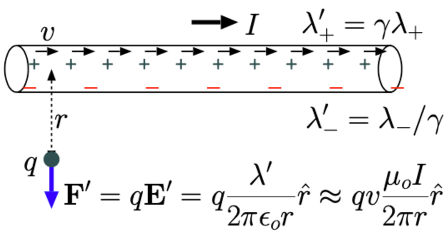
Lecture 14

1 Magnetic force

(a) In the "electron frame" the wire appears positively charged and repelsa test charge q with force F′=qE′

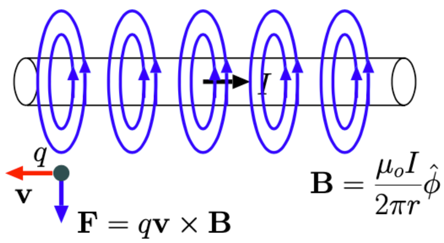


The electrostatic force $\vec{F}' = q\vec{E}'$ on a stationary charge q is

$$\vec{F}' = q\vec{E}' = q\frac{\lambda'}{2\pi\epsilon_o r}\hat{r} \approx qv\frac{\mu_o I}{2\pi r}\hat{r}$$

Assign magnitude of magnetic field $B = \frac{\mu_o I}{2\pi r}$

v is the speed of moving charge q

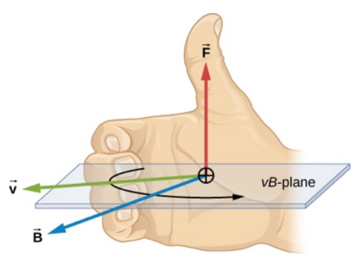


Rewrite the above force with the direction of magnetic field

$$\vec{B} \equiv \frac{\mu_o I}{2\pi r}\hat{\phi}$$

Magnetic field B curls around current I in a right handed direction designated by **azimuthal** $\hat{\phi}$.

Magnetic field lines **close upon themselves** unlike electric field lines which start and stop on point charges.



Right hand rule: point your right thumb in the direction of current flow; your fingers will point in direction $\hat{\phi}$.

$$\vec{F} = q\vec{v} \times \vec{B}$$

2 Biot-Savart law

The Biot–Savart law is fundamental to magnetostatics, playing a role similar to that of Coulomb's law in electrostatics. It describes the magnetic field generated by a constant electric current element $Id\vec{l}$.

$d\vec{B} \equiv$ ______________________________

where $\vec{r} = r\hat{r}$ is a position vector extending from the location of the current increment to the field position where $d\vec{B}$ is being specified.

It is only valid when used in terms of infinitesimal segments $Id\vec{l}$ of time-invarying current loops (static conditions).

3 Ampere's law
3.1 Integral form

The integral form of the Ampere's law

$$\oint_C \vec{B} \cdot d\vec{l} = \mu_o I_C$$

I_C stands for the net sum of all filament currents I_n crossing any surface S bounded by path C.

- Linked current. Current filaments not "linked" to path C — should not be included on the right hand side of Ampere's law.
- When calculating net current I_C, determine the sign of filament currents I_n by the "right-hand-rule":
- Contour path C can be any shape.
- Surface S can be flat, convex, or concave.

Define $\vec{H} \equiv \mu_o^{-1}\vec{B}$, unit of $\vec{H}$ is A/m, unit of $\vec{B}$ is Tesla.

Ampere's law can also be expressed as

$$\oint_C \vec{H} \cdot d\vec{l} = I_C = \int_S \vec{J} \cdot d\vec{S}$$

$\vec{J}$ is the volumetric current density measured in A/m^2 unit.

3.2 Differential form

Using Stoke's theorem $\oint_C \vec{H} \cdot d\vec{l} = \int_S \nabla \times \vec{H} \cdot d\vec{S}$ across any surface S bounded by a path C, we get $\int_S \nabla \times \vec{H} \cdot d\vec{S} = \int_S \vec{J} \cdot d\vec{S}$

So the differential form of Ampere's law should be

$$\nabla \times \vec{H} = \vec{J}$$

This differential relation is accompanied by the Gauss's law for magnetic field

$$\nabla \cdot \vec{B} = 0$$

Note that the Ampere's law above works for magnetostatics and magneto-quasistatics, not for electromagnetics.

4 Volumetric current density $\vec{J}$

Using surface current density $\vec{J}_s$: $\vec{J}(x,y,z) = \vec{J}_s(y,z)\delta(x - x_0)$

Using line current density $I(z)$: $\vec{J}(x,y,z) = \hat{z}I(z)\delta(x - x_0)\delta(y - y_0)$

Using a point charge Q: $\vec{J}(x,y,z) = Q\vec{v}\delta(x - x_0)\delta(y - y_0)\delta(z - z_0)$, which represents the time-varying current density of a point charge Q at coordinates $(x,y,z) = (x_0(t), y_0(t), z_0(t))$ moving with velocity $v = (x(t), y(t), z(t))$.