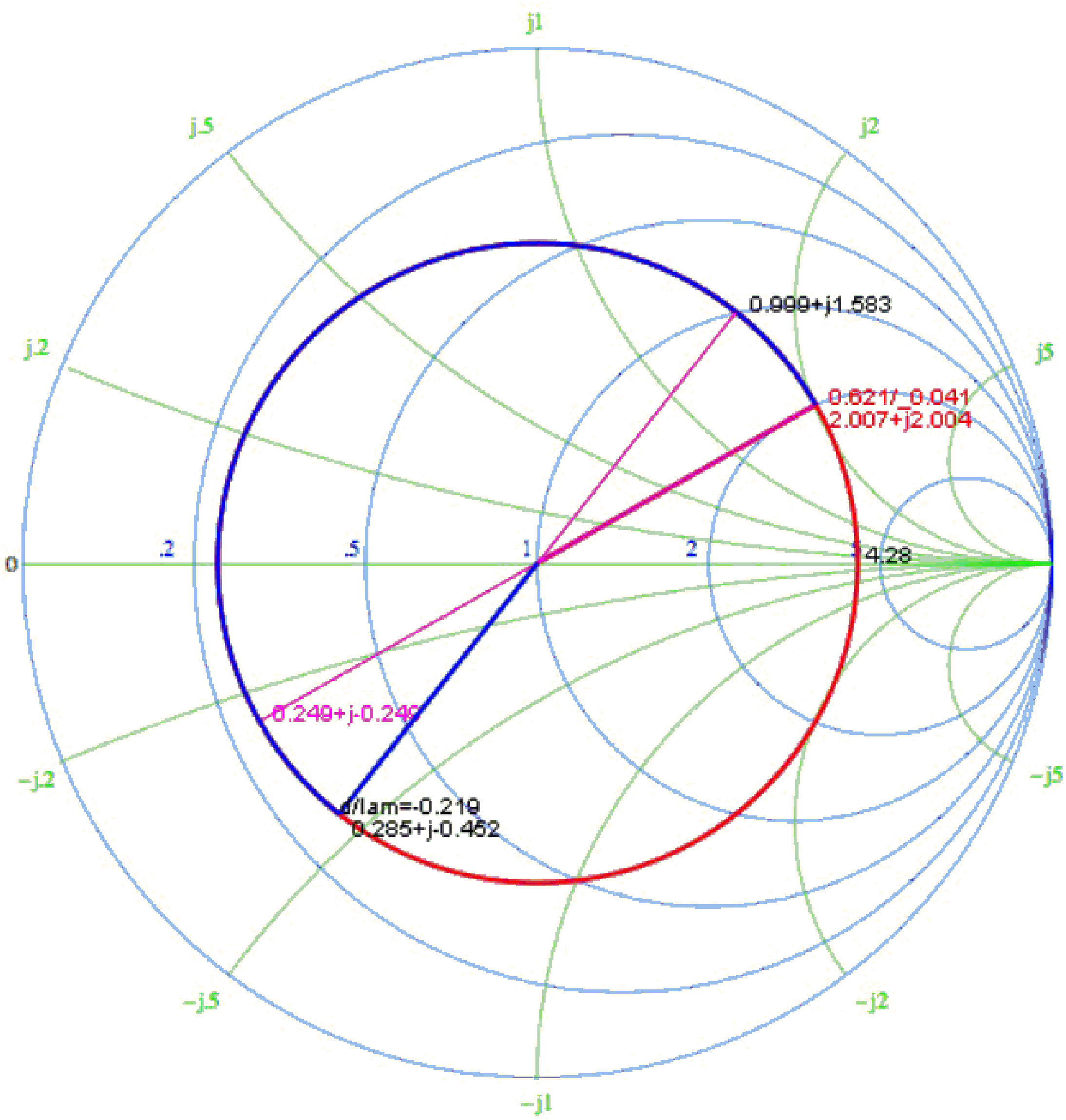
i. (3 points) In the region $0 < d < d_1$, the reflection coefficient is $|\Gamma_L| = \left|\frac{Z_L - Z_0}{Z_L + Z_0}\right| \approx 0.62$. Therefore,

