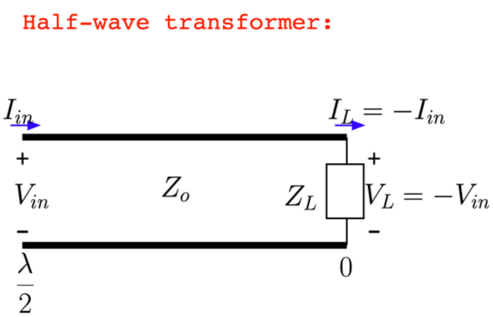
Lecture 35

1 Half-wave transformer

We consider the co-sinusoidal steady state solution of the TL, when the load impedance is connected to an arbitrary ckt. In general, $Z_L = R_L + jX_L$.

The TL length can be varying. But let's consider two special cases, when the TL length l is either $\lambda/2$ or $\lambda/4$.

1.1 Half-wave transformer



When the TL length $l = \lambda/2$

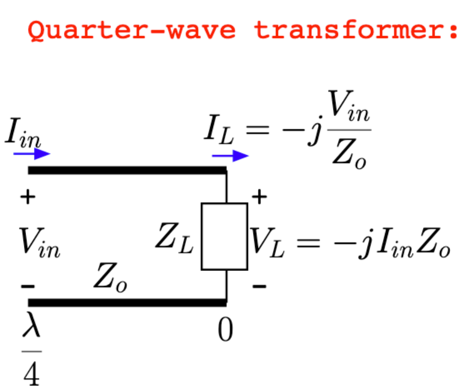
- inverts the algebraic sign of its voltage and current inputs at the load end

$$V_{in} = -V_L$$

$$I_{in} = -I_L$$

- Input impedance is identical to the load impedance $Z_{in} = Z_L$

1.2 Quarter-wave transformer



When the TL length $l = \lambda/4$

- Quarter-wave current-forcing equation

$$I_L = -j\frac{V_{in}}{Z_o}$$

The load current is independent of load impedance.

- Input impedance is

$$Z_{in}Z_L = Z_o^2 \qquad Z_{in} = Z_o^2/Z_L$$

If $Z_L = 0$ (short), quarter-wave transformer will have an input impedance $Z_{in} = \infty$ (open).

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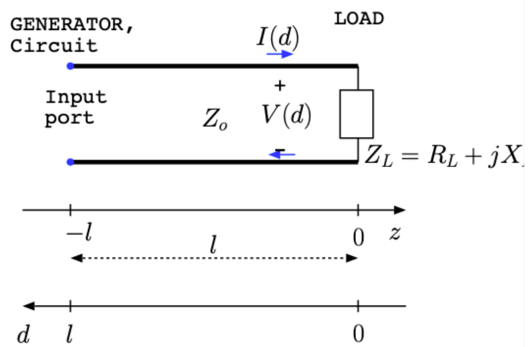
2 Generalized reflection coefficient and input impedance definition



In this class, we will extend the TL to an arbitrary length l, which is terminated by an arbitrary load $Z_L = R_L + jX_L$. The TL is excited with an cosinusoidal signal, so the phasor form of voltage and current on the TL are

$$V(d) \equiv V^+ e^{j\beta d} + V^- e^{j\beta d}$$

$$= V^+ e^{j\beta d}\left(1 + \frac{V^-}{V^+} e^{-2j\beta d}\right)$$

$$I(d) \equiv \frac{I^+}{Z_o} e^{j\beta d} - \frac{I^-}{Z_o} e^{j\beta d} = \frac{I^+}{Z_o} e^{j\beta d}\left(1 - \frac{V^-}{V^+} e^{-2j\beta d}\right)$$

So we define a **generalized reflection coefficient**

$$\Gamma(d) \equiv \frac{V^-}{V^+} e^{-2j\beta d} = \Gamma_L e^{-2j\beta d}$$

Then the V to I ratio is the **line impedance**

$$Z(d) = Z_o \frac{1 + \Gamma(d)}{1 - \Gamma(d)}$$

We have the relationship between $\Gamma(d)$ and $Z(d)$ (or simply Γ and Z) as

$$\frac{Z}{Z_o} = \frac{1 + \Gamma(d)}{1 - \Gamma(d)}$$

And

$$\Gamma = \frac{Z - Z_o}{Z + Z_o}$$

The above relationship is called bilinear transformations or Möbius transformation.