Lecture 32

1 Steady state in TL circuit

We use bounce diagram to indicate the transient response of $V(z,t)$ and $I(z,t)$ along the TL.

When $t \to \infty$, both $V(z,t)$ and $I(z,t)$ will converge to their DC steady state values. Then, TL becomes a pair of wires (short wires) in the lumped circuit sense.

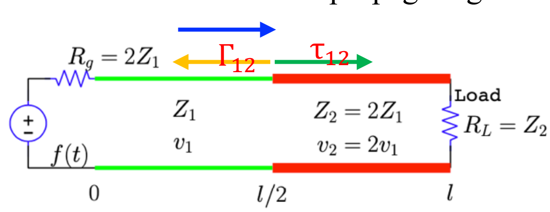
2 Multi-line circuits

We will use bounce diagram to solve multiple lossless TLs. These segments will have different characteristic impedances Z, and different propagation velocities v.

Where these different transmission line segments are connected, there are discontinuities that will cause reflection and transmission, where we need to calculate the new reflection and transmission coefficients.

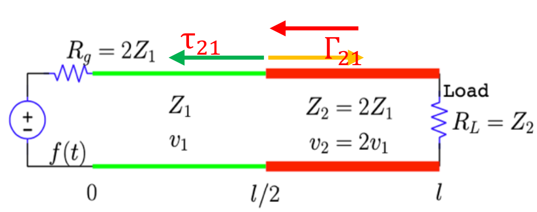
In general, the reflection and transmission coefficients will be different for wave to propagate from line 1 to line 2, and from line 2 to line 1. So, in additional to injection coefficient τ_g, generator reflection coefficient Γ_g, and load reflection coefficient Γ_L we need four new coefficients at the interconnection.

- For forward wave propagating in $+z$ direction from line 1 to line 2 (blue arrow)



  - Wave is reflected and become backward travelling wave in $-z$ direction in line 1, reflection coefficient Γ_{12} (green arrow)
  - Wave is injected and become forward travelling wave in $+z$ direction in line 2, transmission coefficient τ_{12} (orange arrow)

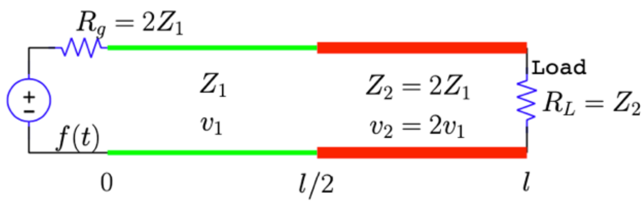
- For backward wave propagating in $-z$ direction from line 2 to line 1 (red arrow)



  - Wave is reflected and become forward travelling wave in $+z$ direction in line 2, reflection coefficient Γ_{21} (orange arrow)
  - Wave is injected and become backward travelling wave in $-z$ direction in line 1, transmission coefficient τ_{21} (green arrow)

3 Multi-line circuits example

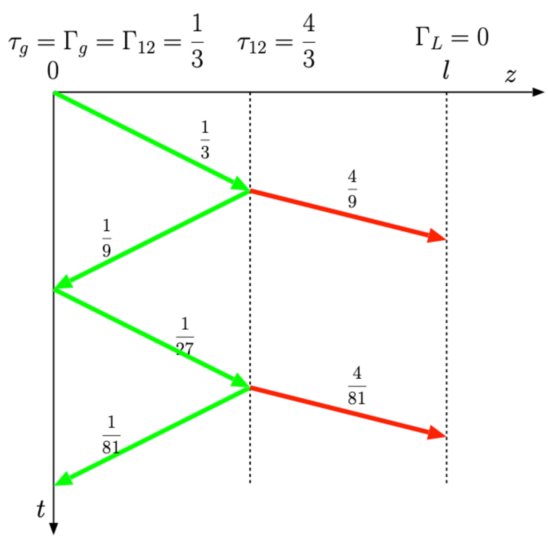
Two distinct TL's of equal lengths have been joined directly at a distance away from the generator. $Z_2 = 2Z_1$, and $v_2 = 2v_1$.



This circuit has matched load ($Z_L = Z_2$), so there's no reflection at load end.

So we only need two new coefficients, reflection coefficient Γ_{12} and transmission coefficient τ_{12}. There's no Γ_{21} or τ_{21}.

$$\Gamma_{12} = \frac{Z_2 - Z_1}{Z_2 + Z_1}, \qquad \tau_{21} = 1 + \Gamma\_12$$



Draw its bounce diagram. Notice that in line 1, we have both forward and backward traveling waves (two green arrows), while in line 2, there's only forward traveling wave (red arrow), since $\Gamma_L = 0$.

3.1 Determine $\mathcal{L}_2$ and $\mathcal{C}_2$ in terms of $\mathcal{L}_1$ and $\mathcal{C}_1$

Characteristic impedance $Z = \sqrt{\frac{\mathcal{L}}{\mathcal{C}}}$, velocity $v = \frac{1}{\sqrt{\mathcal{L}\mathcal{C}}}$

3.2 Determine $V(z,t)$ and $I(z,t)$

In line 1, $z < \frac{l}{2}$

$$V(z,t) = \tau_g \sum_{n=0}^{\infty} (\Gamma_g \, \Gamma_{12})^n \left[f_i\left(t - \frac{z}{v_1} - n\frac{l}{v_1}\right) + \Gamma_{12} f_i\left(t + \frac{z}{v} - (n+1)\frac{l}{v_1}\right)\right]$$

Forward wave backward wave

$$I(z,t) = \frac{\tau_g}{Z_1}\sum_{n=0}^{\infty}(\Gamma_g\,\Gamma_{12})^n\,\left[f_i\left(t-\frac{z}{v_1}-n\frac{l}{v_1}\right) - \Gamma_{12}f_i\left(t+\frac{z}{v}-(n+1)\frac{l}{v_1}\right)\right]$$

Forward wave backward wave

In line 2, $\frac{l}{2} < z < l$

$$V(z,t) = \tau_g\sum_{n=0}^{\infty}(\Gamma_g\,\Gamma_{12})^n\,\tau_{12}\,f_i\left(t-\frac{z}{v_2}-(4n+2)\frac{l/2}{v_2}\right)$$

Forward wave only

$$I(z,t) = \frac{\tau_g}{Z_2}\sum_{n=0}^{\infty}(\Gamma_g\,\Gamma_{12})^n\,\tau_{12}\,f_i\left(t-\frac{z}{v_2}-(4n+2)\frac{l/2}{v_2}\right)$$

Forward wave only