

37 Smith Chart and impedance matching

- In lossless TL circuits the average power input P_{in} at the generator end precisely matches the average power delivered to the load, P_L .

In fact, P_{in} and P_L also match the average power $P(d)$ transported on the line at an arbitrary d .

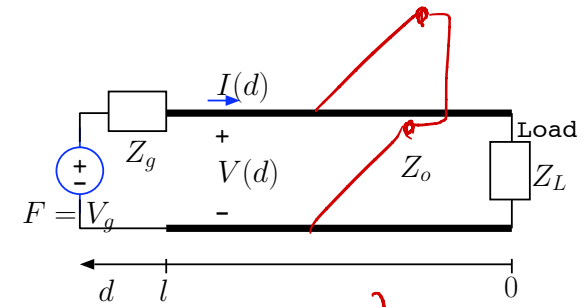
- We have in general

$$\begin{aligned}
 P(d) &= \frac{1}{2} \text{Re}\{V(d)I^*(d)\} \\
 &= \frac{1}{2} \text{Re}\{(V^+ e^{j\beta d} + V^- e^{-j\beta d}) \left(\frac{V^+ e^{j\beta d} - V^- e^{-j\beta d}}{Z_o} \right)^*\} \\
 &= \frac{1}{2} \text{Re}\left\{ \frac{|V^+|^2}{Z_o} - \frac{|V^-|^2}{Z_o} + \frac{V^- V^{+*} e^{-j2\beta d} - (V^- V^{+*} e^{-j2\beta d})^*}{Z_o} \right\} \\
 &\quad \frac{1}{2} \text{Re}\{V^+ I^{+*}\} \frac{|V^+|^2}{2Z_o} - \frac{|V^-|^2}{2Z_o}.
 \end{aligned}$$

- Note that $P(d)$ is the difference of power transported $\frac{|V^+|^2}{2Z_o}$ **toward the load** by the “forward-going” wave, and $\frac{|V^-|^2}{2Z_o}$ **toward the generator** by the reflected wave.
- Also note that

$$P(d) = \frac{|V^+|^2}{2Z_o} - \frac{|V^-|^2}{2Z_o} = \frac{|V^+|^2}{2Z_o} (1 - |\Gamma_L|^2)$$

so that $|\Gamma_L|^2$ is an effective **power reflection coefficient**.



$Z_L = R_L \rightarrow \frac{1}{4}$ transformer
 $Z_L = R_L + jX_L \rightarrow$ single stub

Power tx'ed toward the load:

$$\frac{|V^+|^2}{2Z_o}.$$

Power tx'ed toward the generator:

$$\frac{|V^-|^2}{2Z_o}.$$

Power reflection coefficient:

$$|\Gamma_L|^2.$$

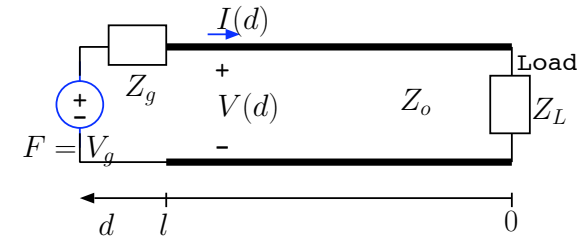
Power transmission coeff.:

$$1 - |\Gamma_L|^2.$$

- In TL circuits with load impedances Z_L **unmatched** to the characteristic impedance Z_o , the **reflected power**

$$\frac{|V^+|^2}{2Z_o} |\Gamma_L|^2$$

will be non-zero and the $\text{VSWR} > 1$.



This a condition not favored by practical signal generators used in TL circuits.

- Most generators are *designed* (in their biasing arrangements) to operate in circuits with low VSWR (close to unity), requiring Z_{in} closely matched to R_g , most frequently 50Ω , an optimal characteristic impedance value for coax-lines (when line losses are taken into account).
- Thus a standard procedure is to use TL's with $Z_o = R_g$, and utilize a *lossless impedance matching network* on the TL if the load impedance $Z_L \neq Z_o$.
 - This practice is called **impedance matching**.

Impedance matching achieves $\text{VSWR}=1$ between the generator and the matching network inserted at a location between the load and the generator.

- The inserted network should be designed to yield an input impedance equal Z_o at its input terminals.

The following examples illustrate different ways of achieving an impedance match.

