Lecture 30

1 Transmission line circuit

The voltage and current variations on TL's are d'Alembert solutions

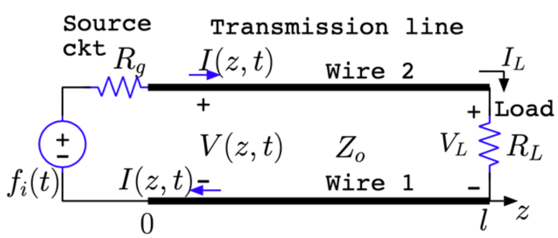
$$V(z,t) = f\left(t - \frac{z}{v}\right) + g\left(t + \frac{z}{v}\right) \tag{1}$$

$$I(z,t) = \frac{f\left(t - \frac{z}{v}\right)}{Z_o} - \frac{g\left(t + \frac{z}{v}\right)}{Z_o} \tag{2}$$

Where propagation speed $v \equiv \frac{1}{\sqrt{\mathcal{L}\mathcal{C}}}$ and characteristic impedance $Z_o \equiv \sqrt{\frac{\mathcal{L}}{\mathcal{C}}}$.

Here $f\left(t - \frac{z}{v}\right)$ represents the forward traveling wave in $+z$ direction, and $g\left(t + \frac{z}{v}\right)$ represents the forward traveling wave in $-z$ direction.

We want to find out the V and I distributions on the TL, which are functions of z and t.



Let's consider the distributed problem as the TL segments, the source ckt (with Thervnin source) and load ckt (with load impedance R_L).

We will apply the usual rules of lumped circuit analysis at element terminals and treat the TL's in terms of d'Alembert solutions above.

2 Matched TL

2.1 Matched impedance at load end

We want to determine voltage and current signals $V(z,t)$ and $I(z,t)$ on the TL in terms of source signal $f_i(t)$. Let's first consider the matched case, where the load impedance $R_L = Z_o$.

From the V and I at the load end $z = l$,

$$\frac{V(l,t)}{I(l,t)} = \frac{V_L}{I_L} = R_L = Z_o$$

Substitute equation (1) and (2) and we get

$$\frac{V(l,t)}{I(l,t)} = \frac{f\left(t - \frac{l}{v}\right) + g\left(t + \frac{l}{v}\right)}{\frac{f\left(t - \frac{l}{v}\right)}{Z_o} - \frac{g\left(t + \frac{l}{v}\right)}{Z_o}} = Z_o \frac{f\left(t - \frac{l}{v}\right) + g\left(t + \frac{l}{v}\right)}{f\left(t - \frac{l}{v}\right) - g\left(t + \frac{l}{v}\right)} = R_L = Z_o$$

Which is only possible if $g\left(t + \frac{l}{v}\right) = 0$, for all t.

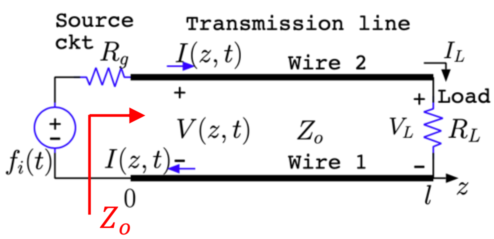
Therefore, for the matched TL, $R_L = Z_o$

- There's no backward traveling wave, $g\left(t+\frac{l}{v}\right) = 0$.
- Reflection coefficient $\Gamma_L = \frac{g}{f} = 0$.
- Voltage distribution on the TL, $V(z,t) = f\left(t-\frac{z}{v}\right)$
- Current distribution on the TL, $I(z,t) = \frac{f\left(t-\frac{z}{v}\right)}{Z_o}$

2.2 Injection coefficient at source end

Then, at the input end of the TL, where $z = 0$,

$V(0,t) = f(t)$ and $I(0,t) = \frac{f(t)}{Z_o}$



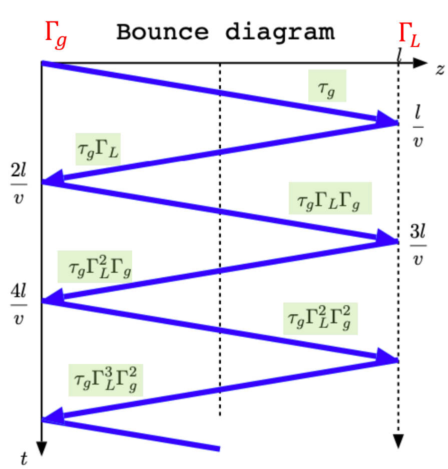
At $z = 0$, we can view the effect of TL as an input impedance Z_o, which is connected in series with generator resister R_g and voltage source $f_i(t)$.

Then by the voltage division rule,

$$f(t) = \tau_g f_i(t) = \frac{Z_o}{R_g + Z_o} f_i(t)$$

τ_g is called the injection coefficient.

3 Bounce diagram overview



A convenient tool for understanding TL transients is a bounce diagram. The horizontal axis of the diagram is the z coordinate, and the downward axis is time t.

The load reflection coefficient Γ_L is put in red at $z = l$, and generator reflection coefficient Γ_g is in red font at $z = 0$.

$\frac{l}{v}$ is the time for a wave to propagate across the line.

The blue arrows are the wave that is bouncing back and forth in between the source end and load end.

The light green values give the wave magnitude after each reflection.

The first time the wave propagates into the TL, its magnitude is injection coefficient τ_g.

Then, each time it bounces, either at load or source end, its magnitude is multiplied with reflection coefficient, Γ_L or Γ_g.

4 Coefficients in Bounce diagram

At the source end, injection coefficient $\tau_g = \frac{Z_o}{R_g+Z_o}$. This is the coefficient of the forward traveling wave when it was injected into the TL (for the 1st time).

When the wave reaches at the load end, reflection coefficient $\Gamma_L = \frac{R_L-Z_o}{R_L+Z_o}$.

When the backward traveling wave comes back from the load to source end, we will define a generator coefficient Γ_g.