$$VSWR = \frac{1 + |\mathrm{G}_L|}{1 - |\mathrm{G}_L|} = 4.266$$

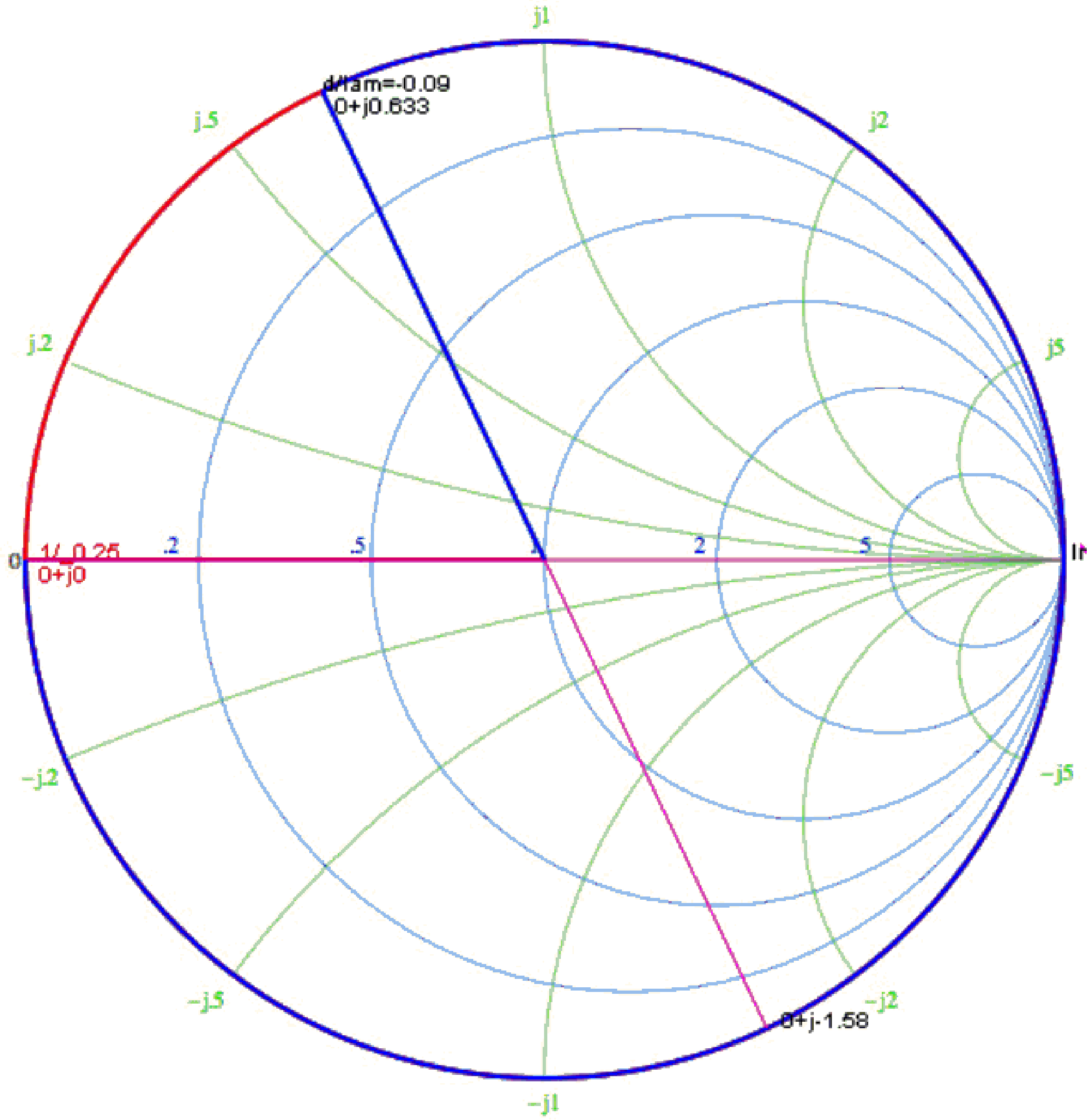
ii. (3 points) In the region $d_1 < d < d_1 + \frac{\lambda}{4}$, the reflection coefficient is $\Gamma(d_1) = \frac{Z(d_1) - Z_{qo}}{Z(d_1) + Z_{qo}} = \frac{213 - 103.19}{213 + 103.19} = 0.348$. Therefore,

