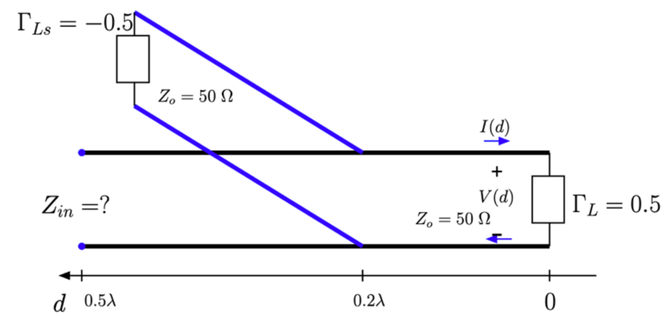
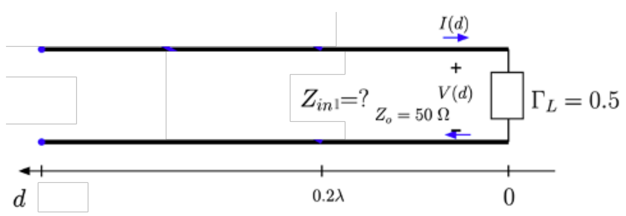
Lecture 37

1 Example – shunt connect TL problem

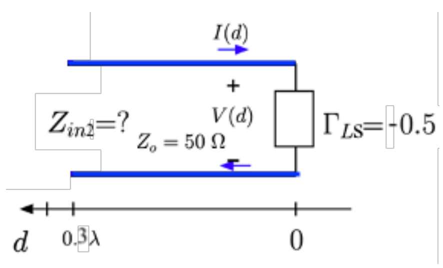
Example 4: A TL of length $l = 0.5\,\lambda$ and $Z_o = 50\ \Omega$ has a load reflection coefficient $\Gamma_L = 0.5$ and a shunt connected TL at $d = 0.2\,\lambda$. The shunt connected TL has $l = 0.3\,\lambda$, $Z_o = 50\ \Omega$, and a load reflection coefficient $\Gamma_L = -0.5$. Determine the input impedance of the line.



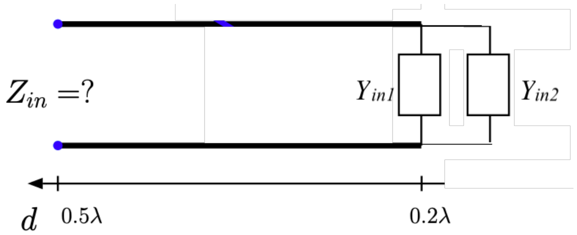
Step 1: Black TL with length $l = 0.2\,\lambda$ and $Z_o = 50\ \Omega$, load reflection coefficient $\Gamma_L = 0.5$, find input impedance Z_{in1} at $l = 0.2\,\lambda$.



Step 2: Blue TL with length $l = 0.3\,\lambda$ and $Z_o = 50\ \Omega$, load reflection coefficient $\Gamma_{Ls} = -0.5$, find input impedance Z_{in2} at $l = 0.3\,\lambda$.



Step 3: Black TL with length $l = 0.3\,\lambda$ and $Z_o = 50\ \Omega$, at load side it has parallel impedance Z_{in1} and Z_{in2}, find input impedance Z_{in}



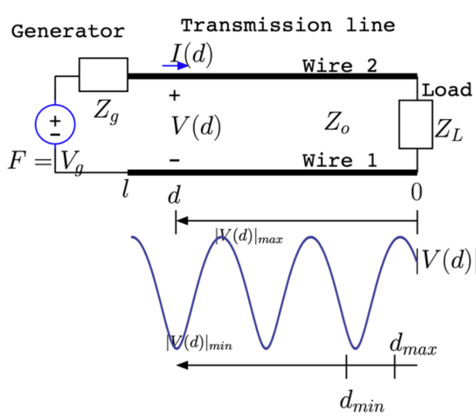
2 Example – From Γ to Z at one location

Example 5: What is the load impedance Z_{Ls} terminating the shunt connected stub in Example 4?

$$Z_{Ls} = \frac{1 + \Gamma_{LS}}{1 - \Gamma_{LS}}$$

3 VSWR definition

When the input signal V_g is cosinusoidal signal, we will see standing wave (or galloping wave) on TL line, for an arbitrary load impedance Z_L.



Voltage phasor is

$$V(d) \equiv V^+e^{j\beta d} + V^-e^{-j\beta d} = V^+e^{j\beta d}(1 + \Gamma(d))$$

Where $\Gamma(d)$ is the generalized reflection coefficient.

We see voltage wave form in the left figure, where the voltage will have a maximum magnitude at d_{max} location, and it has a minimum magnitude at d_{min} location.

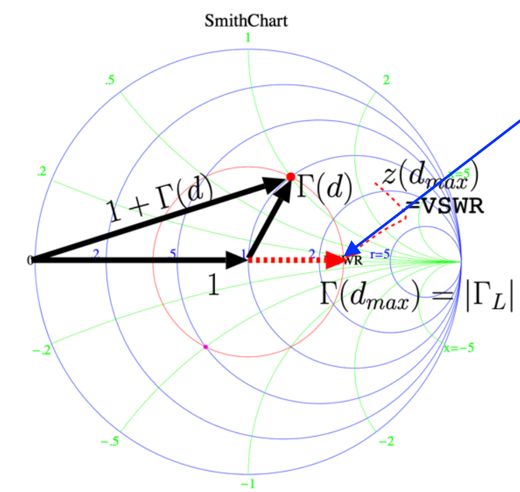
$|V(d_{max})| = |V^+|(1 + |\Gamma_L|)$ and $|V(d_{min})| = |V^+|(1 - |\Gamma_L|)$

We define VSWR (voltage standing wave ratio) as the ratio of the maximum voltage magnitude vs. minimum voltage magnitude.

$$\text{VSWR} \equiv \frac{|V(d_{max})|}{|V(d_{min})|} = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|}$$

VSWR and $|\Gamma_L|$ form a bilinear transform pair $|\Gamma_L| = \frac{\text{VSWR}-1}{\text{VSWR}+1}$

4 VSWR on Smith Chart



Since $\Gamma(d_{max}) = |\Gamma_L|$ (reflection coefficient at d_{max} location is the radius of red circle $|\Gamma_L|$.

$$\text{VSWR} = \frac{1+\Gamma(d_{max})}{1-\Gamma(d_{max})}$$

Also, from previous lecture we know that normalized impedance z and generalized reflection coefficient Γ also form a bilinear pair, such that

$$z(d_{max}) = \frac{1+\Gamma(d_{max})}{1-\Gamma(d_{max})}$$

So we can directly read VSWR on Smith Chart at d_{max} location.

$$\text{VSWR} = z(d_{max}) = \Gamma(d_{max})$$

Steps to find VSWR for given TL:

1. Calculate normalized impedance z
2. Put z on Smith Chart and draw a constant $|\Gamma_L|$ circle
3. Read VSWR at $z(d_{max})$ (right half of the horizontal axis)

Consider three special cases:

- Matched load $Z_L = Z_o$, constant $|\Gamma_L|$ circle has a radius of 0, VSWR = 1 (center point)
- Short and open circuit, constant $|\Gamma_L|$ circle has a radius of 1, VSWR = ∞ (Open ckt location)
- Purely reactive load $Z_L = jX_L$, VSWR = ∞ (Open ckt location)