Lecture 19

1 Maxwell's Equations in free space

Maxwell's equations for time-varying electric and magnetic fields:

$$\nabla \cdot \vec{D} = \rho \qquad \text{Gauss's law}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \text{Faraday's law}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \qquad \text{Ampere's law}$$

Where

$$\vec{D} = \epsilon_o \vec{E} \text{ and } \vec{B} = \mu_o \vec{H}$$

provided that ρ and $\vec{J}$ describe the distributions of all charges and currents associated with *free and bound charge carriers*.

2 Boundary condition equations

The full set of Maxwell's boundary condition equations concerning any interface with a normal unit vector $\hat{n}$ are

$$\hat{n} \cdot \left(\vec{D}^+ - \vec{D}^-\right) = \rho_s$$

$$\hat{n} \cdot \left(\vec{B}^+ - \vec{B}^-\right) = 0$$

$$\hat{n} \times \left(\vec{E}^+ - \vec{E}^-\right) = 0$$

$$\hat{n} \times \left(\vec{H}^+ - \vec{H}^-\right) = \vec{J}_s$$

You can remember the boundary condition equations from the full set of Maxwell's Equation, by applying the transformation rules discussed in class.

3 Maxwell's Equations in material media

Maxwell's equations for time-varying electric and magnetic fields:

$$\nabla \cdot \vec{D} = \rho \qquad \text{Gauss's law}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \text{Faraday's law}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \qquad \text{Ampere's law}$$

Where $\vec{D} = \epsilon \vec{E}$ and $\vec{B} = \mu \vec{H}$

provided that ρ and $\vec{J}$ describe the distributions of all charges and currents associated with *free charge carriers* only.

3.1 Free and bound charge carriers

In material media, the charge density ρ is a combination of the free charge density ρ_f and bound charge density $-\nabla \cdot \vec{P}$:

$$\rho = \rho_f - \nabla \cdot \vec{P}$$

Also, the current density $\vec{J}$ is a combination of the free current density $\vec{J}_f$, the polarization current density $\frac{\partial \vec{P}}{\partial t}$, and the magnetization current density $\nabla \times \vec{M}$:

$$\vec{J} = \vec{J}_f + \frac{\partial \vec{P}}{\partial t} + \nabla \times \vec{M}$$

3.2 Redefinition of $\vec{D}$ and $\vec{H}$

Consider the Gauss's and Ampere's laws in free space

$\nabla \cdot \epsilon_o \vec{E} = \rho$ and $\nabla \times \mu_o^{-1} \vec{B} = \vec{J} + \frac{\partial \epsilon_o \vec{E}}{\partial t}$

Using the polarization field $\vec{P}$ and the magnetization field $\vec{M}$, we obtain

$\nabla \cdot (\epsilon_o \vec{E} + \vec{P}) = \rho_f$ and $\nabla \times (\mu_o^{-1} \vec{B} - \vec{M}) = \vec{J}_f + \frac{\partial}{\partial t}(\epsilon_o \vec{E} + \vec{P})$

So we can redefine displacement flux density $\vec{D}$ as

$$\vec{D} = \epsilon_o \vec{E} + \vec{P} = \epsilon \vec{E}$$

And magnetic field intensity $\vec{H}$ as

$$\vec{H} = \mu_o^{-1} \vec{B} - \vec{M} = \mu^{-1} \vec{B}$$

With these new definitions of $\vec{D}$ and $\vec{H}$, ρ and $\vec{J}$ are due to free charge carriers only.

3.3 Magnetic susceptibility and permeability

$$\vec{B} = \mu_o(1 + \chi_m)\vec{H} = \mu\vec{H}$$

where χ_m is a dimensionless parameter called magnetic susceptibility, $\vec{M} = \chi_m \vec{H}$;

and $\mu = \mu_o(1 + \chi_m)$ is called the permeability of the medium.

- For diamagnetic materials, $-1 \ll \chi_m < 0$. These are materials that we ordinarily think of being non-magnetic (copper, water, wood, glass, etc.)
- For paramagnetic materials, χ_m slightly larger than 0 ($1 \gg \chi_m > 0$). Paramagnetic materials are very weakly attracted to permanent magnets (aluminum, platinum, liquid oxygen, etc.)
- For ferromagnetic materials, χ_m is much larger than 1 ($\chi_m \gg 1$), where it has high magnetic susceptibility χ_m and $\vec{B}$ is non-linear with $\vec{H}$. Common ferromagnets includes iron, nickel, and cobalt and their alloys.