Example 1: *Quarter-wave matching* of resistive loads:

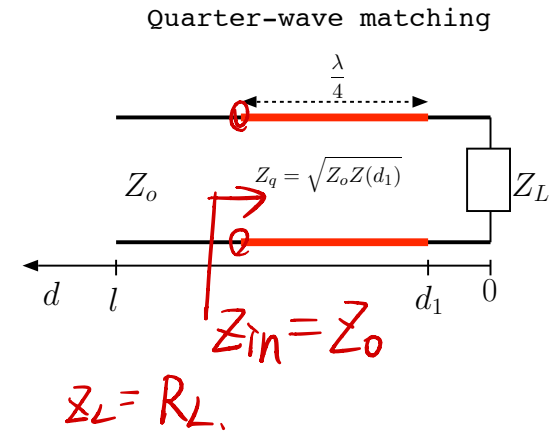
Consider a TL with $Z_L = 25\ \Omega$ and $R_g = Z_o = 50\ \Omega$. Since $Z_L \neq Z_o$ the load is unmatched and the $\text{VSWR} > 1$.

To reduce the VSWR on the line connected to the generator to unity, we can insert a **quarter-wave transformer** right after Z_L — i.e., at $d_1 = 0$ in the circuit shown in the margin — with a characteristic impedance

$$Z_q = \sqrt{25 \times 50} = \sqrt{1250} = 35.35\ \Omega.$$

$$= \sqrt{Z_{in} \cdot Z_L} = \sqrt{Z_o \cdot Z_L}$$

The impedance at the input terminals of the quarter-wave transformer (on the left) is then Z_o , i.e., $50\ \Omega$, implying a perfect impedance match.



- Quarter-wave matching illustrated above is a very commonly used matching technique.
- It is a straightforward application of the quarter-wave transformer impedance formula

$$Z_{in} = \frac{Z_q^2}{Z_L}$$

for a transformer with characteristic impedance Z_q .

$$Z_q = \sqrt{Z_{in} \cdot Z_L}$$

$$= \sqrt{Z_o \cdot Z_L}$$

↑

1. select d_1 such that $Z(d_1)$ is real.
2. design $Z_q = \sqrt{Z_o Z(d_1)}$

Example 2: *Quarter-wave matching* of reactive loads:

Consider a TL with $Z_L = 50 + j50 \Omega$ and $R_g = Z_o = 50 \Omega$. Since $Z_L \neq Z_o$ the load is unmatched and the $\text{VSWR} > 1$.

$$\Gamma_L = 1 + j1$$

We cannot insert the quarter-wave transformer right after the load because then we would need a complex valued Z_q implying a lossy matching network.

Instead, we insert a **quarter wave transformer** a distance d_1 to the left of Z_L , where d_1 is selected, using a SC, to have a purely resistive $Z(d_1)$. In that case, the quarter-wave transformer impedance formula

$$Z_q = \sqrt{Z(d_1) \times 50}$$

yields a real valued Z_q as needed. This procedure leads to having $d_1 = d_{max}$ or $d_1 = d_{min}$ corresponding to the positions of voltage maxima and minima on the line.

As shown in the margin,

$$Z(d_1) = 50(2.62 + j0) = 131 \Omega.$$

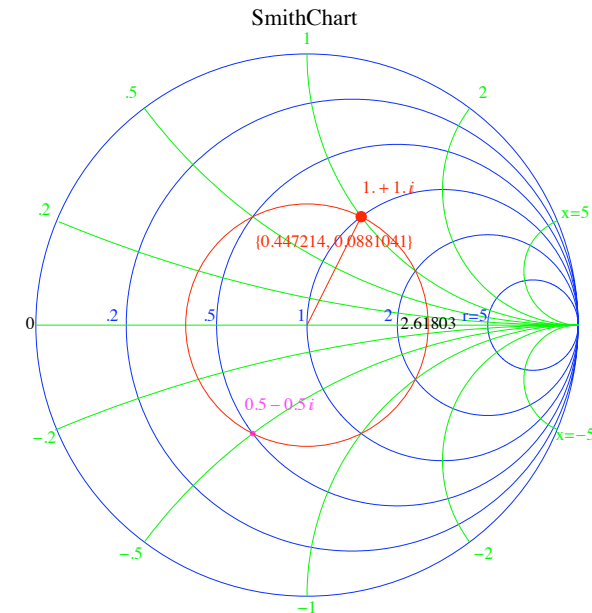
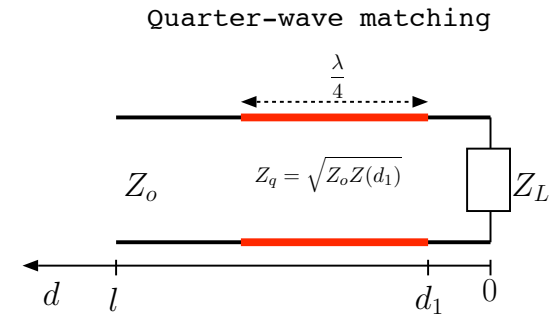
for

$$d_1 \approx 0.250\lambda - 0.162\lambda = 0.088\lambda$$

is a suitable choice for quarter-wave matching. In that case we need

$$Z_q = \sqrt{131 \times 50} = 50 \times \sqrt{2.62} \Omega$$

for the quarter wave transformer in order match to load to a line with $Z_o = 50 \Omega$.



