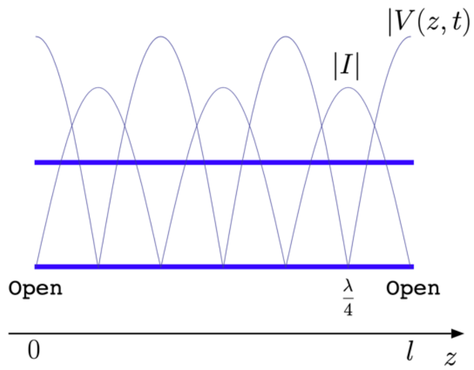
# Lecture 34

## 1 Oscillations in lossless TL ckts

### 1.1 Review open-open stub




A TL segment open circuited at both ends can support voltage and current oscillations such that the current waveform vanishes at both ends.

The figure on the left depicts the absolute values of V and I waveforms that satisfying this boundary condition.

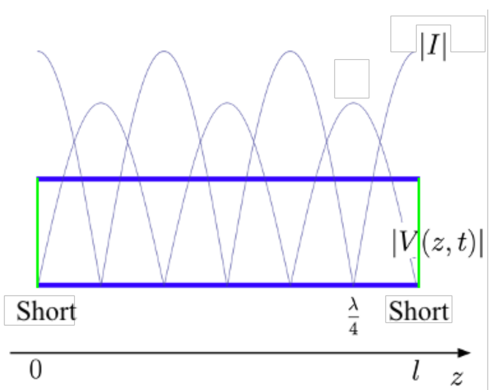
- resonant modes

$$\cos(n\omega_o t + \theta_n)\cos(n\beta_o z) \text{ (for V) and } \sin(n\omega_o t + \theta_n)\sin(n\beta_o z) \text{ (for I)}$$

- resonance frequency $\omega = \frac{\pi v}{l} n$ rad/s or f= $\frac{v}{2l} n$ Hz
- resonance wavelength $\lambda = \frac{v}{f} = \frac{2l}{n}$
- length of the stub $l = n\frac{\lambda}{2}$. Length $l$ is integer multiples of half wavelength.

### 1.2 Short-short stub

A TL segment open circuited at both ends can support voltage and current oscillations such that the voltage waveform vanishes at both ends.



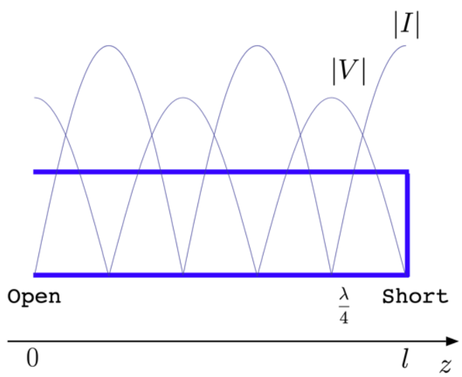

- resonant modes

$$\cos(n\omega_o t + \theta_n)\cos(n\beta_o z) \text{ (for I) and } \sin(n\omega_o t + \theta_n)\sin(n\beta_o z) \text{ (for V)}$$

- resonance frequency $\omega = \frac{\pi v}{l} n$ rad/s or f= $\frac{v}{2l} n$ Hz
- resonance wavelength $\lambda = \frac{v}{f} = \frac{2l}{n}$
- length of the stub $l = n\frac{\lambda}{2}$. Length $l$ is integer multiples of half wavelength.

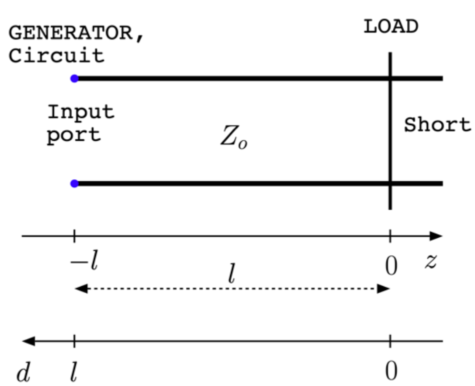
### 1.3 Open-short stub

A TL segment which is open on one end and short on the other end must satisfy different boundary conditions. On the open end, the current must go to zero, while on the short end, the voltage needs to be zero.




- length of the stub $l = n\frac{\lambda}{2} + \frac{\lambda}{4} = \frac{\lambda}{4}(2n+1)$. Length $l$ is odd integer multiple of quarter wavelength.
- resonance frequency $\omega = \frac{\pi v}{l}\left(n + \frac{1}{2}\right)$ rad/s or f= $\frac{v}{2l}\left(n + \frac{1}{2}\right)$ Hz
- resonance wavelength $\lambda = \frac{v}{f} = \frac{4l}{2n+1}, n = 0, 1, 2, \cdots$

## 2 Line impedance




For the TL on the left, we put a 'short' wire at the load end. Then we refer a new 'd' axis, such that 'd' is equal to '-z'.

