Lecture 27

1 Waves propagation example

A monochromatic plane wave propagating in a vacuum in the region $x < 0$ has an electric field phasor given by

$$\tilde{\vec{E}}_i = (j\hat{z} - \hat{y})e^{-j2\pi x} \text{ V/ m}$$

The wave encounters a planar boundary at $x = 0$ which separates the vacuum from a perfect dielectric material in the region $x > 0$which has magnetic permeability $\mu = \mu_0$ and electric permittivity $\epsilon > \epsilon_0$. The electric field phasor of the reflected wave in the $x < 0$ region is given by

$$\tilde{\vec{E}}_r = -\frac{1}{2}(j\hat{z} - \hat{y})e^{-j2\pi x} \text{ V/ m}$$

a) What are the polarizations of the incident and reflected waves?

b) What is the linear frequency f of the wave?

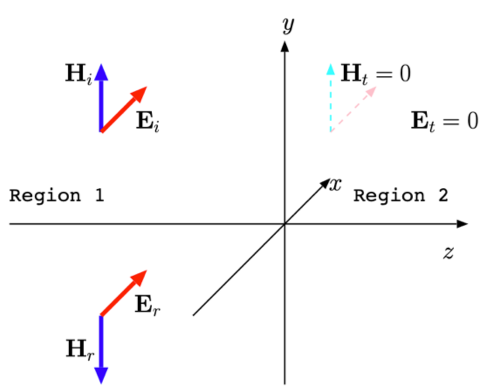
c) What is the permittivity ϵ of the dielectric material?

d) Write the phasor expression for the electric field waveform that is transmitted into the dielectric medium.

e) What percentage of the time-averaged incident power per unit area is transmitted into the dielectric medium?

2 Standing wave

A monochromatic plane wave propagating in a vacuum in the region $x < 0$ has an electric field phasor given by



Incident wave (a traveling wave going in z-direction):

$$\tilde{\vec{E}}_i = \hat{x}E_o e^{-j\beta_1 z} \quad \text{and} \quad \tilde{\vec{H}}_i = \hat{y}\frac{E_o}{\eta_1}e^{-j\beta_1 z}$$

Assuming a perfectly conducting mirror is placed at $z = 0$ surface, we total reflection, reflection coefficient $\Gamma = -1$.

Reflected wave (a traveling wave going in -z-direction):

$$\tilde{\vec{E}}_r = -\hat{x}E_o e^{j\beta_1 z} \quad \text{and} \quad \tilde{\vec{H}}_i = \hat{y}\frac{E_o}{\eta_1}e^{j\beta_1 z}$$

The incident wave and reflected wave in this case forms a standing wave.

$$\tilde{\vec{E}} = \tilde{\vec{E}}_i + \tilde{\vec{E}}_r = \hat{x}E_o\left(e^{-j\beta_1 z} - e^{j\beta_1 z}\right) = -j\hat{x}E_o\, 2\sin(\beta_1 z)$$

$$\tilde{\vec{H}} = \tilde{\vec{H}}_i + \tilde{\vec{H}}_r = \hat{y}\frac{E_o}{\eta_1}\left(e^{-j\beta_1 z} + e^{j\beta_1 z}\right) = \hat{y}E_o\, 2\cos(\beta_1 z)$$

This is due to Euler's identity, $\frac{e^{j\phi}+e^{-j\phi}}{2} = \cos\phi$, and $\frac{e^{j\phi}-e^{-j\phi}}{2} = \sin\phi$.

If we multiply the total electric and magnetic phasor with $e^{j\omega t}$ and take their real part, we get the time and space vary form for $\vec{E}$ and $\vec{H}$ as

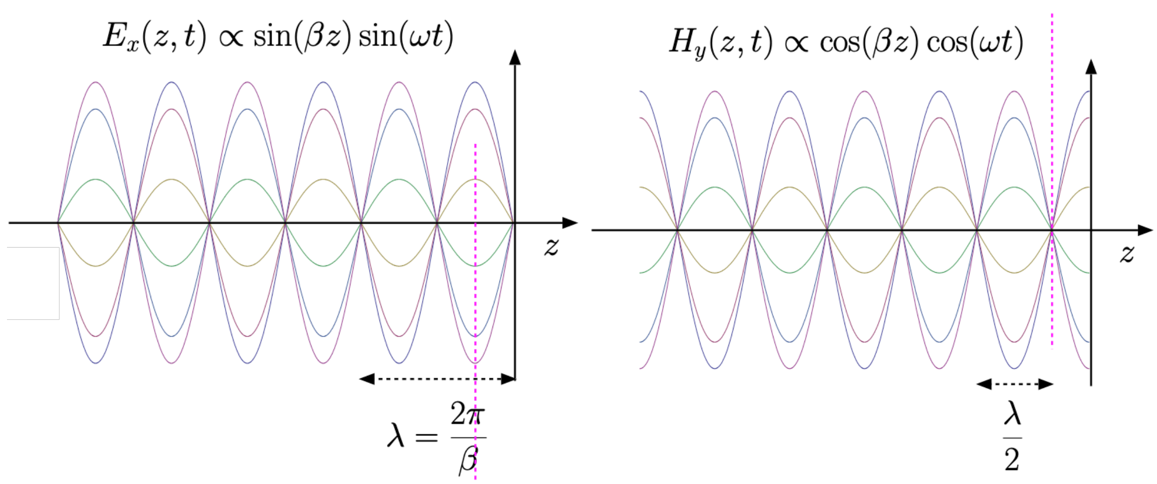
$$\vec{E}(z,t) = \hat{x}2E_o\sin(\beta_1 z)\sin(\omega t)$$

$$\vec{H}(z,t) = \hat{y}2E_o\cos(\beta_1 z)\cos(\omega t)$$

Q: Why is this wave standing wave? Why isn't it propagating?

A: Because the propagating wave has the d'Alembert form as $f(t \mp z)$, or $\cos(\omega t - \beta z)$ type, which describing the moving phase.

The standing wave patterns of $\vec{E}$ and $\vec{H}$ looks as follows:



Here the wavelength is $\lambda = \frac{2\pi}{\beta}$

What is the distance between the zeros (nodes) of the standing wave? ________________

What is the distance between the peaks (antinodes) of the standing wave? _____________

What is the distance between the zeros and peaks of the standing wave? ______________