# 37 Smith Chart and impedance matching

- In lossless TL circuits the average power input $P_{in}$ at the generator end precisely matches the average power delivered to the load, $P_L$.

  In fact, $P_{in}$ and $P_L$ also match the average power $P(d)$ transported on the line at an arbitrary $d$.

- We have in general

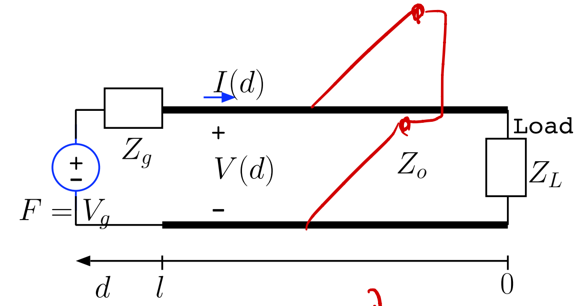
$$\begin{aligned} P(d) &= \frac{1}{2}\mathrm{Re}\{V(d)I^*(d)\} \\ &= \frac{1}{2}\mathrm{Re}\{(V^+e^{j\beta d} + V^-e^{-j\beta d})(\frac{V^+e^{j\beta d} - V^-e^{-j\beta d}}{Z_o})^*\} \\ &= \frac{1}{2}\mathrm{Re}\{\frac{|V^+|^2}{Z_o} - \frac{|V^-|^2}{Z_o} + \frac{V^-V^{+*}e^{-j2\beta d} - (V^-V^{+*}e^{-j2\beta d})^*}{Z_o}\} \\ &= \frac{|V^+|^2}{2Z_o} - \frac{|V^-|^2}{2Z_o}. \end{aligned}$$

$\frac{1}{2}\mathrm{Re}\{V^+ \cdot I^{+*}\}$

  - Note that $P(d)$ is the difference of power transported
    $\frac{|V^+|^2}{2Z_o}$ **toward the load** by the "forward-going" wave, and
    $\frac{|V^-|^2}{2Z_o}$ **toward the generator** by the reflected wave.
  - Also note that

$$P(d) = \frac{|V^+|^2}{2Z_o} - \frac{|V^-|^2}{2Z_o} = \frac{|V^+|^2}{2Z_o}(1 - |\Gamma_L|^2)$$

so that $|\Gamma_L|^2$ is an effective **power reflection coefficient**.




$Z_L = R_L \rightarrow \frac{\lambda}{4}$ transforma

$Z_L = R_L + jX_L \rightarrow$ single stub

**Power tx'ed toward the load:**

$$\frac{|V^+|^2}{2Z_o}.$$

**Power tx'ed toward the generator:**

$$\frac{|V^-|^2}{2Z_o}.$$

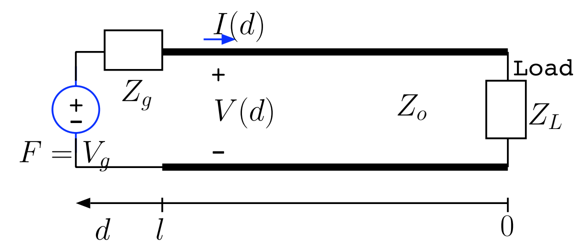
**Power reflection coefficient:**

$$|\Gamma_L|^2.$$

**Power transmission coeff.:**

$$1 - |\Gamma_L|^2.$$

- In TL circuits with load impedances $Z_L$ **unmatched** to the characteristic impedance $Z_o$, the **reflected power**




$$\frac{|V^+|^2}{2Z_o}|\Gamma_L|^2$$

will be non-zero and the VSWR>1.

**This a condition not favored by practical signal generators used in TL circuits.**

- Most generators are *designed* (in their biasing arrangements) to operate in circuits with low VSWR (close to unity), requiring $Z_{in}$ closely matched to $R_g$, most frequently $50\,\Omega$, an optimal characteristic impedance value for coax-lines (when line losses are taken into account).

- Thus a standard procedure is to use TL's with $Z_o = R_g$, and utilize a *lossless* **impedance matching network** on the TL if the load impedance $Z_L \neq Z_o$.

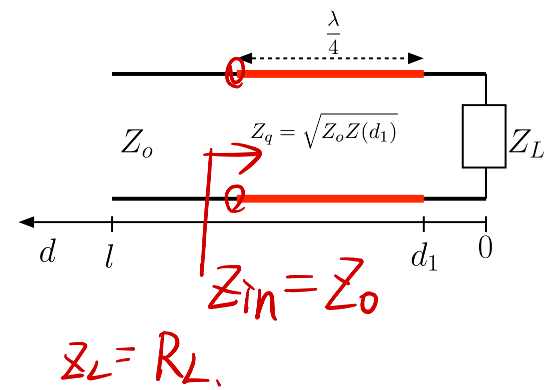
  - This practice is called **impedance matching**.