Note that:

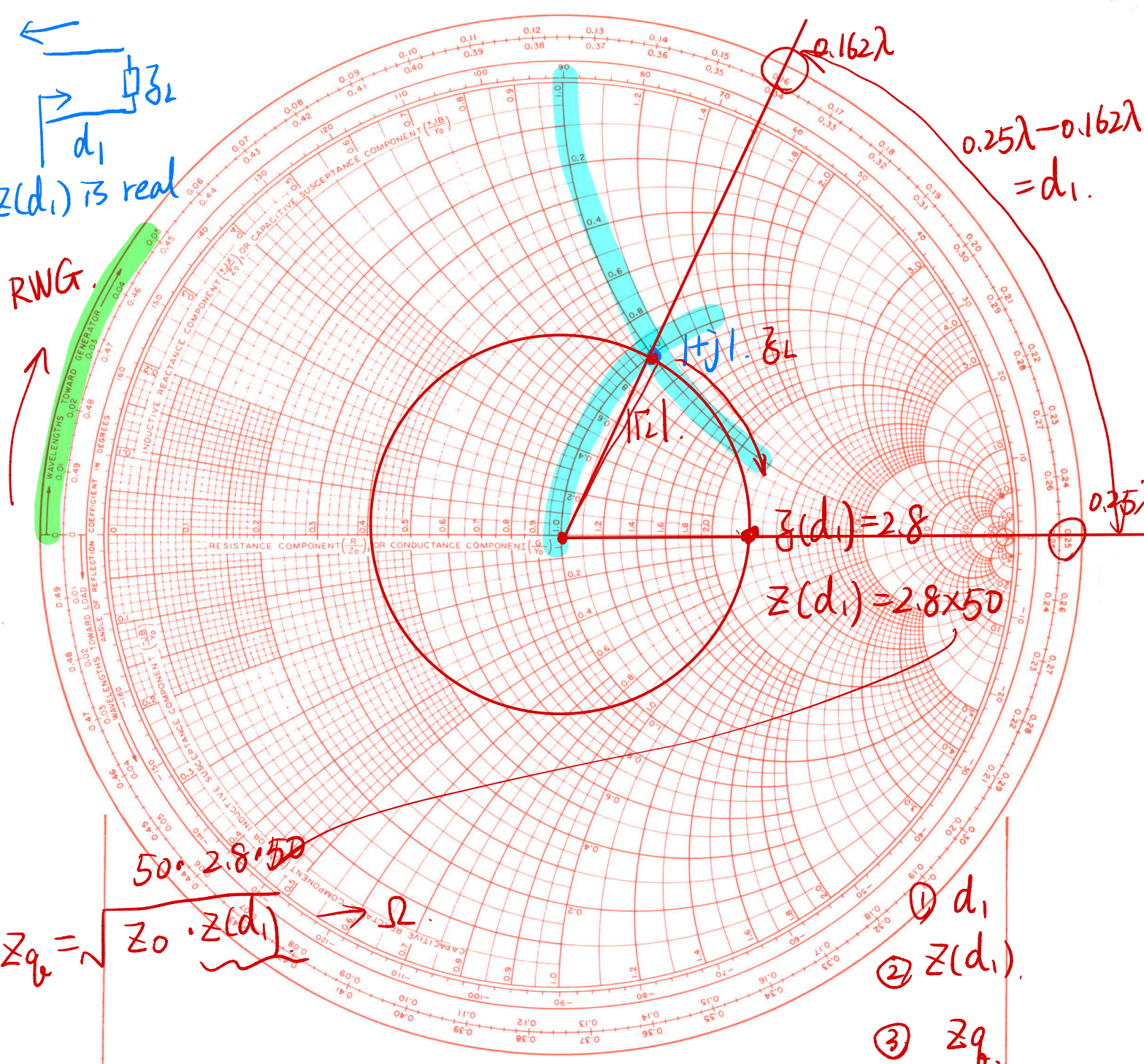
$$z(d_1) = z(d_{max}) = \text{VSWR} \approx 2.62$$

as marked on the SC.

