Lecture 26

1 Wave polarization example

What is the average power density of the circular polarized carrier signal

$$E_c = cos(\omega t - \beta z)\hat{x} + sin(\omega t - \beta z)\hat{y}$$

in the region z > 0, assumed to be vacuum?

Step:

a) Find phasor form of $\tilde{\vec{E}}_c =$ ____________________________

b) Get the phasor form of $\tilde{\vec{H}}_c$, need to consider $x-$ and $y-$ polarized waves separately to find $\vec{H}$ direction.

For $x-$ polarized wave, $\tilde{\vec{H}}_c$ is pointing in ______ direction;

For $y-$ polarized wave, $\tilde{\vec{H}}_c$ is pointing in ______ direction;

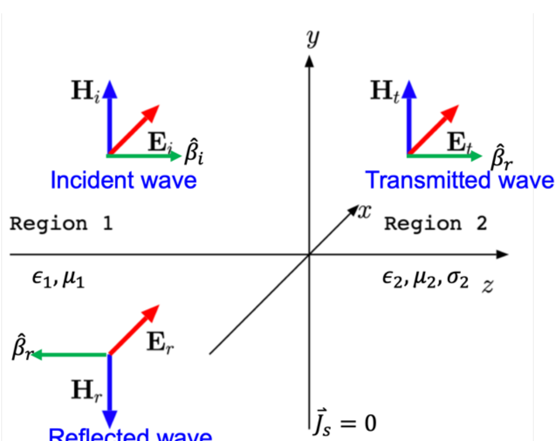
Combine them, the phasor form of $\tilde{\vec{H}}_c =$ ___________________________________

c) Calculate average power density

$\frac{1}{2}Re\{\tilde{\vec{E}}_c \times \tilde{\vec{H}}_c^*\}=$ _________________________________

2 Wave propagation

Suppose we have two homogeneous regions in the space, and a TEM wave (incident) is propagating from the first lossless medium to the second lossy medium, in general, part of the wave is going to be reflected, and the rest will continue to propagate as transmitted wave in the second medium.



Interface $z = 0$

All waves must satisfy Maxwell's equation inside each medium.

At the $z = 0$ interface, the waves must also satisfy boundary conditions.

$$\hat{n} \cdot (\vec{D}^+ - \vec{D}^-) = \rho_s$$
$$\hat{n} \cdot (\vec{B}^+ - \vec{B}^-) = 0$$
$$\hat{n} \times (\vec{E}^+ - \vec{E}^-) = 0$$
$$\hat{n} \times (\vec{H}^+ - \vec{H}^-) = \vec{J}_s$$

Suppose there's no surface current density $\vec{J}_s$ on the interface, then we know from boundary condition that both tangential $\vec{E}$ and the tangential $\vec{H}$ must be continuous on $z = 0$ plane.

Note that there can still be a volumetric current density inside media 2, since $\vec{J} = \sigma\vec{E}$.

2.1 Incident wave

Suppose media 1 is lossless $\sigma = 0$, so it's a perfect dielectric, then the incident wave is propagating in $+z$ direction:

$$\tilde{\vec{E}}_i = \hat{x}E_o e^{-j\beta_1 z} \qquad \text{and} \qquad \tilde{\vec{H}}_i = \hat{y}\frac{E_o}{\eta_1}e^{-j\beta_1 z}$$

Where intrinsic impedance $\eta_1 = \sqrt{\frac{\mu_1}{\epsilon_1}}$, and wave number $\beta_1 = \omega\sqrt{\mu_1\epsilon_1} = \frac{\omega}{v_1}$, $v_1 = \frac{1}{\sqrt{\mu_1\epsilon_1}}$ for media 1.

Note that for perfect dielectric, there's no decay factor α_1.

2.2 Reflected wave

Reflected wave is propagating again in media 1, but in $-z$ direction.

$$\tilde{\vec{E}}_r = \hat{x}\Gamma E_o e^{j\beta_1 z} \qquad \text{and} \qquad \tilde{\vec{H}}_r = -\hat{y}\frac{\Gamma E_o}{\eta_1}e^{j\beta_1 z}$$

We call that Γ as reflection coefficient.