Impedance matching achieves VSWR=1 between the generator and the matching network inserted at a location between the load and the generator.

- The inserted network should be designed to yield an input impedance equal $Z_o$ at its input terminals.

**The following examples illustrate different ways of achieving an impedance match.**

Quarter-wave matching




**Example 1:** ***Quarter-wave matching*** **of resistive loads:**

Consider a TL with $Z_L = 25\,\Omega$ and $R_g = Z_o = 50\,\Omega$. Since $Z_L \neq Z_o$ the load is unmatched and the VSWR>1.

To reduce the VSWR on the line connected to the generator to unity, we can insert a **quarter-wave transformer** right after $Z_L$ — i.e., at $d_1 = 0$ in the circuit shown in the margin — with a characteristic impedance

$$Z_q = \sqrt{25 \times 50} = \sqrt{1250} = 35.35\,\Omega.$$

$$= \sqrt{Z_{in} \cdot Z_L} = \sqrt{Z_o \cdot Z_L}$$

The impedance at the input terminals of the quarter-wave transformer (on the left) is then $Z_o$, i.e., 50 $\Omega$, implying a perfect impedance match.

- Quarter-wave matching illustrated above is a very commonly used matching technique.

- It is a straightforward application of the quarter-wave transformer impedance formula

$$\boxed{Z_{in} = \frac{Z_q^2}{Z_L}}$$

$$Z_q = \sqrt{Z_{in} \cdot Z_L} = \sqrt{Z_o \cdot Z_L}.$$

for a transformer with characteristic impedance $Z_q$.

1. select $d_1$ such that $z(d_1)$ is real.

2. design $Z_q = \sqrt{Z_o z(d_1)}$

**Example 2:** ***Quarter-wave matching*** **of reactive loads:**

Consider a TL with $Z_L = 50 + j50\,\Omega$ and $R_g = Z_o = 50\,\Omega$. Since $Z_L \neq Z_o$ the load is unmatched and the VSWR>1.

$z_L = 1 + j1$

We cannot insert the quarter-wave transformer right after the load because then we would need a complex valued $Z_q$ implying a lossy matching network.

Instead, we insert a **quarter wave transformer** a distance $d_1$ to the left of $Z_L$, where $d_1$ is selected, using a SC, to have a purely resistive $Z(d_1)$. In that case, the quarter-wave transformer impedance formula

$$Z_q = \sqrt{Z(d_1) \times 50}$$

yields a real valued $Z_q$ as needed. This procedure leads to having $d_1 = d_{max}$ or $d_1 = d_{min}$ corresponding to the positions of voltage maxima and minima on the line.

As shown in the margin,

$$Z(d_1) = 50(2.62 + j0) = 131\,\Omega.$$

for

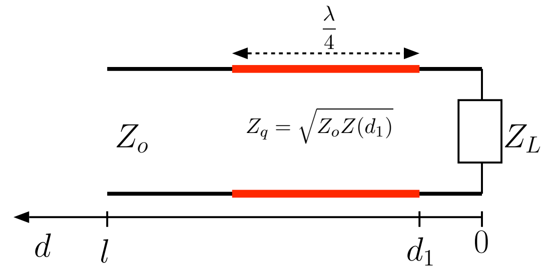
$$d_1 \approx 0.250\lambda - 0.162\lambda = 0.088\lambda$$

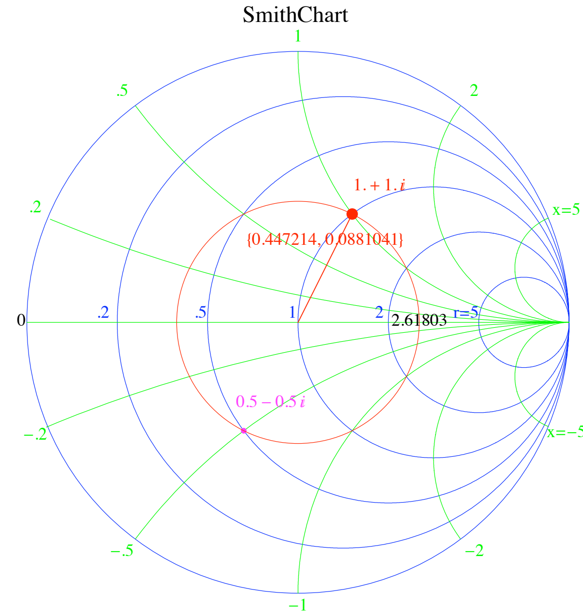
is a suitable choice for quarter-wave matching. In that case we need

$$Z_q = \sqrt{131 \times 50} = 50 \times \sqrt{2.62}\,\Omega$$

for the quarter wave transformer in order match to load to a line with $Z_o = 50\,\Omega$.