Also

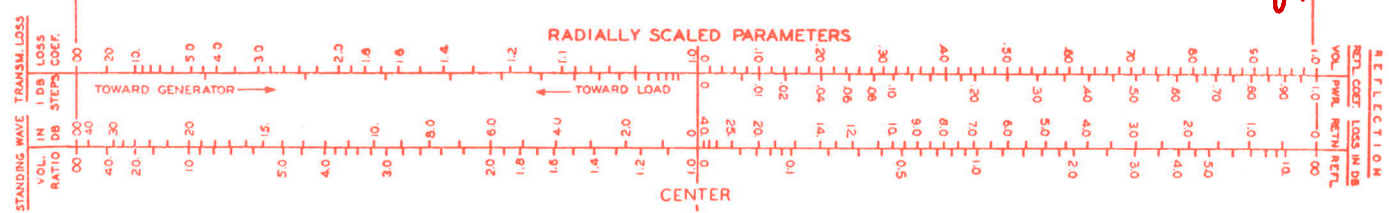
$$d_{max} \approx 0.088\lambda$$

since, as marked on the SC,
the angle of Γ_L is 0.088λ .



$$Z_g = \sqrt{Z_0 \cdot Z(d_1)} \rightarrow \Omega$$

- ① d_1
- ② $z(d_1)$
- ③ zq



① Normalized admittance y_L .
 $z_L = R_L + jX_L$.
 $\tilde{z}_L = \frac{z_L}{z_0}$, $y_L = \frac{1}{\tilde{z}_L}$.

② RWG in clockwise direction such $y(d_1) = 1 + jb$.

Example 3: Single-stub tuning:

Consider a TL with $Z_L = 100 - j50 \Omega$ and $R_g = Z_o = 50 \Omega$. Since $Z_L \neq Z_o$ the load is unmatched and the $VSWR > 1$.

We will insert a **shorted-stub** a distance d_1 to the left of Z_L in parallel with the line to achieve an impedance match.

Distance d_1 will be selected, using a SC, to have a normalized admittance of

$$y(d_1) = 1 + jb$$

so that a stub, with a normalized input admittance

$$y_{stub} = -jb,$$

can be added in parallel to have a combined admittance of

$$y(d_1) + y_{stub} = 1 + j0$$

and achieve a perfect impedance match (i.e., $VSWR=1$).

In specific

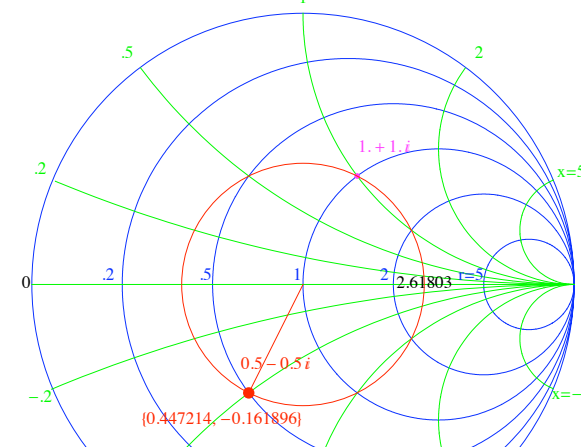
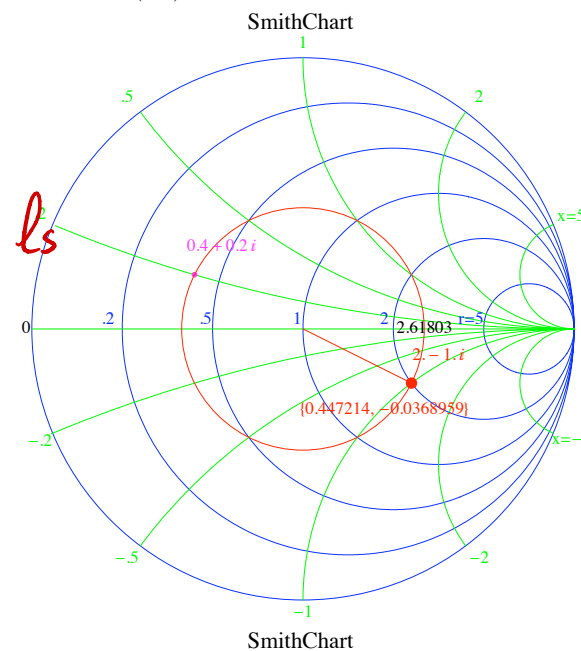
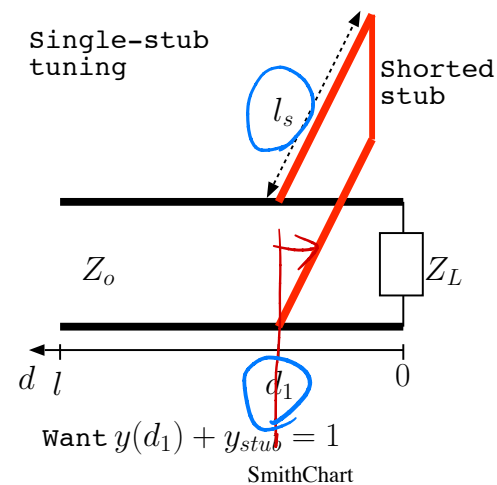
$$z_L = \frac{Z_L}{Z_o} = 2 - j1 \text{ and } y_L = \frac{1}{z_L} = 0.4 + j0.2$$