Reflected wave has 3 differences compared with the incident wave.

- Direction – now in -z.
- Magnitude – ΓE_o, multiply with reflection coefficient
- Direction of magnetic field – now in $-\hat{y}$ direction, so that $\hat{x} \times -\hat{y} = -\hat{z}$ is complying with right hand rule.

2.3 Transmitted wave

Transmitted wave is propagating again in media 2 in the same direction as the incident wave.

$$\tilde{\vec{E}}_t = \hat{x}\tau E_o e^{\gamma_2 z} \qquad \text{and} \qquad \tilde{\vec{H}}_t = \hat{y}\frac{\tau E_o}{\eta_2}e^{\gamma_2 z}$$

We refer τ as the transmission coefficient.

Suppose media 2 is a lossy medium with conductivity σ_2, then

$\eta_2 = \sqrt{\frac{j\omega\mu_2}{\sigma_2 + j\omega\epsilon_2}}$ and $\gamma_2 = \sqrt{(j\omega\mu_2)(\sigma_2 + j\omega\epsilon_2)} = \alpha_2 + j\beta_2$.

3 Reflection and Transmission coefficient

Suppose there's no surface current density $\vec{J}_s = 0$ on $z = 0$ plane, then both tangential $\vec{E}$ and $\vec{H}$ is continuous on the interface (because of boundary condition).

We first substitute $z = 0$ to the $\tilde{\vec{E}}$ and $\tilde{\vec{H}}$ phasor form.

Incident $\tilde{\vec{E}}_i\Big|_{z=0} = \hat{x}E_o$, reflected $\tilde{\vec{E}}_r\Big|_{z=0} = \hat{x}\Gamma E_o$, and transmitted wave $\tilde{\vec{E}}_t\Big|_{z=0} = \hat{x}\tau E_o$

Incident $\tilde{\vec{H}}_i\Big|_{z=0} = \hat{y}\frac{E_o}{\eta_1}$, reflected $\tilde{\vec{H}}_r\Big|_{z=0} = -\hat{y}\frac{\Gamma E_o}{\eta_1}$, and transmitted wave $\tilde{\vec{E}}_t\Big|_{z=0} = \hat{y}\frac{\tau E_o}{\eta_2}$

Then, we apply the boundary condition on $z = 0$ plane.

a) Tangential $\vec{E}$ continuous at $z = 0$: This requires $\tilde{E}_{ix} + \tilde{E}_{rx} = \tilde{E}_{tx}$,

$$(1+\Gamma)E_o = \tau E_o \quad \Longrightarrow \quad 1+\Gamma = \tau \qquad (1)$$

b) Tangential $\vec{H}$ continuous at $z = 0$: This requires $\tilde{H}_{ix} + \tilde{H}_{rx} = \tilde{H}_{tx}$,

$$(1-\Gamma)\frac{E_o}{\eta_1} = \tau\frac{E_o}{\eta_2} \quad \Longrightarrow \quad 1-\Gamma = \frac{\eta_1}{\eta_2}\tau \qquad (2)$$

Solve for equation (1) and (2), we get

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

And

$$\tau = 1 + \Gamma$$

<u>Reflection coefficient formula is what you are banging into, minus what you are coming back to, divided by their sum</u>. (need to memorize)

Transmission coefficient τ is always $1 + \Gamma$.

4 Special cases

4.1 Region 2 is perfect conductor $\sigma_2 \to \infty$, and $\eta_2 \to 0$ (mirrors)

$$\Gamma = -1 \qquad \text{and} \qquad \tau = 0$$

4.2 Region 2 is the same as region 1 (matched impedance)

$$\Gamma = 0 \qquad \text{and} \qquad \tau = 1$$

4.3 Region 2 Lossless (partial reflections)