ECE 329 Fields and Waves I
Homework 13

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Due April 27, 2023, 11:59 PM

Homework Policy:

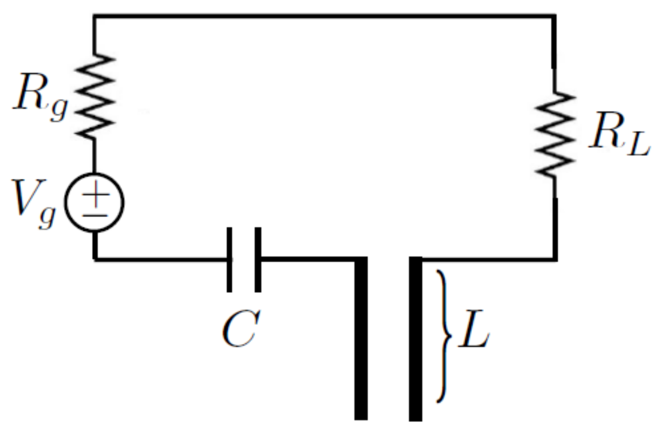
- Write your name and NetID on top of every page. This habit will help you in exams in the event of having loose page(s).
- Tag all the questions in Gradescope. Failure to do so results in a 5 points deduction.
- Cheating results in ZERO and 50% reduction in HW average on first offense. A 100% reduction in HW average on second offense.
- Please show detailed process for each problem instead of just an answer. No partial credits would be given otherwise. All answers should include units wherever appropriate.
- No late HW is accepted.
- Regrade requests are available one week following grade release.

You are allowed to work with anyone else, but the work you submit should only belong to you. Note that if you have knowledge of a violation of the Honor Code, then you are obligated to report it. By submitting this homework, you are agreeing to the Honor Code: "I have neither given nor received unauthorized aid on this homework, nor have I concealed any violations of the Honor Code."

Question	Points	Score
1	10	
2	5	
3	15	
4	6	
5	8	
6	6	
7	10	
Total:	60	

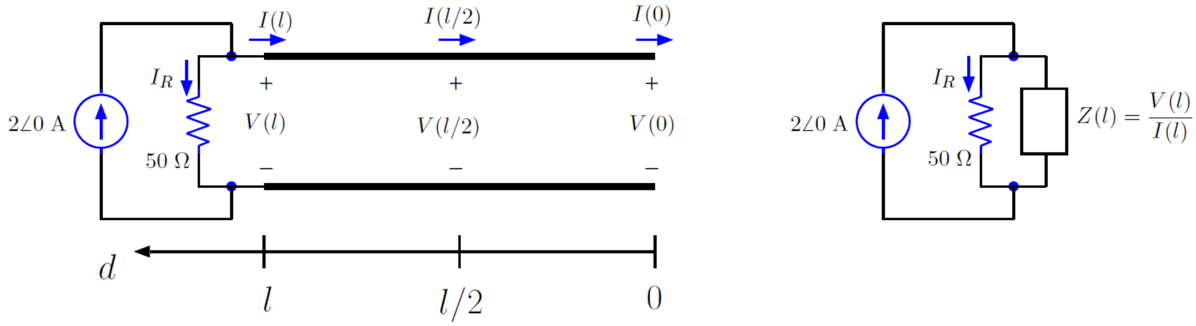
1. Consider a lossless TL which is open circuited at both ends. If $l = 5$ m is the length of the line, and $v = \frac{1}{3}c = 1 \times 10^8$ m/s for the line,
   (a) (4 points) What are all the resonance frequencies of the line — frequencies at which source-free *oscillations* (standing waves) of voltage and current are sustained on the line — expressed in MHz units? **Hint:** to satisfy the "open" boundary conditions at both ends of the line, we want the current phasor to vanish at $z = 0$ and $z = l$ when the oscillation frequency is resonant.
   (b) (3 points) Sketch the shapes of current magnitude $|I(z)|$ vs z for the line corresponding to the *three lowest resonance frequencies.* Label each plot clearly and explain each briefly.
   (c) (3 points) Repeat (b) for voltage magnitude $|V(z)|$ vs z.

2. Consider the circuit shown below, where a resistor, $R_L = 50\ \Omega$, and a capacitor C, are connected in series with an open-circuited transmission line stub with characteristic impedance $Z_0 = R_L$, $v = c$, and length $L = 75$ cm (see diagram).



   (a) (2 points) What is the expression for the input impedance, $Z(L)$, of the open T.L. stub?
   (b) (3 points) The circuit is driven by a voltage source of magnitude V_g at a frequency of 50 MHz, which has internal resistance $R_g = R_L$. If the voltage at the load, V_L, equals $V_g/2$, what is the capacitance C?