Quarter-wave matching



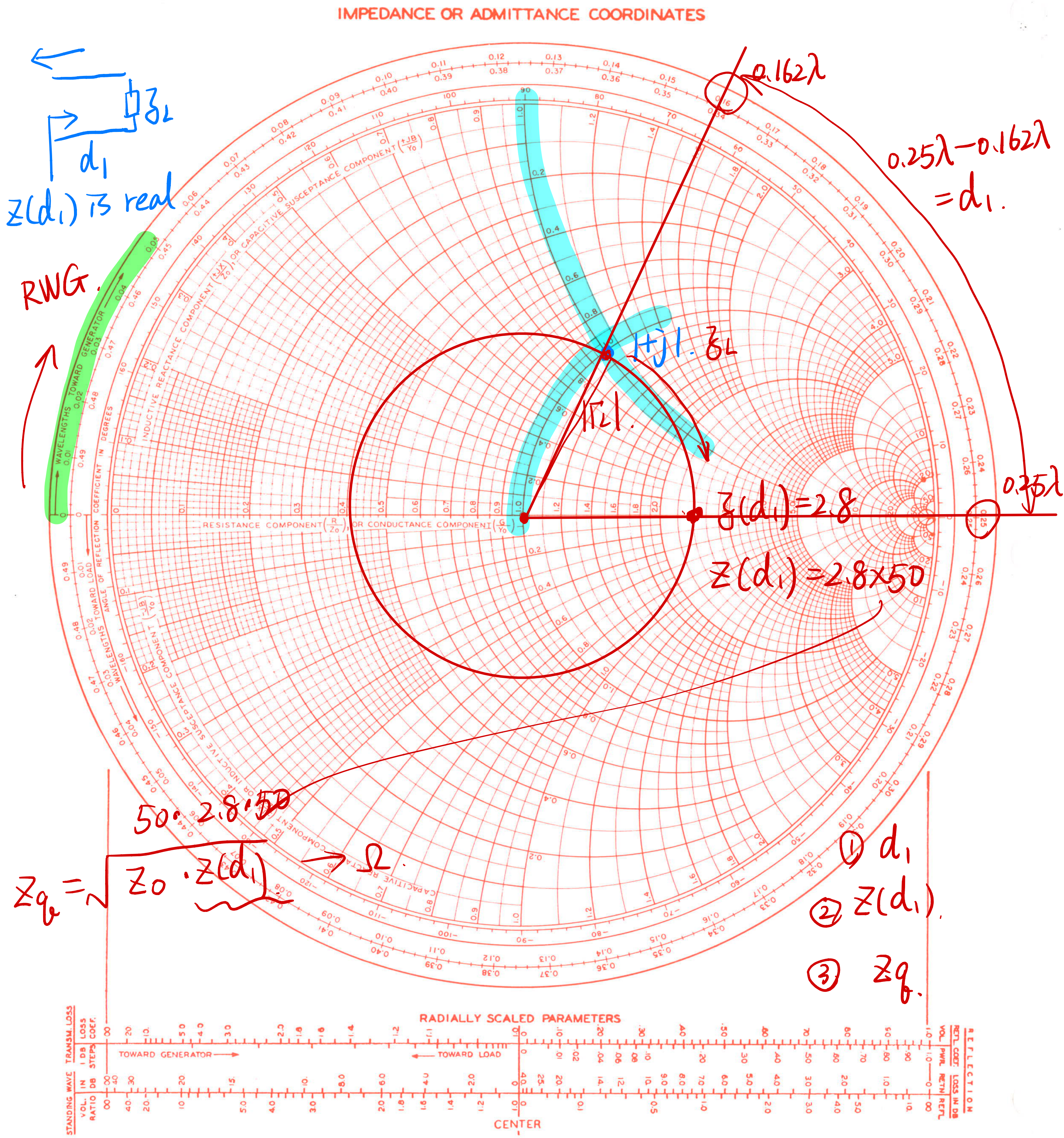

**Note that:**

$$z(d_1) = z(d_{max}) = \textbf{VSWR} \approx 2.62$$

**as marked on the SC.**
**Also**

$$d_{max} \approx 0.088\lambda$$

**since, as marked on the SC, the angle of** $\Gamma_L$ **is** $0.088\lambda$.


IMPEDANCE OR ADMITTANCE COORDINATES
RADIALLY SCALED PARAMETERS
TOWARD GENERATOR
TOWARD LOAD
CENTER
RESISTANCE COMPONENT (R/Zo), OR CONDUCTANCE COMPONENT (G/Yo)
WAVELENGTHS TOWARD GENERATOR
WAVELENGTHS TOWARD LOAD
ANGLE OF REFLECTION COEFFICIENT IN DEGREES
d1
Z(d1) is real
RWG.
0.162λ
0.25λ−0.162λ = d1.
0.25λ
1+j1. zL
|ΓL|.
z(d1)=2.8
Z(d1)=2.8×50
50·2.8·50
Zq = √(Zo·Z(d1)) → Ω.
① d1
② Z(d1).
③ Zq.

