ECE 329 Fields and Waves I
Homework 10

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Due April 6, 2023, 11:59 PM

Homework Policy:

- Write your name and NetID on top of every page. This habit will help you in exams in the event of having loose page(s).
- Tag all the questions in Gradescope. Failure to do so results in a 5 points deduction.
- Cheating results in ZERO and 50% reduction in HW average on first offense. A 100% reduction in HW average on second offense.
- Please show detailed process for each problem instead of just an answer. No partial credits would be given otherwise. All answers should include units wherever appropriate.
- No late HW is accepted.
- Regrade requests are available one week following grade release.

You are allowed to work with anyone else, but the work you submit should only belong to you. Note that if you have knowledge of a violation of the Honor Code, then you are obligated to report it. By submitting this homework, you are agreeing to the Honor Code: "I have neither given nor received unauthorized aid on this homework, nor have I concealed any violations of the Honor Code."

Question	Points	Score
1	15	
2	5	
3	10	
4	5	
5	10	
6	10	
7	15	
Total:	70	

1. For each of the five plane TEM waves (in free space where $\eta_o \approx 120\pi\,\Omega$) described by
   (a) (3 points) $\mathbf{E}_1 = \cos(\omega t - \beta z + \frac{\pi}{4})\hat{x}\,\text{V/m}$
   (b) (3 points) $\mathbf{H}_2 = 2\cos(\omega t + \beta z + \frac{\pi}{3})\hat{x} + 2\sin(\omega t + \beta z + \frac{5\pi}{6})\hat{y}\,\text{A/m}$
   (c) (3 points) $\mathbf{E_3} = 4\cos(\omega t - \beta x)\hat{z} - 3\sin(\omega t - \beta x)\hat{y}\,\text{V/m}$
   (d) (3 points) $\mathbf{E}_4 = 5\cos(\omega t - \beta y)\hat{x} + 5\sin(\omega t - \beta y)\hat{z}\ \text{V/m}$
   (e) (3 points) $\mathbf{H}_5 = 2\sin(\omega t + \beta y)\hat{x} - 2\sin(\omega t + \beta y - \frac{\pi}{4})\hat{z}\,\text{A/m}$

   Determine the following: (i) the phasor electric field $\tilde{\mathbf{E}}$ and phasor magnetic field $\tilde{\mathbf{H}}$, and (ii) the wave polarization, using the following categories: linear polarized (state the polarization direction), circular polarized (state right- or left-circular), or neither (elliptical polarized).

2. (5 points) Assume that a monochromatic wave propagates in the z-direction with $\tilde{\mathbf{E}} = E_o \exp(-\gamma z)\hat{x}$ where $\gamma^2 = \omega^2\mu\epsilon(1 - j\frac{\sigma}{\omega\epsilon})$. Assuming the material through which the wave propagates has material parameters ϵ, μ, and σ, use the phasor form of Faraday's Law $\nabla \times \tilde{\mathbf{E}} = -j\omega\mu\tilde{\mathbf{H}}$ to derive $\tilde{\mathbf{H}}$. If the relationship between $\tilde{\mathbf{E}}$ and $\tilde{\mathbf{H}}$ is still $\left|\tilde{\mathbf{E}}\right| = |\eta| \left|\tilde{\mathbf{H}}\right|$ what is $|\eta|$?

