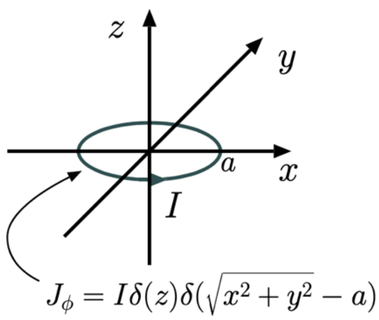
Lecture 16

1 Magnetic field from a loop



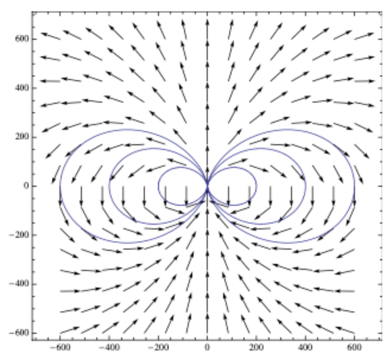
General solution to the vector potential Poisson's equation

$$\nabla^2 \vec{A} = -\mu_o \vec{J}$$

Is

$$\vec{A}(r) = \int \frac{\mu_o \vec{J}(\vec{r}')}{4\pi|\vec{r}-\vec{r}'|} d^3\vec{r}'$$

Then solve for $\vec{B} = \nabla \times \vec{A}$ and $\vec{B} = \mu\vec{H}$ to get $\vec{B}$ and $\vec{H}$.



The far field structure of the magnetic field a loop is the same as the far field pattern of the electric field of a dipole.

2 Faraday's Law

Michael Faraday discovered (in 1831, less than 200 years ago) that a changing current in a wire loop induces current flows in nearby wires.

Recall that electrostatic fields produced by static charge distributions are unconditionally curl-free $\nabla \times \vec{E} = 0$

However, time-varying electric fields produced by cur- rent distributions with time-varying components are found to have, in accordance with Faraday's observations, non-zero curls

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Using Stoke's theorem, the same constraint can also be expressed in integral form

$$\oint_C \vec{E} \cdot d\vec{l} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}$$

for 'all' surfaces S bounded by all closed paths C (with the directions of C and $d\vec{S}$ related by right hand rule).

2.1 For stationary contour C

Faraday's Law can be written as

$$\oint_C \vec{E} \cdot d\vec{l} = -\frac{\partial}{\partial t}\int_S \vec{B} \cdot d\vec{S}$$

2.2 For moving countour C

Faraday's Law can be written as

$$\oint_C (\vec{E} + \vec{v} \times \vec{B}) \cdot d\vec{l} = -\frac{\partial}{\partial t}\int_S \vec{B} \cdot d\vec{S}$$

2.3 Magnetic Flux

We define the magnetic flux through a surface S as

$$\psi \equiv \int_S \vec{B} \cdot d\vec{S}$$

Then the right hand side of Faraday's law is the rate of change of magnetic flux.

2.4 Electro-motive Force (EMF)

We define the electro-motive force (EMF): $\mathcal{E} = -\frac{d\psi}{dt}$

representing the work done per unit charge to move it once around path C.

EMF acts like the voltage rises around the loop, similar to the voltage source in circuit theory. But it's a distributed voltage along the entire loop due to the external magnetic field.

If C is a physical conducting path with a total resistance R, then the EMF will drive a current around C in circulation direction

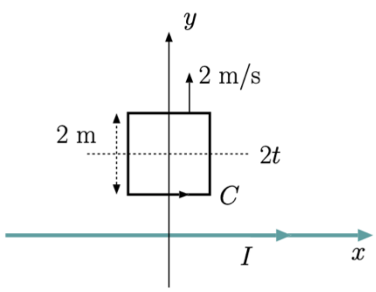
$I =$____________

2.5 Lenz's Law

The minus sign present in Faraday's law assures that induced current I produces an induced magnetic field that **opposes** the flux change producing the EMF.

3 Examples

3.1 Example 2: Moving loop example



Consider the magnetic flux density $\vec{B} = \frac{\mu_o I}{2\pi r}\hat{\phi}$ produced by current I flowing along the x axis. What is the EMF $\mathcal{E}$ of a square loop C of area $4\ m^2$ moving on xy-plane with edges parallel to x- and y-axes, if its center is located at $y = 2t\ m$ as a function of time?

Two ways of computing EMF

1) $\mathcal{E} = -\frac{d\psi}{dt}$
2) $\mathcal{E} = \oint_C (\vec{v} \times \vec{B}) \cdot d\vec{l}$

In general, we want to use method 1 because it's easier.

Note that we didn't use $\mathcal{E} = \oint_C (\vec{E} + \vec{v} \times \vec{B}) \cdot d\vec{l}$, because a static field $\vec{E}$ doesn't contribute to the line integral.

3.2 Read the rest of the examples in Lect. note 14 at home.