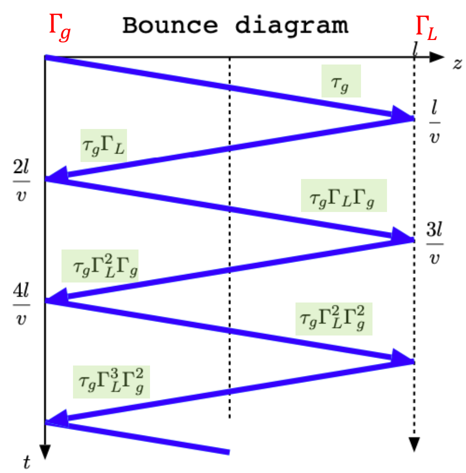
Lecture 31

1 Bounce diagram summary

We can use bounce diagram to express the voltage and current variations on TL circuit.

1.1 Step 1: Assume the input voltage is impulse function $\delta(t)$.



Calculate three coefficients.

a) Injection coefficient $\tau_g = \frac{Z_o}{R_g+Z_o}$

b) Load end reflection coefficient $\Gamma_L = \frac{R_L-Z_o}{R_L+Z_o}$

c) Source end reflection coefficient $\Gamma_g = \frac{R_g-Z_o}{R_g+Z_o}$

Then, the 1st forward traveling wave is $\tau_g\delta\left(t-\frac{z}{v}\right)$.

The wave is bounced back to source with Γ_L. The 2nd backward traveling wave is $\tau_g\Gamma_L\delta\left(t-\frac{z}{v}-\frac{2l}{v}\right)$

The 3rd wave is bounced again at source end with Γ_g, which is $\tau_g\Gamma_L\,\Gamma_g\delta\left(t-\frac{z}{v}-\frac{2l}{v}\right)$

The 4th backward traveling wave is bounced at load end, which is $\tau_g\Gamma_L^2\,\Gamma_g\delta\left(t-\frac{z}{v}-\frac{4l}{v}\right)$

…

The voltage impulse response function is

$$V(z,t) = \tau_g\sum_{n=0}^{\infty}(\Gamma_L\,\Gamma_g)^n\;\delta\left(t-\frac{z}{v}-n\frac{2l}{v}\right) \qquad \text{- forward traveling wave}$$

$$+\tau_g\Gamma_L\sum_{n=0}^{\infty}(\Gamma_L\,\Gamma_g)^n\;\delta\left(t+\frac{z}{v}-(n+1)\frac{2l}{v}\right) \qquad \text{- backward traveling wave}$$

- Green terms above: magnitude of the wave, the same shown in bounce diagram.
- Red − sign: forward traveling wave in $+z$ direction; red + sign: backward traveling wave in $-z$ direction
- Orange delay time: every forward traveling wave add $\frac{2l}{v}$ delay time to the original forward traveling wave; the 1st backward traveling wave has $\frac{2l}{v}$ delay time, then every backward traveling wave add $\frac{2l}{v}$ delay time to the original backward traveling wave.

The current impulse response function is

$$I(z,t) = \frac{\tau_g}{Z_o}\sum_{n=0}^{\infty}(\Gamma_L\,\Gamma_g)^n\;\delta\left(t-\frac{z}{v}-n\frac{2l}{v}\right)$$ - forward traveling wave

$$-\frac{\tau_g}{Z_o}\Gamma_L\sum_{n=0}^{\infty}(\Gamma_L\,\Gamma_g)^n\;\delta\left(t+\frac{z}{v}-(n+1)\frac{2l}{v}\right)$$ - backward traveling wave

Two tips for deriving current $I(z,t)$ from $V(z,t)$

- Divide each voltage term by characteristic impedance Z_o
- Negate the sign in front of every BACKWARD traveling term.

1.2 Step 2: derive the V and I wave form for arbitrary input function $f_i(t)$.

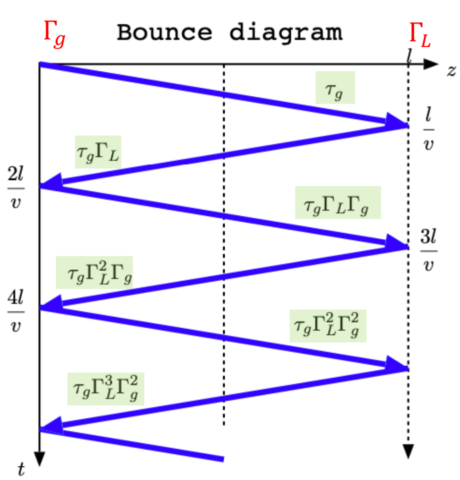
By convolution, we can derive the output of the TL circuit from the above voltage and current impulse response functions $y(t) = x(t) * h(t)$, because $f_i(t) * \delta(t - t_o) = f_i(t - t_o)$.

$$V(z,t) = \tau_g\sum_{n=0}^{\infty}(\Gamma_L\,\Gamma_g)^n\;f_i\left(t-\frac{z}{v}-n\frac{2l}{v}\right)+\tau_g\Gamma_L\sum_{n=0}^{\infty}(\Gamma_L\,\Gamma_g)^n\;f_i\left(t+\frac{z}{v}-(n+1)\frac{2l}{v}\right)$$

$$I(z,t) = \frac{\tau_g}{Z_o}\sum_{n=0}^{\infty}(\Gamma_L\,\Gamma_g)^n\;f_i\left(t-\frac{z}{v}-n\frac{2l}{v}\right)-\frac{\tau_g}{Z_o}\Gamma_L\sum_{n=0}^{\infty}(\Gamma_L\,\Gamma_g)^n\;f_i\left(t+\frac{z}{v}-(n+1)\frac{2l}{v}\right)$$