as shown on the SC on the top in the margin. We rotate clockwise on the SC by an amount corresponding to d_1 to obtain

$$y(d_1) = 1 + jb$$

on the “ $g = 1$ ” or “ $y = 1 + jb$ ” circle as shown in the bottom SC. From the amount of rotation we determine

$$d_1 \approx 0.162\lambda - 0.037\lambda = 0.125\lambda.$$



IMPEDANCE OR ADMITTANCE COORDINATES

$$Z_L = \frac{100 - j50}{50} = 2 - j1$$

$$y_L \rightarrow y(d_1) = 1 + j1$$

$$\textcircled{1} d_1 = 0.164 - 0.037\lambda$$

$$= 0.127\lambda$$

$$\textcircled{2} l_s = 0.375\lambda - 0.25\lambda = 0.125\lambda$$

$$0.037\lambda$$

$$y(d_1) = 1 + j1$$

$$r=1$$

short

$$Z_L = 0$$

$$0.25\lambda$$

$$y_{Ls} = \infty$$

$$Z_L = 2 - j1$$

$$y(d_2) = 1 - j1$$

a

$y_{Ls} = \infty$ $Z_{Ls} = 0$
RWG with l_s

$$y_{stub} = -j1$$

$$d_2 = 0.342\lambda - 0.037\lambda$$

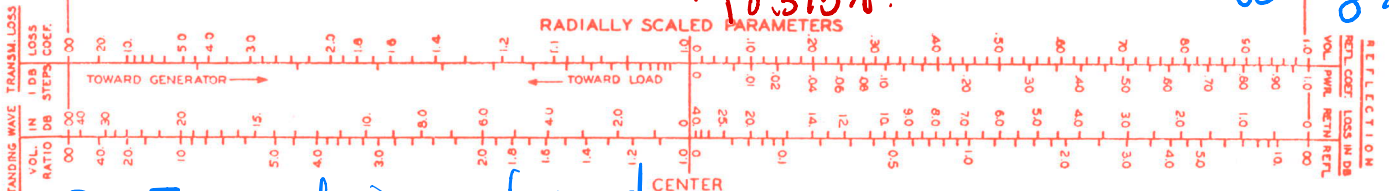
$$= 0.305\lambda$$

$$l_s' = \frac{3}{8}\lambda$$

$$y_{stub} = -j1$$

$$0.375\lambda$$

$$0.342\lambda$$



① Two choice for d_1 .

② Two choices of $d_1 \rightarrow 2$ l_s lengths.

The required input impedance of the shorted stub to achieve

$$y(d_1) + y_{stub} = 1 + j0$$

is

$$y_{stub} = -1j.$$

To achieve this input admittance the required stub length is

$$l_s = \frac{\lambda}{8} = 0.125\lambda$$

as determined from the SC — start at $y = \infty$ point on the SC on the far right (corresponding to the short termination), and then rotate clockwise (toward the generator) until the normalized admittance reads $-j1$; the amount of rotation indicates the required l_s .

- Another matching technique called ***double-stub tuning*** uses *two* shorted stubs of lengths l_1 and l_2 located at fixed values of d_1 and d_2 .

- Typically d_1 is zero or $\frac{\lambda}{4}$, and
- $d_2 = d_1 + 3\frac{\lambda}{8}$.

Vary l_1 and l_2 until VSWR is reduced to 1 near the generator end.

The advantage of double-stub tuning is avoiding changes of stub locations when Z_L is changed. It's implementation on a SC is considerably more complicated than single-stub tuning.