RWG

$Z_L = R_L + jX_L$.

$y(d_1)$. $\leftarrow d_1 \rightarrow$

① Normalized admittance $y_L$.

$z_L = \frac{Z_L}{Z_0}$, $y_L = \frac{1}{z_L}$.

② RWG in clockwise direction. such $y(d_1) = 1 + jb$.

**Example 3:** ***Single-stub tuning:***

Consider a TL with $Z_L = 100 - j50\,\Omega$ and $R_g = Z_o = 50\,\Omega$. Since $Z_L \neq Z_o$ the load is unmatched and the VSWR>1.

We will insert a **shorted-stub** a distance $d_1$ to the left of $Z_L$ in parallel with the line to achieve an impedance match.

Distance $d_1$ will be selected, using a SC, to have a normalized admittance of

$$y(d_1) = 1 + jb$$

so that a stub, with a normalized input admittance

$$y_{stub} = -jb,$$

can be added in parallel to have a combined admittance of

$$y(d_1) + y_{stub} = 1 + j0$$

and achieve a perfect impedance match (i.e., VSWR=1).

$y_L = \infty$.

RWG with $l_s$

such that

$y_{stub} = -jb$.

$y_{stub}$ $\leftarrow l_s \rightarrow$

$y(d_1) + y_{stub} = 1$.

In specific

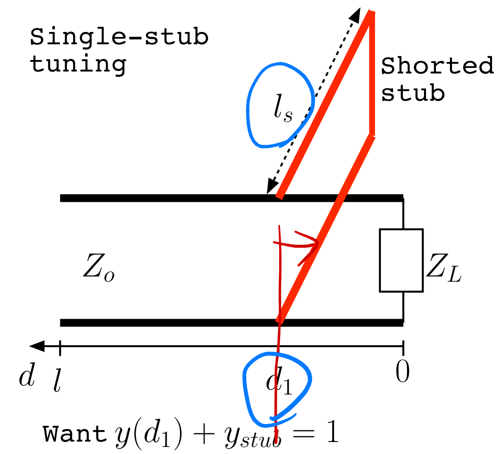
$$z_L = \frac{Z_L}{Z_o} = 2 - j1 \;\text{ and }\; y_L = \frac{1}{z_L} = 0.4 + j0.2$$

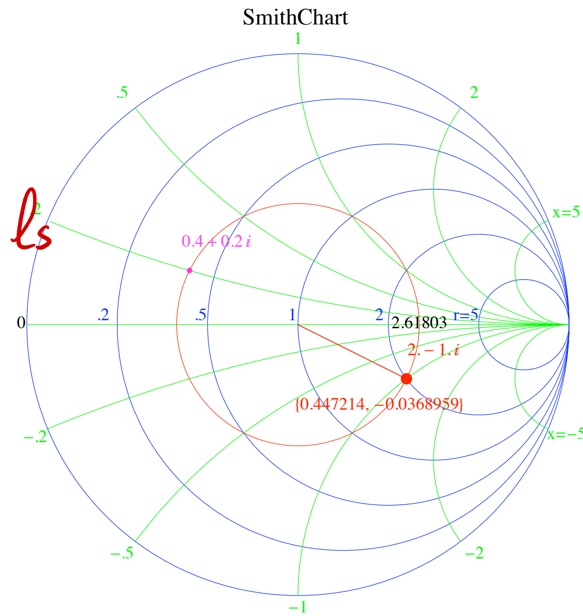
as shown on the SC on the top in the margin. We rotate clockwise on the SC by an amount corresponding to $d_1$ to obtain

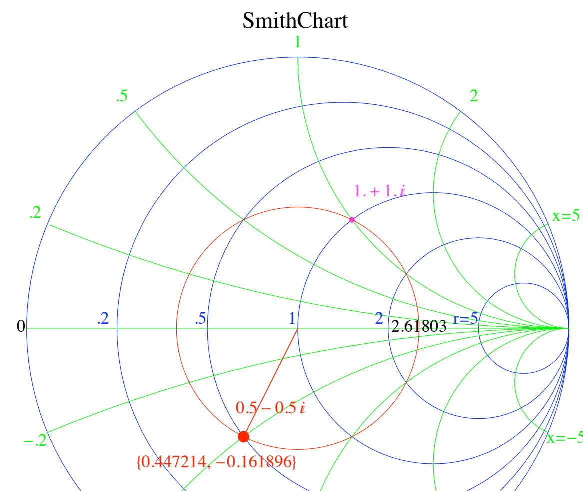
$$y(d_1) = 1 + j1$$

on the "$g = 1$" or "$y = 1 + jb$" circle as shown in the bottom SC. From the amount of rotation we determine

$$d_1 \approx 0.162\lambda - 0.037\lambda = 0.125\lambda.$$



$z_L = \frac{100-j50}{50} = 2-j1$

$y_L \to y(d_1) = 1+jb$

$r=1$ circle.