$$VSWR = \frac{1 + |\mathrm{G}_L|}{1 - |\mathrm{G}_L|} = 2.065$$

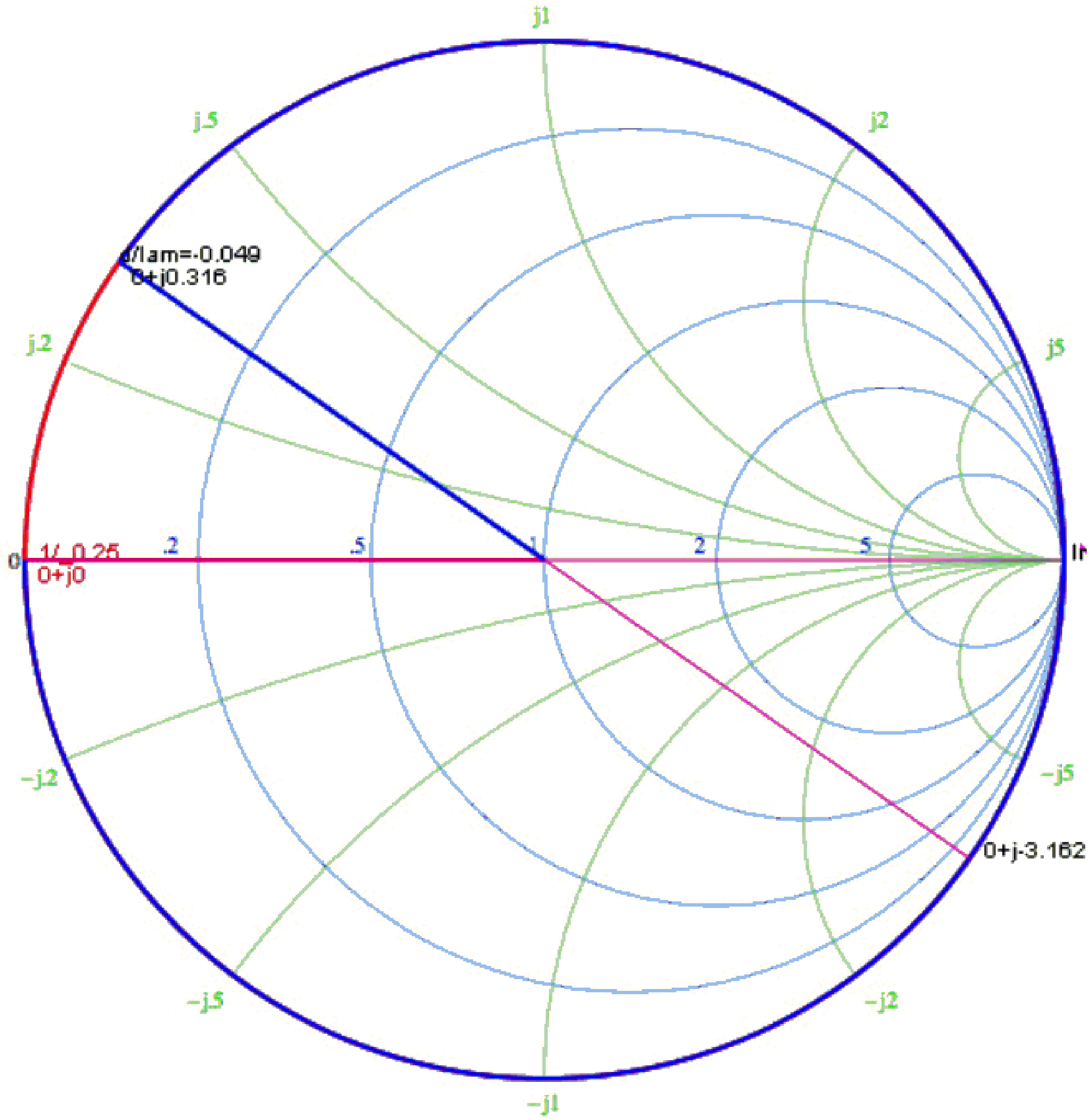
7. (a) (6 points) Referring to Example 3 in course notes Lecture 37, d_1 needs to be selected such that $y(d_1) = 1+jb$, and then the shorted stub which has a purely imaginary impedance will provide $y_s = -jb$, so that $y(d_1) + y_s = 1 + j0$, which means that the impedance is matched at d_1. Then, the normalized load impedance and admittance are $z_L = Z_L/Z_o = 2 + j2$, and $y_L = 1/z_L = 0.25 - j0.25$.
We rotate from point y_L towards the generator (clockwise) until we reach the circle $g = 1$, and the intersection point reads $y(d_1) = 1 + j1.58$. The rotated distance is $d_1 = 0.219\lambda$, and the admittance of the stub is $y_s = 1 + j0 - y(d_1) = -j1.58$.