3. Consider a transmission line segment of propagation velocity $v = \frac{1}{3}c = 1\times10^8$ m/s, characteristic impedance $Z_o = 50\,\Omega$, and length l. As shown in figure (a) below, the segment is connected *in parallel* with a 50 Ω **resistor** and an ideal **current source** $i(t) = \text{Re}\{Ie^{j\omega t}\}$, where $I = 2\angle 0$ A is the source current phasor and $\omega = \pi \times 10^8$ rad/s; figure (b) depicts an *equivalent circuit* in terms of input impedance of the transmission line at $d = l$, namely $Z(l) \equiv \frac{V(l)}{I(l)}$.



(a) Transmission line circuit (b) Equivalent circui

In answering the following questions assume that the circuit above is in sinusoidal steady-state:

(a) (1 point) What is the signal wavelength λ (in m) on the transmission line?

(b) (1 point) Since an "open termination" is located on the line at $d = 0$, what is the pertinent "boundary condition" involving the phasor $I(0)$ for all possible line lengths l?

(c) (2 points) Does the transmission line support a *standing wave* in the above circuit for all values of non-zero l? Justify your answer.

(d) (2 points) What is the smallest non-zero value of l (in m) if phasor $I_R = 2\angle 0$ A? Explain.

(e) (2 points) For l determined in part (d), what is phasor $V(l/2)$? Explain.

(f) (2 points) For l determined in part (d), is $I(l/2) = 0$ possible? Explain.

(g) (1 point) What is the smallest non-zero value of l if $I_R = 0$?

(h) (2 points) Given that $I(l/2) = 0$, what is the smallest possible value of l? Explain

(i) (2 points) For l determined in part (h), what is $V(l)$? Explain.

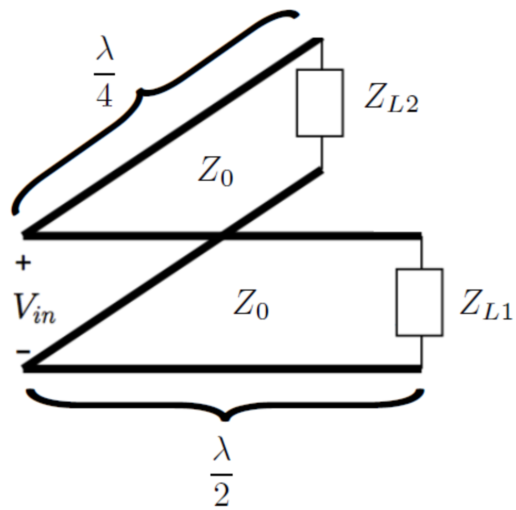
4. A quarter-wavelength long transmission line section having characteristic impedance $Z_o = 100\,\Omega$ is terminated by an unknown impedance Z_L at one end. The input current phasor at the other end is $I_{in} = 0.5\angle 0°$ A. Let V_{in} and V_L denote input and load voltage phasors, respectively, with directions defined in a compatible way with one another and with I_{in}.

(a) (2 points) Assuming that the load is not shorted (i.e., $Y_L = \frac{1}{Z_L} \neq 0$) what is V_L?

(b) (2 points) Is it possible for $I_{in} = 0.5\angle 0°$ A if the load is a short? Discuss your answer.

(c) (2 points) What is V_{in} if $Z_L = 50\,\Omega$?

5. Two transmission line segments each having characteristic impedance $Z_0 = 100\ \Omega$ are connected in parallel and share input voltage $V_{in} = j10$ V. One transmission line has electrical length $l = \frac{\lambda}{2}$ and is terminated by a resistive load $Z_{L1} = 200\ \Omega$, while the other has electrical length $l = \frac{\lambda}{4}$ is terminated by $Z_{L2} = 50\ \Omega$.



(a) (1 point) What is the voltage V_{L1} across Z_{L1}?

(b) (1 point) What is the voltage V_{L2} across Z_{L2}?

(c) (1 point) What is the current I_{L1} through Z_{L1}?

(d) (1 point) What is the current I_{L2} through Z_{L2}?