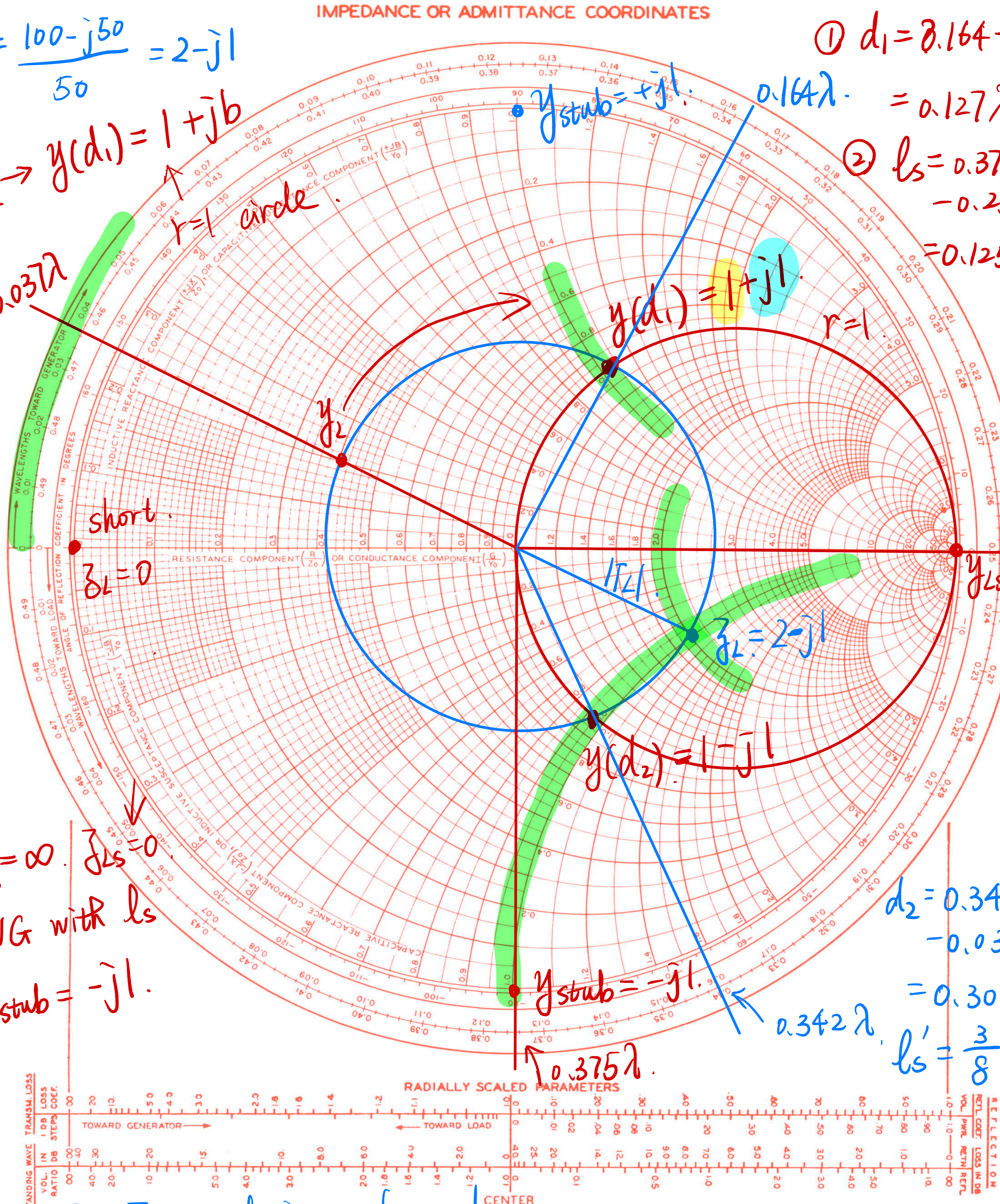
① $d_1 = 0.164 - 0.037\lambda$

$= 0.127\lambda$.

② $l_s = 0.375\lambda$

$-0.25\lambda$

$= 0.125\lambda$.




ⓐ $y_{Ls} = \infty$. $z_{Ls} = 0$

RWG with $l_s$

$y_{stub} = -j1$.

$d_2 = 0.342\lambda$

$-0.037\lambda$

$= 0.305\lambda$.

$l_s' = \frac{3}{8}\lambda$.

① Two choice for $d_1$.

② Two choices of $d_1 \to 2$ $l_s$ lengths.

