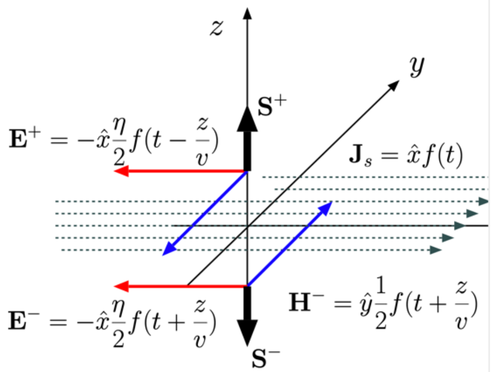
Lecture 22

1 Poynting theorem example of infinite current sheet



On $z = 0$ plane we have a time-harmonic surface current specified as $\vec{J}_s = \hat{x}f(t) = \hat{x}2\cos(\omega t)\frac{A}{m}$ where ω is some frequency of oscillation.

(a) Determine the radiated TEM wave fields $\vec{E}(z,t)$ and $\vec{H}(z,t)$ in the regions $z \gtrless 0$,

(b) The associated Poynting vectors $\vec{E} \times \vec{H}$, and

(c) $\vec{J}_s \cdot \vec{E}$ on the current sheet.

2 Time-averaged power

In the example above, when the field $\vec{E}$ and $\vec{H}$ are cosinusoidal, the Poynting vector $\vec{S} = \vec{E} \times \vec{H} \propto \cos^2(\omega t \mp \beta z)$. It varies from 0 to 1 in time.

Using the trig identity: $\cos^2(\omega t + \phi) = \frac{1}{2}[1 + \cos(2\omega + 2\phi)]$, so time average of $< \cos^2(\omega t + \phi) >= \frac{1}{2}$

Suppose the Poynting vector is $\vec{E} \times \vec{H} = \pm\eta\cos^2(\omega t \mp \beta z)\,\hat{z}\frac{W}{m^2}$

Then the time average Poynting vector: $< \vec{E} \times \vec{H} >= \pm\eta\frac{1}{2}\hat{z}\frac{W}{m^2}$

3 Phasor Notation

The solution to wave equations can be expressed as the superposition of various cosinusoidal functions: $\cos(\omega t \mp \beta z)$, where ω is the angular frequency, and β is the wave number.

Here, $\omega = 2\pi f = 1/T$, T is the time period

Wave number is defined as $\beta = \omega/v$ as the spatial frequency.

Wavelength $\lambda = \frac{2\pi}{\beta} = vf$

A monochromatic (only has one frequency component) $\hat{x}$-polarized electric field wave is

$$\vec{E} = E_o\cos(\omega t \mp \beta z)\,\hat{x}\,\frac{V}{m}$$

It can be expressed in phasor form as $\tilde{\vec{E}} = E_o e^{\mp\beta z}\hat{x}\,\frac{V}{m}$

Remember Euler's identity $e^{j\omega t} = \cos(\omega t) + j sin(\omega t)$

Such that $Re\left\{\tilde{\vec{E}} e^{j\omega t}\right\} = Re\{E_o\, cos(\omega t \mp \beta z) + j\, E_o sin(\omega t \mp \beta z)\} = E_o\, cos(\omega t \mp \beta z) = \vec{E}$

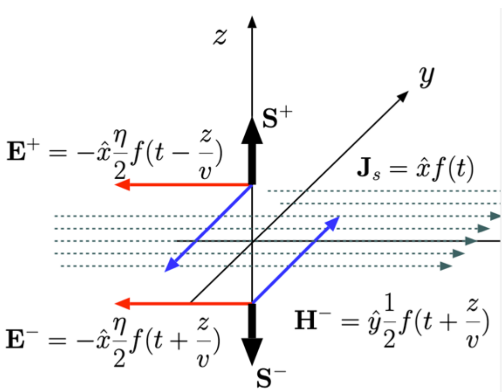
3.1 Phasor Example

Field	Phasor
$\mathbf{E} = \cos(\omega t + \beta y)\,\hat{z}$	$\tilde{\mathbf{E}} = e^{j\beta y}\,\hat{z}$
	$\tilde{\mathbf{H}} = -\frac{e^{j\beta y}}{\eta}\,\hat{x}$
$\mathbf{H} = \sin(\omega t - \beta z)\,\hat{y}$	$\tilde{\mathbf{H}} = -je^{-j\beta z}\,\hat{y}$
$\mathbf{E} = \eta \sin(\omega t - \beta z)\,\hat{x}$	$\tilde{\mathbf{E}} = -j\eta e^{-j\beta z}\,\hat{x}$

Obtain $\tilde{\vec{H}}$ from $\tilde{\vec{E}}$: $\left|\tilde{\vec{H}}\right| = \left|\tilde{\vec{E}}\right| / \eta$, also rotate $\tilde{\vec{E}}$ direction by 90° such that $\tilde{\vec{E}} \times \tilde{\vec{H}}$ points in the propagation direction (using right hand rule).

Obtain $\tilde{\vec{E}}$ from $\tilde{\vec{H}}$: $\left|\tilde{\vec{E}}\right| = \eta \left|\tilde{\vec{H}}\right|$, also rotate $\tilde{\vec{H}}$ direction by 90° such that $\tilde{\vec{E}} \times \tilde{\vec{H}}$ points in the propagation direction (using right hand rule).

3.2 Example of infinite current sheet using phasor form



For the infinite current sheet example that we have demonstrated in previous lecture, given $\vec{J}_s = \hat{x} f(t) = \hat{x} 2\, cos(\omega t) \frac{A}{m} = Re\{\hat{x} 2 e^{j\omega t}\}$, $\vec{E} = -\eta\, cos(\omega t \mp \beta z)\, \hat{x}\, \frac{V}{m}$, and $\vec{H} = \mp\, cos(\omega t \mp \beta z)\, \hat{y}\, \frac{A}{m}$.

a) Find the phasor form of $\tilde{\vec{E}}$ and $\tilde{\vec{H}}$.

b) Calculate the time average Poynting vector: $\langle \vec{E} \times \vec{H} \rangle = \frac{1}{2} Re\{\tilde{\vec{E}} \times \tilde{\vec{H}}^*\}$

c) Calculate the time average dissipated power by $\langle -\vec{J} \cdot \vec{E} \rangle = \frac{1}{2} Re\{-\tilde{\vec{J}} \cdot \tilde{\vec{E}}^*\}$