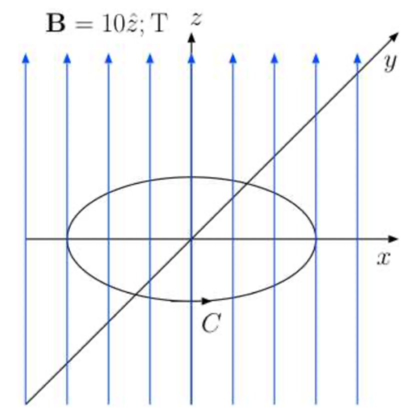
Lecture 17

1 Example of rotating loop



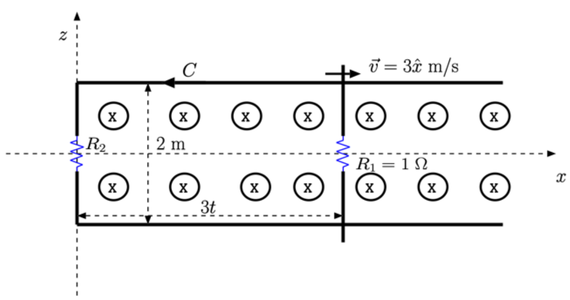
A conducting loop of a radius $r = 0.1$ m (see figure in the margin) is being rotated about the x axis with frequency of $f = \omega/2\pi = 60$ Hz in a region with a DC magnetic field of $B = 10\hat{z}$ T. Determine the induced current in the loop if the loop resistance is 12 Ω.

a) Find magnetic flux $\Psi \equiv \int_S \vec{B} \cdot d\vec{S}$

b) Calculate EMF $\mathcal{E} = -\frac{d\Psi}{dt}$

c) Current $I = \frac{\mathcal{E}}{R}$

2 Example of motional EMF



A conducting bar of resistance $R_1 = 1\ \Omega$ ohms is moved in the x-direction with a velocity $v = 3\hat{x}\ m/s$ on a pair of perfect conducting ($R = 0$) stationary rails 2 m apart terminated with a load resistance R_2 at $x = 0$, all constituting a rectangular contour C to be taken counterclockwise. A constant magnetic field of $B = 1\hat{y}\ T$ is linked through contour C.

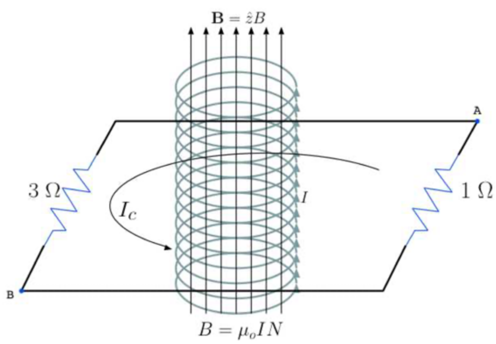
a) Find magnetic flux $\Psi \equiv \int_S \vec{B} \cdot d\vec{S}$, from that, calculate EMF $\mathcal{E} = -\frac{d\Psi}{dt}$

b) EMF can also be written as $\mathcal{E} = \oint_C (\vec{E} + \vec{v} \times \vec{B}) \cdot d\vec{l}$

c) $\vec{v}$ and $\vec{B}$ are given, find $\vec{E}$ relationship

d) If R_2 is known, calculate $\vec{E}$

3 Example of infinite solenoid



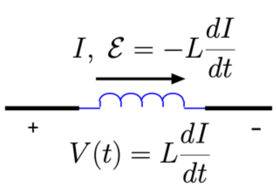
An infinite solenoid producing a constant $-\frac{d\psi}{dt} = 8\,V$, passes through a small loop consisting of a 1 Ω resistor on the right and a 3 Ω resistor on the left, connected in series — see margin plot. What is the current Ic through this resistor loop, and what voltages would be measured (by a voltmeter) across the individual resistors?

a) EMF $\mathcal{E} = -\frac{d\psi}{dt}$
b) Current $I = \frac{\mathcal{E}}{R}$
c) Voltmeter measurement across points A and B, is it 6 V or $-2\,V$?

→ Depends on the voltmeter connection

4 Inductance

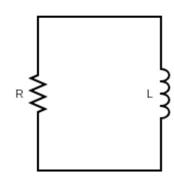
4.1 Self inductance L



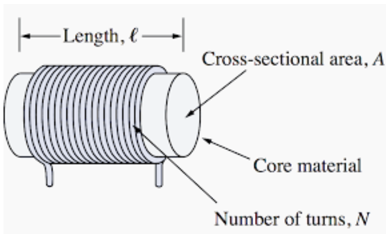
- If we have a linear flux-current relation $\Psi = LI$, then constant $L = \frac{\Psi}{I}$ is the **self-inductance** of path C, an elementary inductor.
- EMF of inductor L is $\mathcal{E} = -L\frac{dI}{dt}$, which is the voltage rise across the inductor in the direction of current I

4.2 Mutual inductance M

- If we have an external loop with current I_e, whose magnetic field lines are piecing through the original loop, then flux $\Psi = LI + MI_e$
- Differencing flux with time, we get $\frac{d\Psi}{dt} = L\frac{dI}{dt} + M\frac{dI_e}{dt} = -\mathcal{E}$
- Or, the differential equation can be written as $L\frac{dI}{dt} + RI = -M\frac{dI_e}{dt}$
  - If there's no external source, right hand side equals zero. $L\frac{dI}{dt} + RI = 0$
  - If self inductance L is small, we get $RI = -M\frac{dI_e}{dt} = \mathcal{E}$, same as last lecture



4.3 Multi-loop inductance



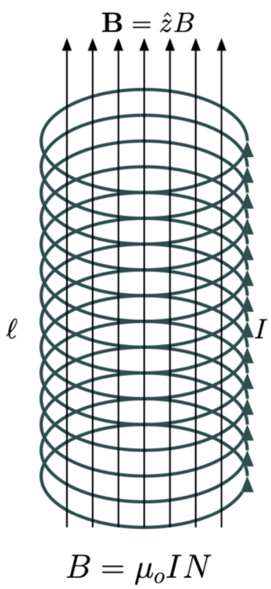
EMF $\mathcal{E}$ measured around n-loops is

$$\mathcal{E} = n\left(-\frac{d\Psi}{dt}\right) = -\frac{d}{dt}n\Psi = -L\frac{dI}{dt}$$

And the inductance is $L = \frac{n\Psi}{I}$

Note: lowercase n is number of loops.

5 Example of infinite solenoid



Consider a long solenoid with length l, cross-sectional area A, and a density of N loops per unit length.

The magnetic flux density in the interior of the solenoid is

$$\vec{B} = \mu_o IN\hat{z}$$

The magnetic flux is $\Psi = \int_S \vec{B} \cdot d\vec{S} = \mu_o INA$

Note: Uppercase N is number density (number of loops per unit length

So $n = Nl$

The inductance of the solenoid is

$$L = \frac{n\Psi}{I} = \underline{\hspace{6cm}}$$

L increases quadratically with number density N or number of loops n