The required input impedance of the shorted stub to achieve

$$y(d_1) + y_{stub} = 1 + j0$$

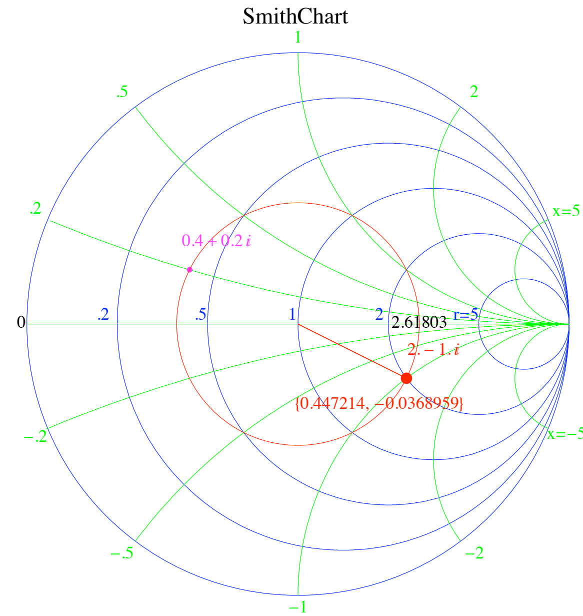
is

$$y_{stub} = -1j.$$

To achieve this input admittance the required stub length is

$$l_s = \frac{\lambda}{8} = 0.125\lambda$$

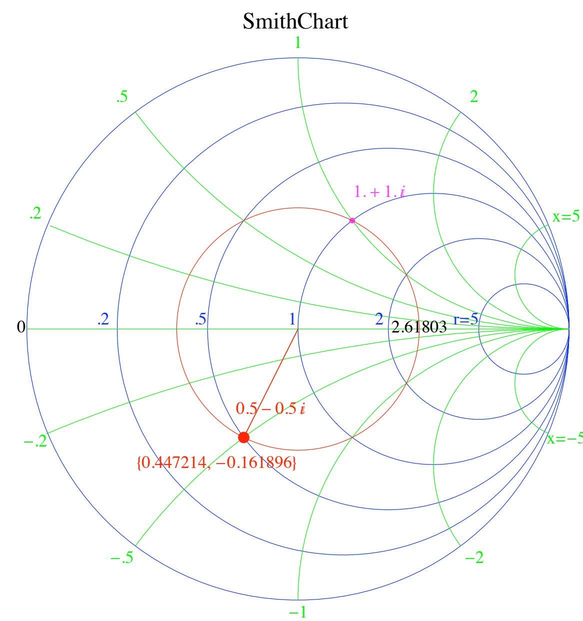
as determined from the SC — start at $y = \infty$ point on the SC on the far right (corresponding to the short termination), and then rotate clockwise (toward the generator) until the normalized admittance reads $-j1$; the amount of rotation indicates the required $l_s$.




- Another matching technique called ***double-stub tuning*** uses *two* shorted stubs of lengths $l_1$ and $l_2$ located at fixed values of $d_1$ and $d_2$.
  - Typically $d_1$ is zero or $\frac{\lambda}{4}$, and
  - $d_2 = d_1 + 3\frac{\lambda}{8}$.

    Vary $l_1$ and $l_2$ until VSWR is reduced to 1 near the generator end.

  The advantage of double-stub tuning is avoiding changes of stub locations when $Z_L$ is changed. It's implementation on a SC is considerably more complicated than single-stub tuning.



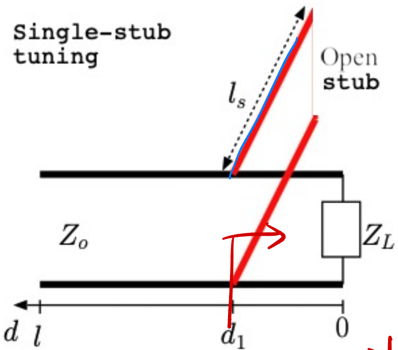
$y(d_1) + y_{stub} = 1. \Rightarrow y_{stub} = -j1. \quad z_{stub} = +j1.$

