Lecture 18

1 Stored energy in inductors

Inductor L such as the solenoid coil considered above can be used to store energy.

An inductor connected to an external circuit with a quasi-static current I develops a voltage drop $V = L\frac{dI}{dt}$ across its terminals.

Instantaneous power that it absorbed is $P = VI = \frac{d}{dt}\left(\frac{1}{2}LI^2\right)$ [W]

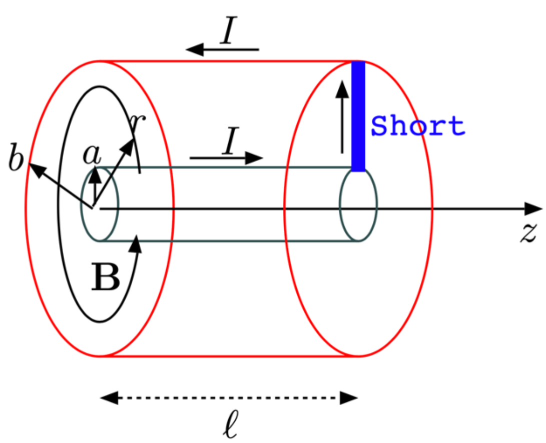
Stored energy $W = \frac{1}{2}LI^2$ [J]

For a long solenoid with inductance $L = N^2\mu_o Al$, stored energy is $W = \frac{1}{2}LI^2 = \frac{1}{2}\mu_o|H_z|^2 Al$

Stored magnetostatic energy per unit volume is $w = \frac{1}{2}\mu_o\vec{H}\cdot\vec{H}$

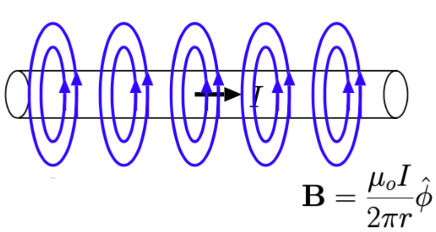
This is similar to electrostatic energy per unit volume

2 Inductance of shorted coax



Consider a coaxial cable of some length l which is "shorted" at one end, and a steady current I can flow on the inner conductor of radius a to return on the interior surface of the outer conductor at radius b after having circulated through the short.

a) Calculate magnetic field $\vec{B}$ in between the conductors



Consider the inner conductor with current I, use Ampere's Law

$$\oint_C \vec{B}\cdot d\vec{l} = \mu_o I$$

We get the magnetic field in $\hat{\phi}$ direction

$$\vec{B} = \frac{\mu_o I}{2\pi r}\hat{\phi}$$

b) Calculate magnetic flux

$$\Psi = \int_S \vec{B} \cdot d\vec{S} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Note that $\vec{B}$ is position dependence $\left(\vec{B} \propto \frac{1}{r}\right)$

c) Find inductance L

$$L = \frac{\Psi}{I} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Discussion

1) The inductance of the coax per unit length is

$$\mathcal{L} = \frac{\ln\frac{b}{a}}{2\pi}\mu_o$$

2) The capacitance of the coax per unit length is

$$\mathcal{C} = \frac{2\pi}{\ln\frac{b}{a}}\epsilon_o$$

3) The conductance of the coax per unit length is

$$\mathcal{G} = \frac{2\pi}{\ln\frac{b}{a}}\sigma$$

In summary,

- Inductance, capacitance, and conductance are associated with μ_o, ϵ_o, and σ, respectively.
- The geometric factor (blue part) for the capacitance, and conductance is the same; while the geometric factor for the inductance $\mathcal{L}$ is the inverse of the geometric factor of the capacitance $\mathcal{C}$. Or, $\mathcal{LC} = \mu_o\epsilon_o$

3 Inductance of parallel plates

4 Conservation of charge

5 Continuity equation

6 Ampere’s Law and displacement current