1.3 $V(z,t)$ at mid-point on the TL

For an impulse input, the voltage arrives at mid-point of the TL at $\frac{l}{2v}$ time; then it takes an $\frac{3l}{2v}$ time to bounce back from load end to mid-point again, …



Compile with the three coefficients τ_g, Γ_L, and Γ_g, together with the green font magnitude for the waves in the bounce diagram, we can write the $V(z,t)$ at the mid-point of the TL as

$$V(z,t)\big|_{z=\frac{l}{2}} = \tau_g\delta\left(t-\frac{l}{2v}\right)+\tau_g\Gamma_L\delta\left(t-\frac{3l}{2v}\right)$$
$$+\,\tau_g\Gamma_L\,\Gamma_g\delta\left(t-\frac{5l}{2v}\right)+\tau_g\Gamma_L^2\,\Gamma_g\delta\left(t-\frac{7l}{2v}\right)$$
$$+\cdots$$

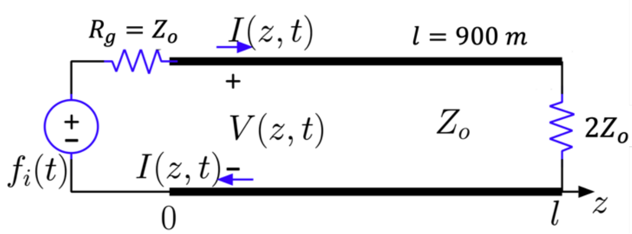
For the current, pay attention to the signs for backward waves

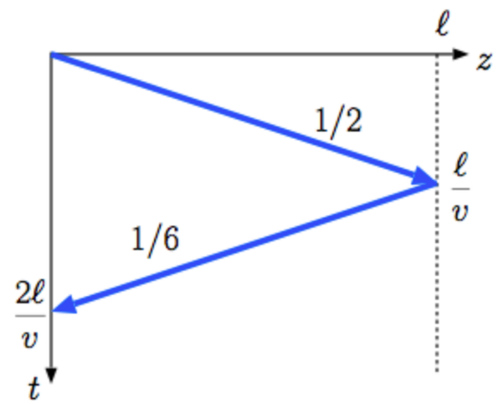
$$I(z,t)|_{z=\frac{l}{2}} = \frac{\tau_g}{Z_o}\delta\left(t-\frac{l}{2v}\right) - \frac{\tau_g}{Z_o}\Gamma_L\delta\left(t-\frac{3l}{2v}\right) + \frac{\tau_g}{Z_o}\Gamma_L\,\Gamma_g\delta\left(t-\frac{5l}{2v}\right) - \frac{\tau_g}{Z_o}\Gamma_L^2\,\Gamma_g\delta\left(t-\frac{7l}{2v}\right) + \cdots$$

↑ Forward ↑ Backward ↑ Forward ↑ Backward

2 Bounce diagram example with matched source end

Example 1: A transmission line of length $l = 900$ m and speed $v = c$ has $R_L = 2Z_o$. It is excited by a generator having an open circuit voltage $f_i(t) = sin(\omega t)u(t)$ and Thevenin resistance $R_g = Z_o$. The source frequency is $\frac{\omega}{2\pi} = 1$ MHz. Determine the voltage response $V(z,t)$ in the circuit after first determining the impulse response function $h_z(t)$.



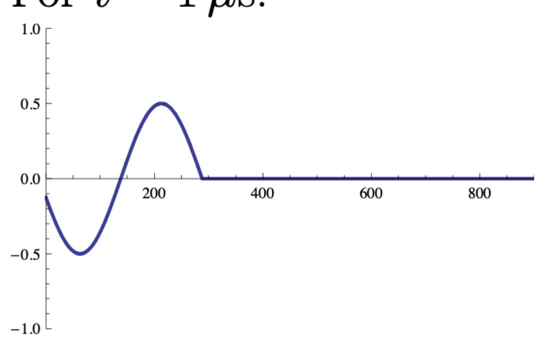


The TL has matched impedance at the source end $R_g = Z_o$, so $\Gamma_g = 0$.

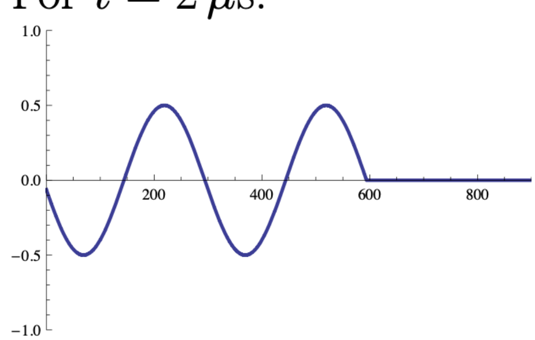
The bounce diagram on the right only has two waves, and the backward traveling wave is totally absorbed at source end.

The position plots for $t = 1\ us$, and $t = 2\ us$ are the following:

For $t = 1\ \mu$s:



For $t = 2\ \mu$s:



3 Bounce diagram example

Example2: Consider a TL circuit where $Z_o = 50\ \Omega$, $v = c$, $l = 2400$ m, $R_g = 0$, and $R_L = 100$ Ω. Determine and plot $V(1200, t)$ if $f_i(t) = u(t)$.