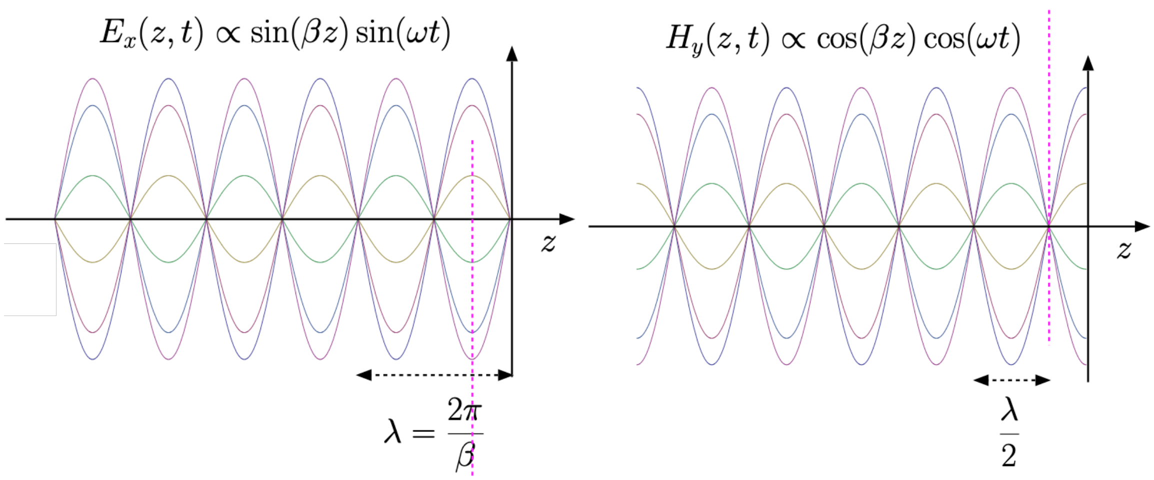
Lecture 28

1 Standing wave and power density

1.1 Standing wave pattern

For standing wave, $\tilde{\vec{E}} = -j\hat{x}E_o\, 2\sin(\beta_1 z)$ and $\tilde{\vec{H}} = \hat{y}E_o\, 2\cos(\beta_1 z)$



Here the wavelength is $\lambda = \frac{2\pi}{\beta}$

What is the distance between the zeros (nodes) of the standing wave? $\frac{\lambda}{2}$

What is the distance between the peaks (antinodes) of the standing wave? $\frac{\lambda}{2}$

What is the distance between the zeros and peaks of the standing wave? $\frac{\lambda}{4}$

1.2 Standing wave power density

The time average Poynting vector $\langle \vec{E} \times \vec{H} \rangle = 0$

Standing waves carry no net energy.

1.3 Standing wave boundary condition

Why is electric field equals zero at the interface $E_x(z,t)|_{z=0} = 0$?

Due to boundary condition, $\hat{n} \times (\vec{E}^+ - \vec{E}^-) = 0$. For the region $z > 0$, we have $\vec{E}^+ = 0$. So $\vec{E}^- = 0$ for $z < 0$.

Why is magnetic field magnitude $H_y(z,t)|_{z=0}$ reaches maximum at the interface?

> Due to boundary condition, $\hat{n} \times (\vec{H}^+ - \vec{H}^-) = \vec{J}_s$. For the region $z > 0$, we have $\vec{H}^+ = 0$. So magnetic field is experiencing a jump at interface $z = 0$ due to surface current density $\vec{J}_s$.

$$\vec{H}(0,t) = \hat{x}\frac{2E_o}{\eta_1}\cos(\omega t)\ A/m \qquad \text{and} \qquad \vec{J}_s = \hat{x}\frac{2E_o}{\eta_1}\cos(\omega t)\ A/m$$

2 Effective surface current

Previously we examined the surface current density $\vec{J}_s$ on interface, in the perfect conductor case.