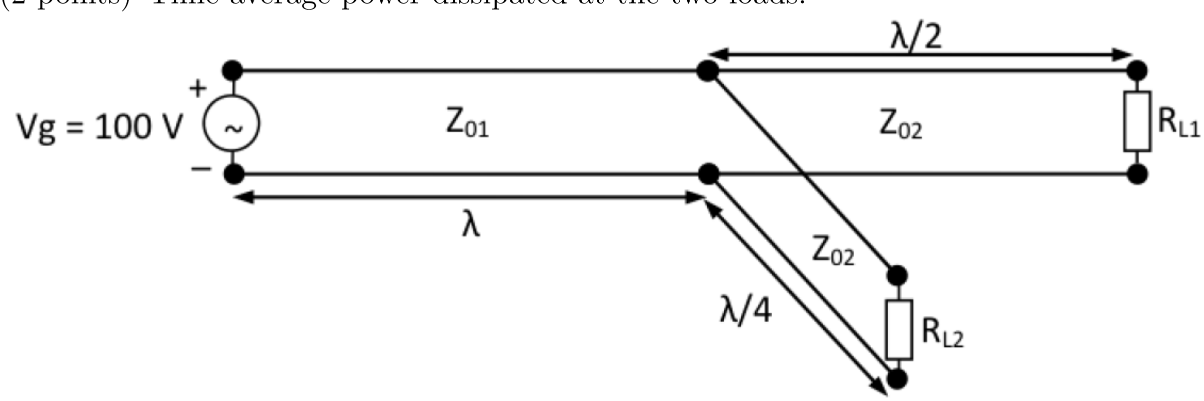
(e) (2 points) What is the input impedance Z_{in} associated with the combined TL network?

(f) (2 points) What is the total time-averaged power absorbed by the two resistive loads?

6. In the transmission-line circuit shown below all lines are lossless, $2Z_{01} = Z_{02} = 100\,\Omega$ and $R_{L1} = R_{L2} = 50\,\Omega$. Calculate the following:

(a) (4 points) Voltage and current phasors at the two loads,

(b) (2 points) Time-average power dissipated at the two loads.



7. **BONUS PROBLEM:** Consider an air filled (i.e., $\epsilon = \epsilon_o$ and $\mu = \mu_o$) parallel-plate transmission line with length $\ell = 1$ m, width $w = 5$ cm, and plate separation $d = 0.5$ cm. The transmission line is shorted at both ends.

   (a) (1 point) Determine the lowest resonant frequency f of the shorted transmission line segment such that the resonant voltage waveform $V(z,t)$ vanishes at both $z = 0$ and $z = \ell$ at all times t. **Hint:** at the lowest resonant frequency ℓ should be half a wavelength long.

   (b) (2 points) Denoting the maximum voltage of the resonant mode as V_o, write the expression of the resonant voltage waveform $V(z,t)$.

   (c) (2 points) Determine the corresponding resonant current waveform $I(z,t)$ by utilizing the Telegrapher's equation
   $$-\frac{\partial V}{\partial z} = \mathcal{L}\frac{\partial I}{\partial t}$$
   and the fact that $\mathcal{L} = \mu_o \frac{d}{w}$.

   (d) (3 points) Given that in the parallel plate transmission line $V = E_x d$ and $I = H_y w$, utilize the $V(z,t)$ and $I(z,t)$ expressions from (b) and (c) to calculate the stored energy of the mode, namely
   $$W = \int_{x=0}^{d}\int_{y=0}^{w}\int_{z=0}^{\ell}(\frac{1}{2}\epsilon_o E_x^2 + \frac{1}{2}\mu_o H_y^2)\, dx\, dy\, dz.$$
   The result should be
   $$W \propto \mathcal{C}\ell V_o^2$$
   where $\mathcal{C} = \epsilon_o \frac{w}{d}$ is the capacitance per unit length of the transmission line. You are asked to find the numerical value of the proportionality constant in this relation. **Hint:** the result will be independent of time t because $\sin^2 + \cos^2 = 1$.

   (e) (2 points) According to *statistical mechanics*, if a slightly lossy transmission line is in *thermal equilibrium* at some temperature T, then the *expected value* of W of each resonant mode should be $k_B T$, where $k_B = 1.38 \times 10^{-23}$ J/K is Boltzmann's constant.[1]Assuming that $T = 300$ K, calculate the square root of the expected value of V_o^2, i.e., the rms value of voltage V_o for such a transmission line. **Hint:** if $W = AV_o^2$, where A is some constant, then the expected value of V_o^2 is the expected value of W divided by A (as you should have learned in ECE 313).

[1] Lossy transmission lines resonate just like lossless lines we are studying in class, except that resonant oscillations are damped out at some rate and dissipated as heat — i.e., become random motions of charge carriers (colliding with one another) in the transmission line wires. However, because random charge carrier motions are in effect random currents that regenerate the resonant modes (via radiation), a transmission line in thermal equilibrium (i.e., with a fixed temperature) is found to have its resonances excited with randomly varying rms amplitudes (and variances proportional to temperature) as examined in this problem. Each resonant mode can be thought of as being analogous to a randomly moving and colliding molecule of a volume of gas at a given temperature — all molecules (as all resonances) will have randomly varying energies with well defined mean values.