Lecture 24

1 Plane waves propagating in lossy media

In the lossy media, we can have x-polarized plane wave propagating in z direction in terms of phasors as

$\tilde{\tilde{E}}(z) = \hat{x} E_o e^{\mp \gamma z}$ and $\tilde{\tilde{H}}(z) = \pm \hat{y} \frac{E_o}{\eta} e^{\mp j\beta z}$

Where $\gamma = \sqrt{(j\omega\mu)(\sigma + j\omega\epsilon)} \equiv \alpha + j\beta$, $\alpha = Re\{\gamma\}$ is the damping factor, and $\beta = Im\{\gamma\}$ is the wave number,

And $\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} \equiv |\eta| e^{j\tau}$

We summarize TEM wave parameters in homogeneous conducting media into four categories in the table below

	Condition	β	α	$\lvert\eta\rvert$	τ	$\lambda = \frac{2\pi}{\beta}$	$\delta = \frac{1}{\alpha}$
Perfect dielectric	$\sigma = 0$	$\omega\sqrt{\epsilon\mu}$	0	$\sqrt{\frac{\mu}{\epsilon}}$	0	$\frac{2\pi}{\omega\sqrt{\epsilon\mu}}$	∞
Imperfect dielectric	$\frac{\sigma}{\omega\epsilon} \ll 1$	$\sim \omega\sqrt{\epsilon\mu}$	$\beta\frac{1}{2}\frac{\sigma}{\omega\epsilon} = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}$	$\sim \sqrt{\frac{\mu}{\epsilon}}$	$\sim \frac{\sigma}{2\omega\epsilon}$	$\sim \frac{2\pi}{\omega\sqrt{\epsilon\mu}}$	$\frac{2}{\sigma}\sqrt{\frac{\epsilon}{\mu}}$
Good conductor	$\frac{\sigma}{\omega\epsilon} \gg 1$	$\sim \sqrt{\pi f \mu\sigma}$	$\sim \sqrt{\pi f \mu\sigma}$	$\sqrt{\frac{\omega\mu}{\sigma}}$	$45°$	$\sim \frac{2\pi}{\sqrt{\pi f \mu\sigma}}$	$\sim \frac{1}{\sqrt{\pi f \mu\sigma}}$
Perfect conductor	$\sigma = \infty$	∞	∞	0	-	0	0

2 Perfect dielectric ($\sigma = 0$)

$\gamma = j\beta = j\omega\sqrt{\mu\epsilon}$ and $\eta = \sqrt{\frac{\mu}{\epsilon}}$, $\alpha = \tau = 0$

3 Perfect conductor ($\sigma \to \infty$)

There's no field existing in perfect conductors.

4 Imperfect dielectric

Imperfect dielectric condition: $\frac{\sigma}{\omega\epsilon} \ll 1$

$$\gamma = \sqrt{(j\omega\mu)(\sigma + j\omega\epsilon)} = \sqrt{(j\omega\mu)(j\omega\epsilon)\left(1 - j\frac{\sigma}{\omega\epsilon}\right)}$$

Q: why can't we just ignore σ in the equation above, since $\sigma \ll j\omega\epsilon$?

Using the Taylor theories expansion $(1 \pm a)^p \approx 1 \pm pa$ for $|a| \ll 1$, we have

$$\gamma \approx j\omega\sqrt{\mu\epsilon}\left(1 - j\frac{\sigma}{2\omega\epsilon}\right) = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}} + j\omega\sqrt{\mu\epsilon}$$

Where damping factor $\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}$, and wave number $\beta = \omega\sqrt{\mu\epsilon}$

So the electric field phasor is $\tilde{\bar{E}}(z) \approx \hat{x}E_o e^{\mp(\alpha + j\beta)z}$

Also, the intrinsic impedance $\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = \sqrt{\frac{j\omega\mu}{j\omega\epsilon}}\sqrt{\frac{1}{1 - j\frac{\sigma}{\omega\epsilon}}} = \sqrt{\frac{\mu}{\epsilon}}\left(1 - j\frac{\sigma}{\omega\epsilon}\right)^{-\frac{1}{2}} \approx \sqrt{\frac{\mu}{\epsilon}}e^{j\tan^{-1}\frac{\sigma}{2\omega\epsilon}}$

Such that magnitude $|\eta| \approx \sqrt{\frac{\mu}{\epsilon}}$ and phase $\tau = \angle\eta \approx \frac{\sigma}{2\omega\epsilon}$

And the magnetic field phasor $\tilde{\bar{H}}(z) = \pm\hat{y}\frac{E_o}{\eta}e^{\mp(\alpha + j\beta)z}$

5 Good conductor

Good conductor condition: $\frac{\sigma}{\omega\epsilon} \gg 1$, so $\sigma \gg j\omega\epsilon$, we can ignore $j\omega\epsilon$ term

$$\gamma = \sqrt{(j\omega\mu)(\sigma + j\omega\epsilon)} \approx \sqrt{j\omega\mu\sigma}$$

Q: what is $\sqrt{j}$?

We can rewrite j as $j = 1e^{j\frac{\pi}{2}}$, so $\sqrt{j} = 1e^{j\frac{\pi}{4}} = \frac{1+j}{\sqrt{2}}$

$$\gamma \approx (1+j)\sqrt{\frac{\omega\mu\sigma}{2}} = (1+j)\sqrt{\pi f \mu\sigma}$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} \approx \sqrt{\frac{j\omega\mu}{\sigma}} = \sqrt{\frac{\omega\mu}{\sigma}}\, e^{j\frac{\pi}{4}}$$

Such that $\alpha \approx \beta \approx \sqrt{\pi f \mu\sigma}$ while $|\eta| \approx \sqrt{\frac{\omega\mu}{\sigma}}$ and $\tau = 45°$

6 Penetration depth

Penetration depth is defined as

$$\delta \equiv \frac{1}{\alpha}$$

It is the distance for the field strength to be reduced from e^0 by e^{-1} factor in its direction of propagation.

- For an imperfect dielectric with a fixed σ and a sufficiently large ω, the penetration depth is

$$\delta \approx \frac{2}{\sigma\sqrt{\frac{\mu}{\epsilon}}}$$

- For a good conductor with a fixed σ and a sufficiently small ω, the penetration depth (skin depth) is

$$\delta \approx \sqrt{\frac{1}{\pi f \mu\sigma}}$$