Now, if we have a very good conductor on region 2 instead of perfect conductor, then the transmitted field $\tilde{\vec{E}}$ and $\tilde{\vec{H}}$ in the region 2 is

$$\tilde{\vec{E}}_t = \hat{x}\tau E_0 e^{-\gamma_2 z} \qquad \text{and} \qquad \tilde{\vec{H}}_t = \hat{y}\frac{\tau E_0}{\eta_2}e^{-\gamma_2 z}$$

The volumetric current density inside the good conductor is

$$\tilde{\vec{J}}_t = \sigma_2 \tilde{\vec{E}}_t = \hat{x}\sigma_2 \tau E_0 e^{-\gamma_2 z}$$

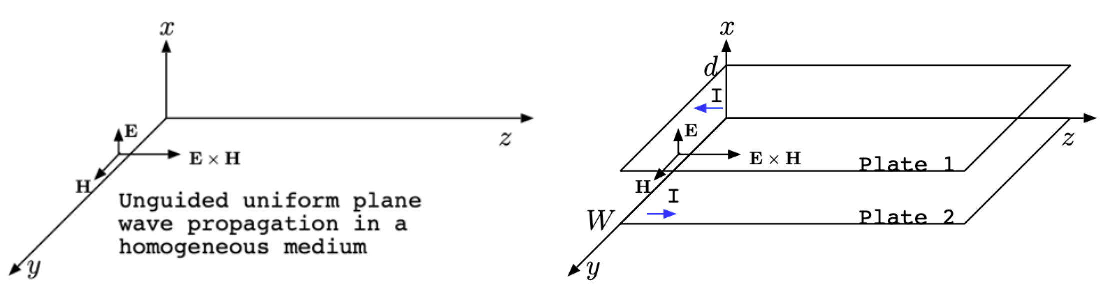
The depth integral of the volumetric current density in Region 2, that is, the effective surface current of the region in the phasor form is

$$\int_0^\infty \tilde{\vec{J}}_t\, dz = \hat{x}\frac{2E_o}{\eta_o}\ A/m$$

Which matches the surface current density in the perfect conductor case.

3 Transmission Lines

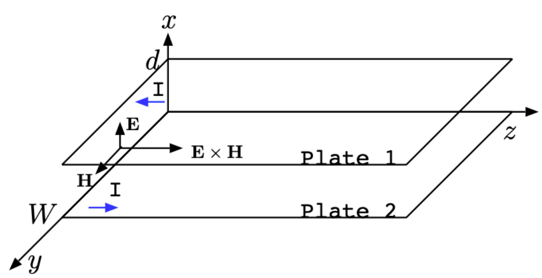
We suppose there's an unguided uniform plane wave propagates in the 3-D space in a homogeneous medium. Then we bring in two metallic plates to $x = 0$ and $x = d$.

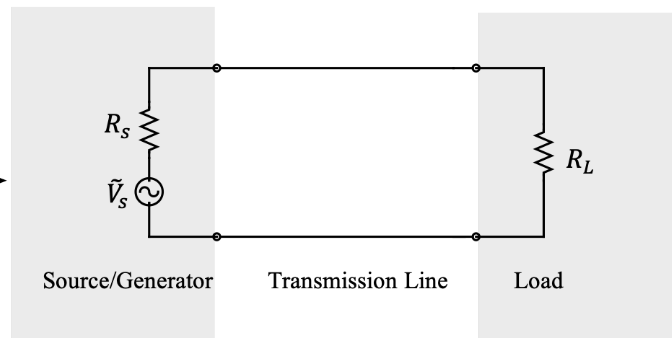


Then it will introduce positive charges on the upper surface of bottom plate, and negative charges on the bottom surface of top plate (two inner surfaces), so that the electric field is pointing from positive charges to negative charges (in the $+x$ direction).

Also, there's current I flowing to the right on bottom plate, and the same current I flows back to the left on top plate. By the right-hand rule, the surface current is introducing a magnetic field pointing in $+y$ direction in between the plates.

We assume the distance between the two plates d is small, and the width of the plates in y direction W is large, so that the fields can be idealized as if they are all trapped in between the plates, and there's no fringing field.





In general, we can excite the transmission line using a Thevenin or Norton circuit on the source/generator end, and terminate the transmission line with any resister, inductor, or capacitor on the load end.

Then, we will use small circuit analysis on both source and load ends, using KCL & KVL; and use transmission line theory for the long transmission line.