ECE 329 Fields and Waves I
Homework 11

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Due April 13, 2023, 11:59 PM

Homework Policy:

- Write your name and NetID on top of every page. This habit will help you in exams in the event of having loose page(s).
- Tag all the questions in Gradescope. Failure to do so results in a 5 points deduction.
- Cheating results in ZERO and 50% reduction in HW average on first offense. A 100% reduction in HW average on second offense.
- Please show detailed process for each problem instead of just an answer. No partial credits would be given otherwise. All answers should include units wherever appropriate.
- No late HW is accepted.
- Regrade requests are available one week following grade release.

You are allowed to work with anyone else, but the work you submit should only belong to you. Note that if you have knowledge of a violation of the Honor Code, then you are obligated to report it. By submitting this homework, you are agreeing to the Honor Code: "I have neither given nor received unauthorized aid on this homework, nor have I concealed any violations of the Honor Code."

Question	Points	Score
1	5	
2	15	
3	20	
4	10	
5	10	
Total:	60	

1. (5 points) Twin-lead transmission lines (TLs) are commonly used to connect TV sets and FM radios to their receiving antennas. For the twin-lead, the geometrical factor is

$$\text{GF} = \frac{\pi}{\cosh^{-1}\frac{D}{2a}},$$

where $2a$ is the diameter of each wire (cylindrical conductor) of the twin lead, and D is the distance between the centers of the wires. Assuming $\epsilon = \epsilon_o$, $\mu = \mu_o$, and $2a = 1$ mm, calculate D for twin lead TL's having: (a) $Z_o = 50\,\Omega$ and (b) $Z_o = 300\,\Omega$.

2. Two TLs with characteristic impedances Z_1 and Z_2 are joined at a junction located at $z = 0$ that also includes a series resistance R as shown in the figure below. A source at $z = -\infty$ launches the signal represented by V^+ that travels in the + z-direction and encounters the junction at $t = 0$.

   (a) (2 points) Write the pertinant KVL and KCL equations at the junction that relate V^+, V^-, and V^{++} at time $t = 0$.

   (b) (4 points) Solve the KVL and KCL equations to obtain the reflection and transmission coefficients $\Gamma_{12} \equiv \frac{V^-}{V^+}$ and $\tau_{12} \equiv \frac{V^{++}}{V^+}$ for the junction.

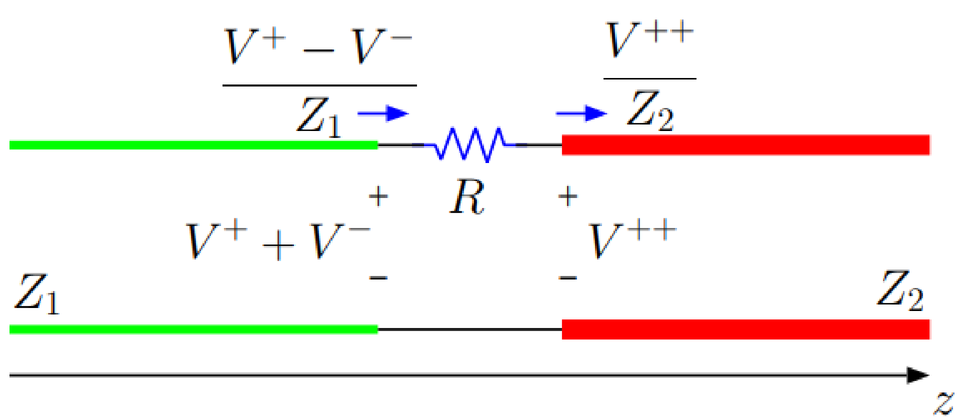
   (c) (3 points) Write the D'Alembert traveling waves solutions of the voltage and current for all z and $t > 0$.

   (d) (2 points) Calculate Γ_{12} and τ_{12} for $Z_1 = 50\ \Omega$, $Z_2 = 100\ \Omega$ and $R = 200\ \Omega$.

