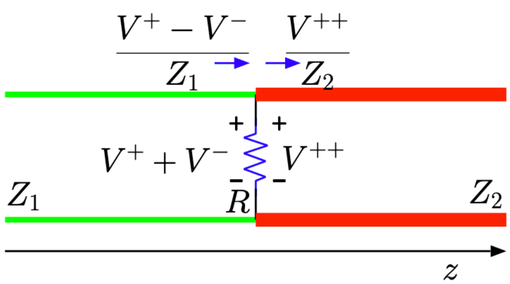
Lecture 33

1 Multi-line circuits example with shunt resister

Two TL's with characteristic impedances Z_1 and Z_2 are joined at a junction that also includes a "shunt" resistance R as shown in the diagram in the margin. Determine the reflection coefficient Γ_{12} and transmission coefficient τ_{12} at the junction.



For wave propagating from line 1 to line 2

- Find the equivalent impedance at the junction.

$$Z_{eq} = R \parallel Z_2$$

- Determine the Γ_{12} and τ_{12}.

$\Gamma_{12} = \frac{Z_{eq}-Z_1}{Z_{eq}+Z_1}$, and $\tau_{12} = 1 + \Gamma_{12}$

For wave propagating from line 2 to line 1

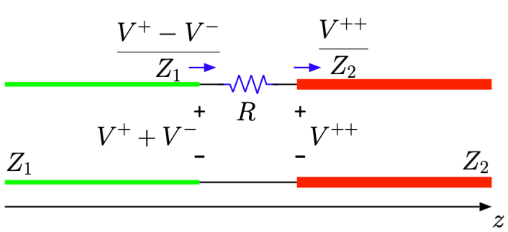
- Find the equivalent impedance at the junction.

$$Z_{eq} = R \parallel Z_1$$

- Determine the Γ_{21} and τ_{21}.

$\Gamma_{21} = \frac{Z_{eq}-Z_2}{Z_{eq}+Z_2}$, and $\tau_{21} = 1 + \Gamma_{21}$

2 Multi-line circuits example with series resister



For wave propagating from line 1 to line 2

- Find the equivalent impedance at the junction.

$$Z_{eq} = R + Z_2$$

- Determine the Γ_{12} and τ_{12}.

$$\Gamma_{12} = \frac{Z_{eq} - Z_1}{Z_{eq} + Z_1}$$

However, $\tau_{12} \neq 1 + \Gamma_{12}$, because $\tau_{12} = \frac{Z_2}{R+Z_2}(1 + \Gamma_2)$

3 Oscillations in lossless TL ckts

If the transmission line ends are terminated with an open or short circuit, the TL (stub) becomes a oscillator, in analogous to a LC circuit.

3.1 Open-open stub

When both ends of the stub are open, the reflection coefficient $\Gamma = 1$.

The standing wave on the stub is $I(z,t) = \frac{f\left(t-\frac{z}{v}\right)}{Z_o} - \frac{g\left(t+\frac{z}{v}\right)}{Z_o}$

The boundary conditions at $z = 0$ and $z = l$ are current $I(z,t) = 0$

From that we know, the forward going wave and the backward going wave are the same; and the function $f(t)$ is a periodic function with

- Period $T = \frac{2l}{v}$
- Fundamental frequency $\omega_o = \frac{2\pi}{T} = \frac{\pi v}{l}$

Using Fourier series, the standing wave current is

$$I(z,t) = \sum_{n=1}^{\infty} \frac{2F_n}{Z_o} \underbrace{\sin(n\omega_o t + \theta_n)\sin(n\beta_o z)}_{\text{resonant modes}}$$

Where the fundamental wavenumber $\beta_o \equiv \frac{\omega_o}{v} = \frac{\pi}{l}$

And the standing wave voltage is

$$I(z,t) = \sum_{n=1}^{\infty} 2F_n \underbrace{\cos(n\omega_o t + \theta_n)\cos(n\beta_o z)}_{\text{resonant modes}}$$

So the current and voltage on stub has infinite number of resonances.

- resonance frequency $\omega = \frac{\pi v}{l} n$ rad/s or f= $\frac{v}{2l} n$ Hz
- resonance wavelength $\lambda = \frac{v}{f} = \frac{2l}{n}$
- length of the stub $l = n\frac{\lambda}{2}$