$Z_{Ls} = \infty. \quad y_{Ls} = 0.$

RWG

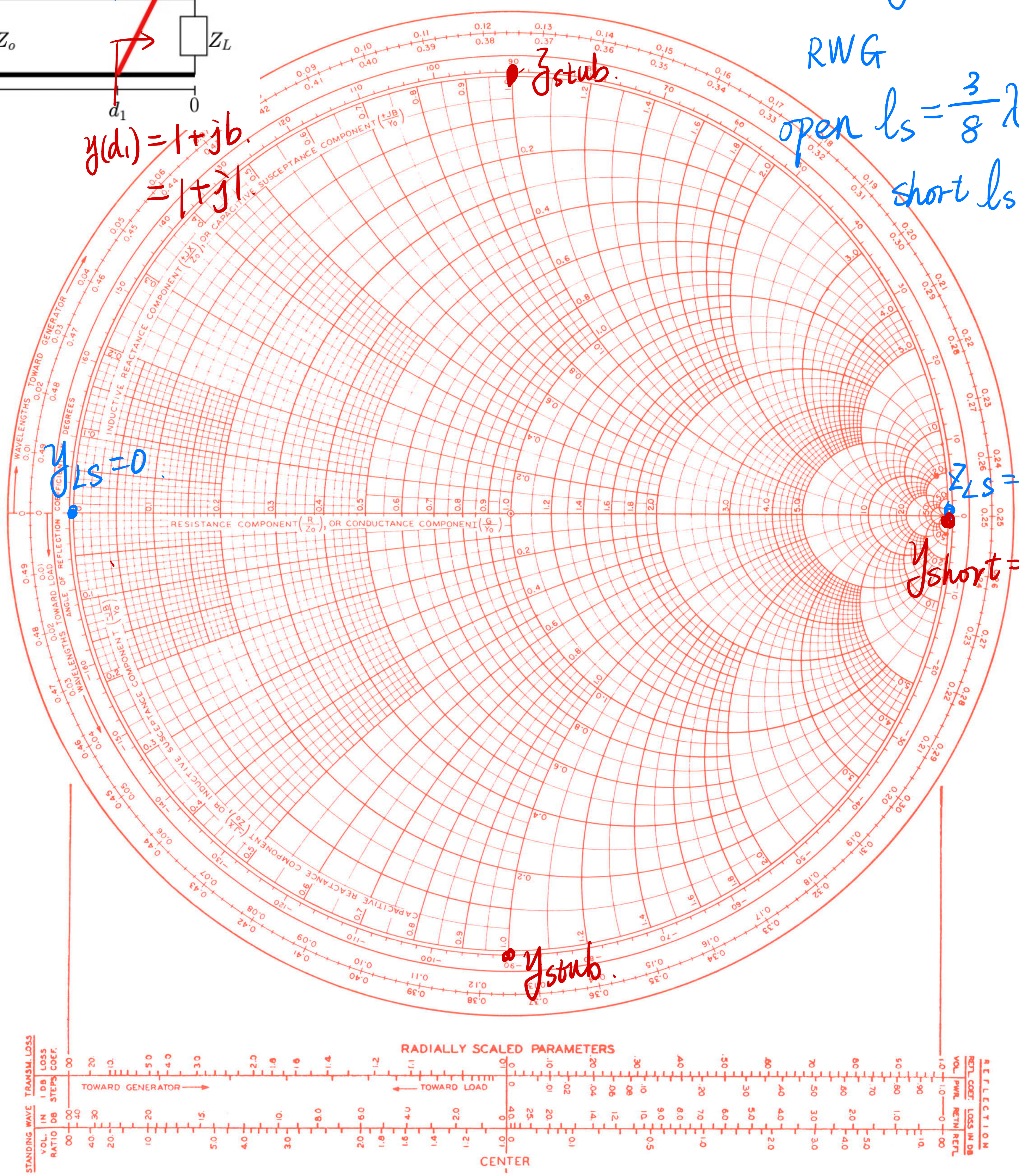
open $l_s = \frac{3}{8}\lambda.$

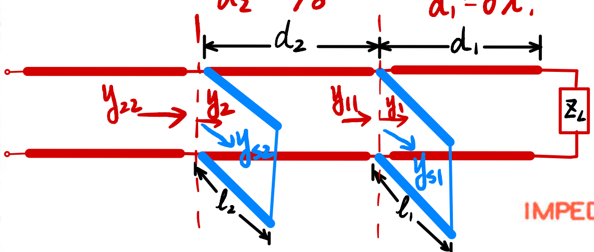
short $l_s = \frac{1}{8}\lambda$




$y(d_1) = 1 + jb.$

$= 1 + j1$






$Z_L = 80 - j50\Omega$

$Z_0 = 50\Omega$.

