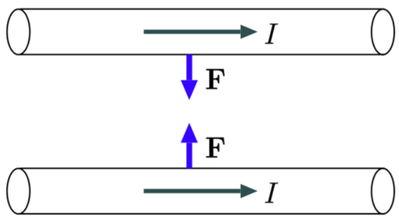
Lecture 13

1 Magnetic force



Pairs of wires carrying currents I running in the same (opposite) direction are known to attract (repel) one another.

2 Brief summary of special relativity

Newtonian transformation is only valid when the speed is slow compared to the speed of light.

Speed: $u' = u - v$ A particle velocity u could appear as u' to an observer approaching the particle with a velocity v

Location: $x' = x - vt$

Time: $t' = t$

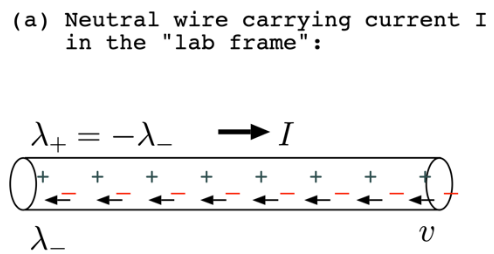
Relativistic formula

Speed: $u' = \frac{u-v}{1-\frac{uv}{c^2}}$ So that if $u = c$, then $u' = c$; when $v \ll c$, relativistic formula turns into Newtonian formula.

Location: $x' = \gamma(x - vt)$ **Lorentz contraction**. $dx = dx'/\gamma$ at a fixed t: since $\gamma > 1$, $dx < dx'$. The moving objects having velocities v appear shorter than they are when viewed from other reference frames where v is determined.

Time: $t' = \gamma(t - \frac{v}{c^2}x)$ **Time dilation**. $dt = dt'/\gamma$ at a fixed t: since $\gamma > 1$, $dt' < dt$. The moving clocks having velocities v and fixed x′ run slower than clocks in other reference frames where v is determined.

3 A current carrying stationary wire
3.1 In lab frame

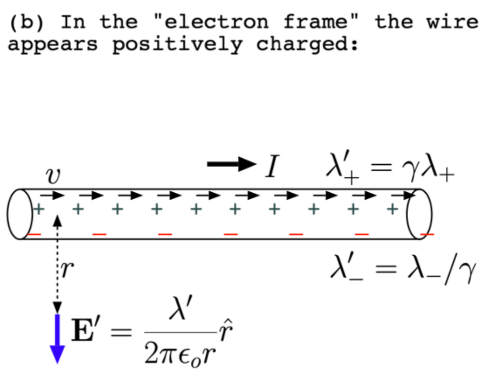


In the lab frame, electrons are moving to the left, while protons are moving to the right at the same speed v.

Define Line charge density for electron as λ_+, and Line charge density for electron as λ_-.

Line is net neutral, as $\lambda_+ + \lambda_- = 0$

3.2 In electron frame



In the electron frame, electrons are stationary, while protons are moving to the right.

Due to relativistic effect and Lorentz contraction, since $\gamma > 1$:

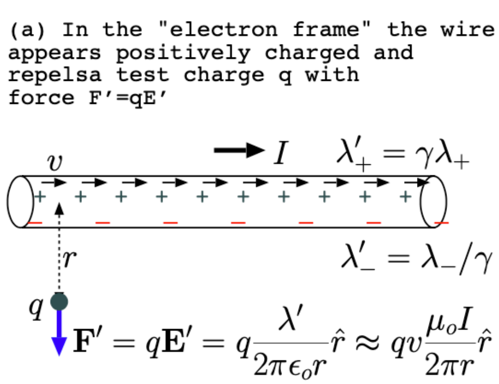
- $\lambda'_- = \lambda_-/\gamma$, electron spacing appears to be larger
- $\lambda'_+ = \gamma\lambda_+$, proton spacing appears to be shorter

Wire appears to be "charged" due to relativistic effect. Total line charge density $\lambda' = \lambda'_+ + \lambda'_- \approx \lambda_+ v^2/c^2$ (see margin in page 3)

Since $\lambda_+ = I/v$, $\lambda' \approx \frac{Iv}{c^2} = Iv\mu_0\epsilon_o$, which is the total line charge density due to relativistic calculation.

Recall we calculated the macroscopic electric field equation due to a line charge in note 2 by Gauss's Law,

$$\vec{E}' = \frac{\lambda'}{2\pi\epsilon_o r}\hat{r}$$



The electrostatic force $\vec{F}' = q\vec{E}'$ on a stationary charge q is

$$\vec{F}' = q\vec{E}' = q\frac{\lambda'}{2\pi\epsilon_o r}\hat{r} \approx qv\frac{\mu_o I}{2\pi r}\hat{r}$$