3. A plane TEM wave is propagating in a meduim with $\epsilon = \epsilon_o$, $\mu = \mu_o$, the free space values, and a finite conductivity σ. Let's examine the propagation characteristics of a wave propagating a good conductor. In general, $\gamma = \sqrt{j\omega\mu(j\omega\epsilon + \sigma)} = \alpha + j\beta$. The assumption that we make in a good conductor is that $\dfrac{\sigma}{\omega\epsilon} \gg 1$.
   (a) (3 points) Using the assumption that $\dfrac{\sigma}{\omega\epsilon} \gg 1$ show that this approximation leads to the relation $\alpha \approx \beta$.
   (b) (3 points) If the conductivity $\sigma = 0.1\,\text{S}$ and the frequency $f = 20\,\text{MHz}$, find the characteristic associated with the propagating part of the solution, namely, β, λ, and v_p (the phase velocity).
   (c) (1 point) Find the complex impedance η.
   (d) (2 points) Find the electric and magnetic field intensity phasors in the region $z > 0$ if the source current at $z = 0$ is $\tilde{\mathbf{J}} = e^{j\pi}\hat{x}$.
   (e) (1 point) Find the location z where the peak amplitude of the electric field is $1/e$ of the value at $z = 0$.

4. Consider the following right-handed circularly polarized electric field propagating in free space:

$$\mathbf{E} = A\cos(\omega t - \beta x)\hat{y} + B\sin(\omega t - \beta x)\hat{z}\ \text{V/m}$$

   (a) (3 points) Determine the time-averaged Poynting vector assuming that $A = B = \dfrac{1}{\sqrt{2}}$ V/m.
   (b) (1 point) Repeat part (a) if $A = 1$ V/m and $B = 0$.
   (c) (1 point) From the results of (a) and (b), what do you conclude about the relative power densities of linear and circular polarized TEM waves having equal instantaneous peak electric field magnitudes?

5. (10 points) A TEM plane wave (with the **E** component given by $\boldsymbol{E}(z,t) = A_1 \cos(\omega t - \beta_1 z)\hat{x}$) propagates along the $+\hat{z}$ direction toward the $z = 0$ plane. In the region $z < 0$ (region 1), the space is filled with a perfect dieletric with material constants of μ_1, and ϵ_1. In the region $z > 0$ (region 2), the space is filled with perfect dieletric with material constants of μ_2, and ϵ_2. The two mediums interface at the $z = 0$ plane. Use the boundary condition equations for both **E** and **H** fields to derive the expression for the reflected TEM wave at the interface in terms of A_1, ω, μ_1, ϵ_1, μ_2, and ϵ_2.

6. A monochromatic plane wave propagating in a vacuum in the region $x < 0$ has an electric field phasor given by

$$\tilde{\mathbf{E}}_{\mathbf{i}} = (j\hat{z} - \hat{y})e^{-j2\pi x} \text{ V/m}$$

The wave encounters a planar boundary at $x = 0$ which separates the vacuum from a perfect dielectric material in the region $x > 0$ which has magnetic permeability $\mu = \mu_0$ and electric permittivity $\epsilon > \epsilon_0$. The electric field phasor of the reflected wave in the $x < 0$ region is given by

$$\tilde{\mathbf{E}}_{\mathbf{r}} = -\frac{1}{2}(j\hat{z} - \hat{y})e^{j2\pi x} \text{ V/m}$$

(a) (2 points) What are the polarizations of the incident and reflected waves?

(b) (2 points) What is the linear frequency f of the wave?

(c) (2 points) What is the permittivity ϵ of the dielectric material?

(d) (2 points) Write the phasor expression for the electric field waveform that is transmitted into the dielectric medium.

(e) (2 points) What percentage of the time-averaged incident power per unit area is transmitted into the dielectric medium?

7. **Bonus question:** Parts (a), (b), and (c) are unrelated.

(a) (5 points) Consider a hollow, longitudinally symmetric, metallic tube of arbitrary cross-section. Prove that such a configuration does not allow for the propagation of TEM waves. **Hint:** Consider a transverse slice, Laplace's equation, and the electrostatic field.

(b) (5 points) Any pair of two plain copper wires can technically be called a transmission line as long as the cable length is longer than the wavelength. What are two problems with using plain wires as transmission lines? What advantage(s) does coax offer? **Hint:** Reflection is usually unwanted.

(c) (5 points) Please go to https://tinyurl.com/t83wab97 to fill out the informal early feedback form. Please take a screenshot that looks like the following picture and submit the screenshot as the solution.