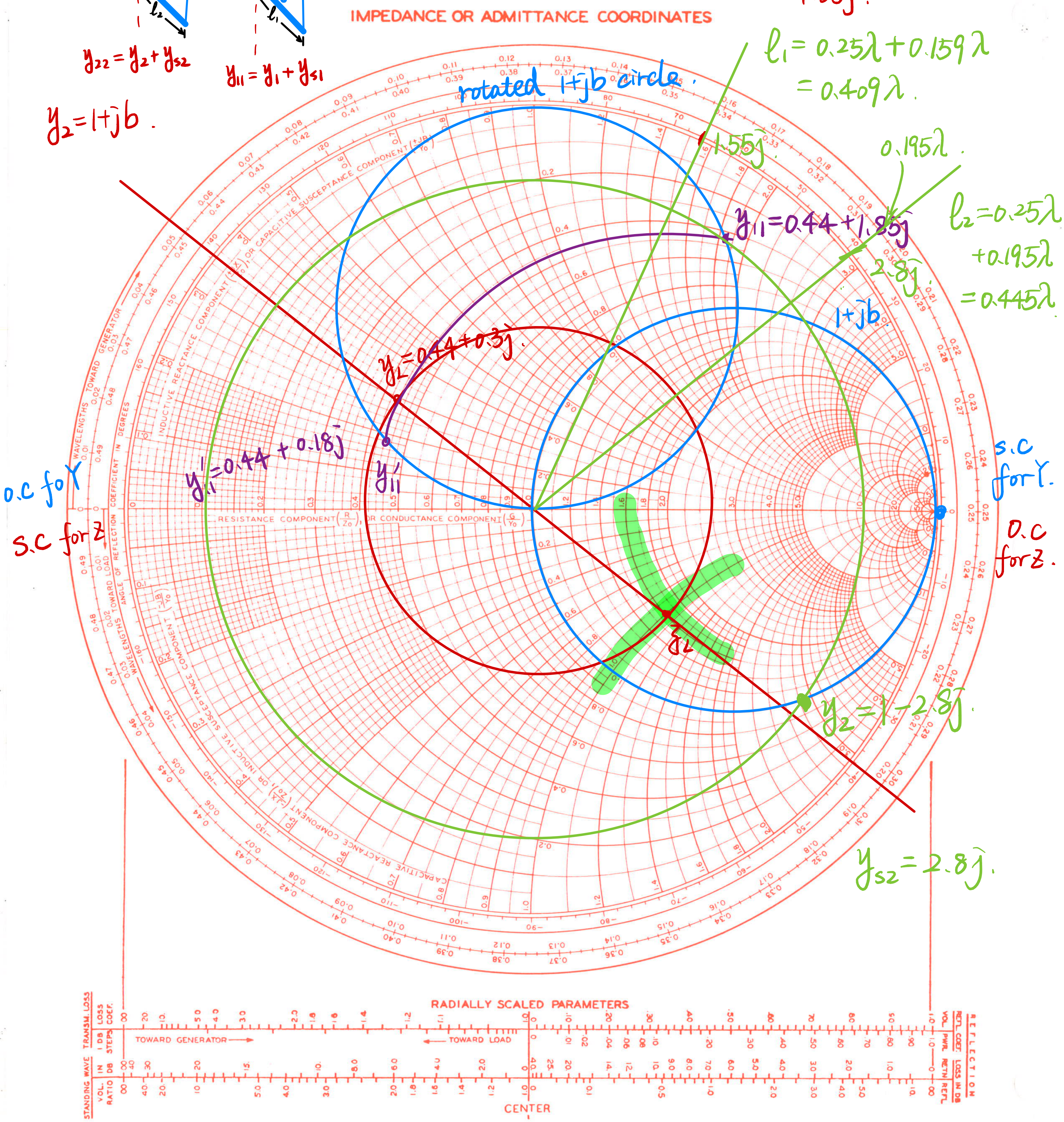
Find $l_1$, $l_2$.

$z_L = \frac{Z_L}{Z_0} = 1.6 - 1j$ $\quad y_L$

$y_{s1} = y_{11} - y_L = (0.44 + 1.85j) - (0.44 + 0.3j)$

$= 1.55j$.

$y_{22} = y_2 + y_{s2}$ $\qquad y_{11} = y_1 + y_{s1}$



Step 1:  normalize load impedance

$$z_L = \frac{Z_L}{Z_0}$$

Step 2: plot ZL on Smith chart

Step 3: Find YL on smith chart

Extend the ZL point through the center of the Smith Chart (in opposite direction, 180 degree, with same distance)

Step 4: Draw (1+ jb) circle and rotate it with the distance d2 chosen for the solution, (d2= $\lambda/8$ )

Step 5: Move on constant real value circle, until it intersect the rotated (1+ jb) circle $\rightarrow y_{11}$

Step 6: Find the difference between YL point and the intersection point

$$y_{s1} = y_{11} - y_L\,(y_1)$$
$$= (0.44 + 1.85j) - (0.44 + 0.3j)$$
$$= 1.55j.$$

Step 7: Calculate the shunt line length l1

From short circuit point to this susceptance value

From $y_{s1} \rightarrow l_1$. from s.c for Y. (right side).

Step8: find the corresponding point for $y_2$ y1l on the original (1+jb) circle

rotate $y_{11}$ RWG by $d_2 = \lambda/8$. to find $y_2 = 1 + jb_2$.

Step 9: find the susceptance of second stub

$$y_{s2} = -jb_2 \qquad y_2 + y_{s2} = \underline{1} = y_{22}.$$

Step 10: calculate the shunt line length l2