Use the phasor notation, we find the total voltage and total current on the TL as

$$\tilde{V}(d) \equiv V^+ e^{j\beta d} + V^- e^{j\beta d}$$

$$\tilde{I}(d) \equiv \frac{I^+}{Z_o} e^{j\beta d} - \frac{I^-}{Z_o} e^{j\beta d}$$

At the load end where $d = 0$, we have a short connection, so the voltage must equal to zero.

$$\tilde{V}(d)|_{d=0} \equiv \left(V^+ e^{j\beta d} + V^- e^{j\beta d}\right)|_{d=0} = V^+ + V^- = 0$$

So we get $V^- = -V^+$.

$$\tilde{V}(d) \equiv V^+\left(e^{j\beta d} - e^{j\beta d}\right) = V^+ \cdot 2j sin(\beta d)$$

$$\tilde{I}(d) \equiv \frac{V^+}{Z_o}\left(e^{j\beta d} + e^{j\beta d}\right) = \frac{V^+}{Z_o} \cdot 2cos(\beta d)$$

We define the line impedance as the V to I ratio anywhere along the TL as

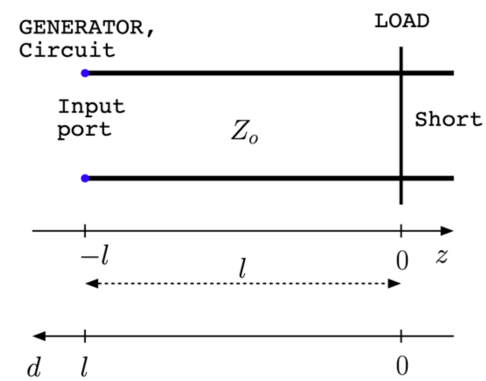
$$Z(d) \equiv \frac{\tilde{V}(d)}{\tilde{I}(d)} = jZ_o tan(\beta d)$$

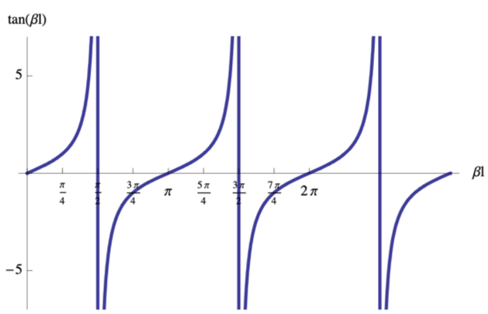
Normalized line impedance

$$z(d) \equiv \frac{Z(d)}{Z_o} = jtan(\beta d)$$

# 3 Input impedance

## 3.1 Short stub



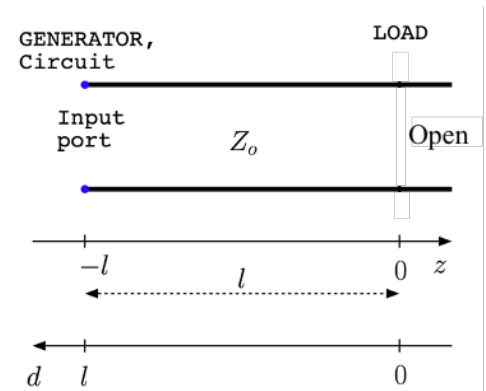

For a TL stub connected with a short ckt on load end, the input impedance for TL with length $l$ is

$$Z(l) = jZ_o tan(\beta l)$$

We plot the normalized reactance part of the input impedance in the figure here. The horizontal axis is $\beta l$.

- When $\beta l = 0$, or $l = 0$, $z(l) = 0$ (because it connects to a short wire on load end.
- When $0 < \beta l < \frac{\pi}{2}$, or $0 < l < \frac{\lambda}{4}$, $z(l)$ is inductive (positive).
- At $\beta l = \frac{\pi}{2}$, or $l = \frac{\lambda}{4}$, $z(l)$ is infinity, which means the TL is open at this point.
- When $\frac{\pi}{2} < \beta l < \pi$, or $\frac{\lambda}{4} < l < \frac{\lambda}{2}$, $z(l)$ is capacitive (negative).
- The input impedance as a period with length $l = \frac{\lambda}{2}$.

## 3.2 Open stub

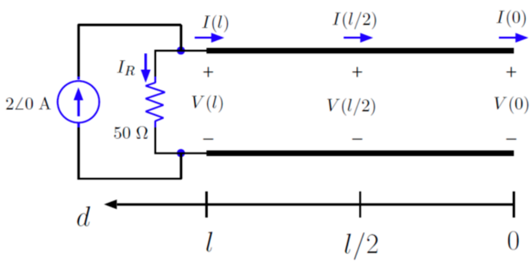



For a TL stub connected with a open ckt on load end, the input impedance for TL with length $l$ is

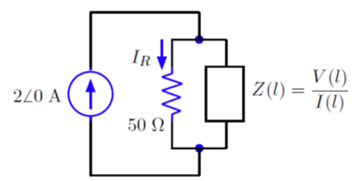
$$Z(l) = -jZ_o cot(\beta l)$$

# 4 Example

Consider a transmission line segment of propagation velocity $v = \frac{1}{3}c = 1 \times 10^8$ m/s, characteristic impedance $Z_o = 50\ \Omega$, and length $l$. As shown in figure (a) below, the segment is connected in parallel with a 50 Ω resistor and an ideal current source $i(t) = Re\{Ie^{j\omega t}\}$, where $I = 2\angle 0$ A is the source current phasor and $\omega = \pi \times 10^8$ rad/s; figure (b) depicts an equivalent circuit in terms of input impedance of the transmission line at $d = l$, namely $Z(l) \equiv \frac{V(l)}{I(l)}$.




(a) Transmission line circuit




(b) Equivalent circuit

In answering the following questions assume that the circuit above is in sinusoidal steady-state:

a) What is the signal wavelength $\lambda$ (in m) on the transmission line?

b) Since an "open termination" is located on the line at $d = 0$, what is the pertinent "boundary condition" involving the phasor $I(0)$ for all possible line lengths $l$?

c) Does the transmission line support a standing wave in the above circuit for all values of non-zero $l$? Justify your answer.

d) What is the smallest non-zero value of $l$ (in m) if phasor $I_R = 2\angle 0$ A? Explain.

e) For l determined in part (d), what is phasor $V\ (l/2)$? Explain.

f) For l determined in part (d), is $I(l/2) = 0$ possible? Explain.

g) What is the smallest non-zero value of $l$ if $I_R = 0$?