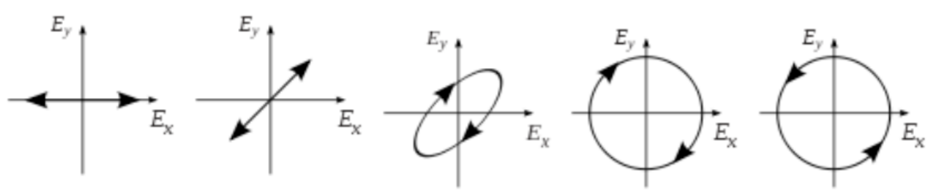
Lecture 25

1 Polarization

Polarization is a property applying to Transverse Electromagnetic (TEM)waves. We have defined the polarization of a TEM wave as the direction of the electric field vector $\vec{E}$ direction.

In general, there are a few possibilities of polarization state. Suppose we have a x polarized wave propagating in $+z$ direction as $\vec{E}(t,z) = \hat{x}E_o \cos(\omega t - \beta z)$ or its phasor $\tilde{\tilde{\vec{E}}}(z) = \hat{x}E_o e^{-j\beta z}$. The field oscillates in the $x - y$ plane, with the wave propagating in the z direction, perpendicular to the $x - y$ plane.

- In linear polarization, the fields oscillate in a single direction. The first two diagrams below trace the electric field vector over a complete cycle for linear polarization at two different orientations.
- Now if one were to introduce an arbitrary phase shift in between those horizontal and vertical polarization components, and/or give it different weights, we would generally obtain elliptical polarization as is shown in the third figure.
- In circular or elliptical polarization, the fields rotate at a constant rate in a plane as the wave travels in 4th and 5th figure.



2 Linear Polarized Waves

Field 1: $\tilde{\tilde{\vec{E}}}(z) = \hat{x}E_o e^{-j\beta z}$ x-polarized

Field 2: $\tilde{\tilde{\vec{E}}}(z) = \hat{y}E_o e^{-j\beta z}$ y-polarized

Field 3: $\tilde{\tilde{\vec{E}}}(z) = (\hat{x} + \hat{y})E_o e^{-j\beta z}$ Linear polarized in 45° angle

3 Circular Polarized Waves

Let's introduce a 90° phase shift in between the linear polarization wave (Field 1 and 2 above), we have two choices

$\tilde{\tilde{\vec{E}}}(z) = (\hat{x} - j\hat{y})E_o e^{-j\beta z}$ or $\tilde{\tilde{\vec{E}}}(z) = (\hat{x} + j\hat{y})E_o e^{-j\beta z}$

One of it is right circular polarized wave, and the other is left circular polarized wave.

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For the 1st case of $\tilde{\vec{E}}(z) = (\hat{x} - j\hat{y})E_o e^{-j\beta z}$, when the x component of E is at 0°, due to the $-j$ term, the y component of E is at -90°.

→ x is leading y by 90°, or, y is lagging x for 90°.

For the 2nd case of $\tilde{\vec{E}}(z) = (\hat{x} + j\hat{y})E_o e^{-j\beta z}$, when the x component of E is at 0°, due to the j term, the y component of E is at 90°.

→ x is lagging y by 90°, or, y is leading x for 90°.

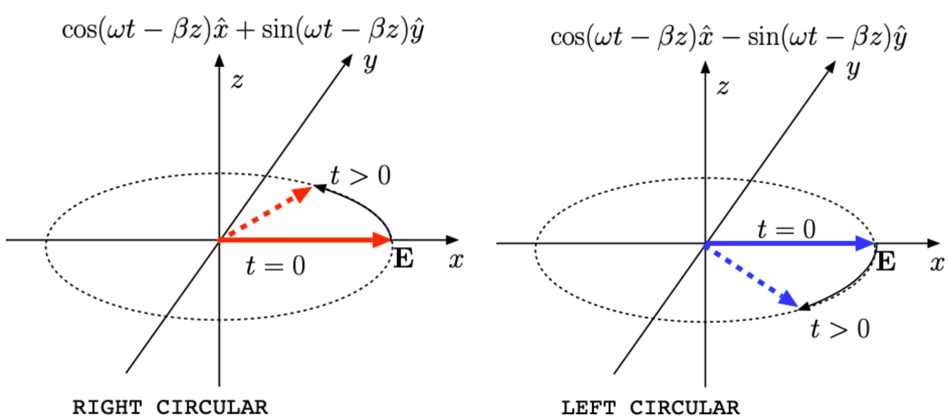
3.1 Right-Circular and Left-Circular Polarized Waves

Those are the phasor form. Let's write them in the space and time variation form. Suppose the wave is propagating in $+z$ direction, then

$$\vec{E}(z) = \hat{x}E_o \cos(\omega t - \beta z) + \hat{y}E_o \sin(\omega t - \beta z)$$

or $$\vec{E}(z) = \hat{x}E_o \cos(\omega t - \beta z) - \hat{y}E_o \sin(\omega t - \beta z)$$

We can check the direction of the vector $\vec{E}$ rotation inside $x - y$ plane,



How to decide which one is right-circular and which one is left-circular?

- Right-circular: When left-hand thumb is pointed along propagation direction z the fingers curl in the rotation direction of the field vector. (check it on the left figure above)
- Left-circular: When left-hand thumb is pointed along propagation direction z the fingers curl in the rotation direction of the field vector. (check it on the right figure above)

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4 Transformation between Linear and Circular Polarized Waves

Any arbitrary polarization can always be expressed as weighted superpositions of any pair of orthogonal polarized fields.

4.1 Linear to Circular

Right- and left- circular waves propagating in z directions are weighted superpositions of orthogonal x- and y-polarized fields

Given $\tilde{\tilde{E}}_1(z) = \hat{x}e^{-j\beta z}$ and $\tilde{\tilde{E}}_2(z) = \hat{x}e^{-j\beta z}$

we superpose them with weight of $1, -j$, and we get the right-circular wave $(\hat{x} - j\hat{y})e^{-j\beta z}$;

Using the weight of $1, j$, we get the left-circular wave $(\hat{x} + j\hat{y})e^{-j\beta z}$

4.2 Circular to Linear

x- and y-polarized waves propagating in z directions are weighted superpositions of orthogonal right- and left-circular fields.

Given $\tilde{\tilde{E}}_1(z) = (\hat{x} - j\hat{y})e^{-j\beta z}$ and $\tilde{\tilde{E}}_2(z) = (\hat{x} + j\hat{y})e^{-j\beta z}$

We add them using the weights $\frac{1}{2}, \frac{1}{2}$, and we get the x- polarized wave $\hat{x}e^{-j\beta z}$;

We add them using the weights -$\frac{1}{2j}, \frac{1}{2j}$, and we get the y- polarized wave $\hat{y}e^{-j\beta z}$.

5 Demo of Fields Propagation in 3-D space

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