To determine the length of the stub, we locate the short point $y = \infty$ and rotate towards the generator (clockwise) until we reach the point $y = -j1.58$. The rotation distance is the length of the stub: $d_s = 0.09\lambda$

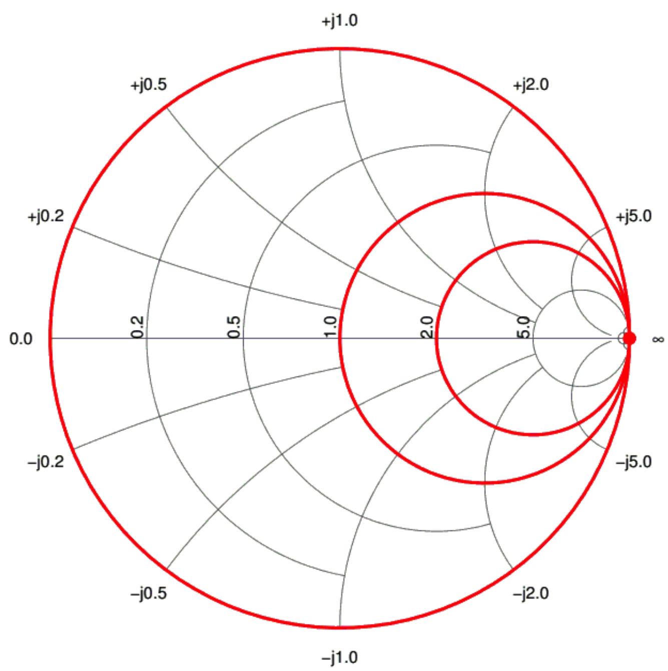


(b) (4 points) The procedure is the same as that in (a), except that now $Z_{os} = 100\,\Omega = 2Z_o$, or $Y_{os} = 0.5Y_o$. For this case, we still find that $y(d_1) = 1 + j1.58$, or $Y(d_1) = \frac{1}{Z_o}(1 + j1.58)$. Thus, the stub admittance should be $Y_s = \frac{1}{Z_0}(-j1.58) = \frac{1}{Z_{OS}}(-j3.16) = -j3.16Y_{os}$, or $y_s = -j3.16$. By rotating from the short point to the ys point, we obtain $d_s = 0.049\lambda$.



8. (10 points) **BONUS**
Your hand-drawn SC should look the same as the SC from course website. Please take your time to try to understand the procedures and think of SC as a graphical mapping tool according to $y = \frac{x-1}{x+1}$ relation, in general.
The red circles in the following figure correspond to normalized impedances $z = \frac{Z}{Z_0} = r + jx$, for fixed values of r. We can fill the following table:

r	center $\left(\frac{r}{r+1}, 0\right)$	radius $\frac{1}{r+1}$
0	$(0, 0)$	1
1	$(0.5, 0)$	0.5
2	$(\frac{2}{3}, 0)$	$\frac{1}{3}$
∞	$(1, 0)$	0



The green circles in the following figure correspond to normalized impedances $z = \frac{Z}{Z_0} = r + jx$, for fixed values of x. We can fill the following table:

x	center $\left(1, \frac{1}{x}\right)$	radius $\left\lvert\frac{1}{x}\right\rvert$
0	$(1, 0)$	∞
± 0.5	$(1, \pm 2)$	2
± 1	$(1, \pm 1)$	1
± 2	$(1, \pm 0.5)$	0.5