   (e) (4 points) Using the values for Γ_{12} and τ_{12} plot the voltage $V(t = 10\text{ns}, z)$. The voltage $V(t = 0^-, z) = u(z + 2) - u(z)$ is the voltage when the V^+ waveform first reaches the boundary $z = 0$, where

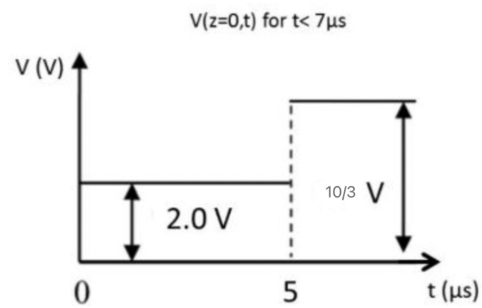
$$u(z) = \begin{cases} 1 & z > 0 \\ 0 & z \le 0 \end{cases}$$

   and the propagation speeds are $v_1 = \frac{c}{3} = 1 \times 10^8\ \frac{\text{m}}{\text{s}}$ and $v_2 = \frac{2c}{3} = 2 \times 10^8\ \frac{\text{m}}{\text{s}}$.



3. Consider a TL with characteristic impedance $Z_o = 50\,\Omega$, length $l = 100$ m, and propagation velocity $v = \dfrac{1}{\sqrt{\mathcal{LC}}} = \dfrac{c}{6} = 0.5 \times 10^8$ m/s. A voltage source $f(t)$ with an internal resistance $R_g = Z_o$ is connected to one end of the TL (at $z = 0$) and the other end ($z = l$) is terminated by a load resistance $R_L = 4Z_o$.
   - (a) (11 points) Construct a "bounce diagram" to determine the voltage, $V(z,t)$, and current, $I(z,t)$, variations on the line for $0 < z < l$ and $t > 0$ for $f(t) = \delta(t)$.
   - (b) (2 points) Write your expressions for $V(\frac{l}{2}, t)$ and $I(\frac{l}{2}, t)$ as weighted sums of appropriately delayed impulses $\delta(t)$.
   - (c) (2 points) Write your expressions for $V(z, 5\mu s)$ and $I(z, 5\mu s)$.
   - (d) (2 points) Plot $V(\frac{l}{2}, t)$ as a function of t for $0 < t < 6\,\mu$s if $f(t) = 60u(t)$ V. — **Hint:** use the convolution of the result of part (b) with $60u(t)$.
   - (e) (2 points) What is the steady state value $V(\frac{l}{2}, \infty)$ for $f(t) = 60u(t)$ V? Thinking of the transmission line as a distributed circuit, explain why this is the case.
   - (f) (1 point) Suppose that a segment of transmission line was added at $z = 100$m with impedance Z_1 of arbitrary length terminated with a load $R_L = Z_1$ so there is no reflection at the termination of the new transmission line segment. What is the value of Z_1 so that the reflection coefficient at $z = 100$m is zero?

4. Time Domain Reflectometry (TDR) is a method by which a defect in a transmission line can be located by injecting a signal at one end of the line and monitoring the reflections that come back. The type of defect and the distance of the defect from the source can be found assuming that the correct response is known. Assume the lossless transmission line to be tested is 2 km long with propagation speed $v = 2 \times 10^8$ m/s. Assume the tested line is terminated by a 100W resistor, given that the source is driven by a voltage source $V(t) = 6u(t)\,$V, in steady-state, $V(z = 0, t = \infty) = 3\,$V and $I(z = 0, t = \infty) = 30\,$mA.
   - (a) (2 points) Find the source impedance R_g.
   - (b) (2 points) If the defect is a short, find $V(z = 0, t = \infty)$ and $I(z = 0, t = \infty)$.
   - (c) (2 points) If the defect is an open, find $V(z = 0, t = \infty)$ and $I(z = 0, t = \infty)$.
   - (d) (4 points) If your test equipment sends out a voltage signal $V(z = 0, t) = 3u(t)\,$V, and receives the voltage at the input as shown in the figure below find the impendance of the transmission line, the location of the defect, and indicate if the damage is a short or an open circuit.



5. (10 points) **Bonus question:** Telegrapher's equations

$$
\begin{aligned}
-\frac{\partial V}{\partial z} &= \mathcal{L}\frac{\partial I}{\partial t} \\
-\frac{\partial I}{\partial z} &= \mathcal{C}\frac{\partial V}{\partial t}
\end{aligned}
$$

govern the voltage and current waves $V(z,t)$ and $I(z,t)$ that propagate on transmission line systems.

If $V(z,t) = 4\cos(\omega t + \beta z)$ on a TL, determine $I(z,t)$ and β (a positive number) by using the telegrapher's equations twice.

Hint: First use one of the telegrapher's equations to determine $I(z,t)$. Then use the other telegrapher's equation to determine $V(z,t)$ from $I(z,t)$ found in the first step. By requiring $V(z,t)$ found in step 2 to equal the original $V(z,t)$ you should be able to identify β.