Lecture 13

1 Magnetic force

Pairs of wires carrying currents $I$ running in the same (opposite) direction are known to attract (repel) one another.

2 Brief summary of special relativity

**Newtonian** transformation is only valid when the speed is slow compared to the speed of light.

Speed: $u' = u - v$ A particle velocity $u$ could appear as $u'$ to an observer approaching the particle with a velocity $v$

Location: $x' = x - vt$

Time: $t' = t$

**Relativistic** formula

Speed: $u' = \frac{u - v}{\sqrt{1 - \frac{v^2}{c^2}}}$ So that if $u = c$, then $u' = c$; when $v \ll c$, relativistic formula turns into Newtonian formula.

Location: $x' = \gamma (x - vt)$ **Lorentz contraction.** $dx = dx'/\gamma$ at a fixed $t$: since $\gamma > 1$, $dx < dx'$. The moving objects having velocities $v$ appear shorter than they are when viewed from other reference frames where $v$ is determined.

Time: $t' = \gamma (t - \frac{v}{c^2}x)$ **Time dilation.** $dt = dt'/\gamma$ at a fixed $t$: since $\gamma > 1$, $dt' < dt$. The moving clocks having velocities $v$ and fixed $x'$ run slower than clocks in other reference frames where $v$ is determined.
3 A current carrying stationary wire

3.1 In lab frame

(a) Neutral wire carrying current $I$ in the "lab frame":

In the lab frame, electrons are moving to the left, while protons are moving to the right at the same speed $v$.

Define Line charge density for electron as $\lambda_+$, and Line charge density for electron as $\lambda_-$. Line is net neutral, as $\lambda_+ + \lambda_- = 0$

3.2 In electron frame

(b) In the "electron frame" the wire appears positively charged:

In the electron frame, electrons are stationary, while protons are moving to the right.

Due to relativistic effect and Lorentz contraction, since $\gamma > 1$:

- $\lambda'_- = \lambda_- / \gamma$, electron spacing appears to be larger
- $\lambda'_+ = \gamma \lambda_+$, proton spacing appears to be shorter

Wire appears to be "charged" due to relativistic effect. Total line charge density $\lambda' = \lambda'_+ + \lambda'_- \approx \lambda_+ v^2 / c^2$ (see margin in page 3)

Since $\lambda_+ = I / v$, $\lambda' \approx \frac{Iv}{c^2} = Iv\mu_0\varepsilon_0$, which is the total line charge density due to relativistic calculation.

Recall we calculated the macroscopic electric field equation due to a line charge in note 2 by Gauss's Law,

$$\vec{E}' = \frac{\lambda'}{2\pi\varepsilon_o r} \hat{r}$$

The electrostatic force $\vec{F}' = q\vec{E}'$ on a stationary charge $q$ is

$$\vec{F}' = q\vec{E}' = q\frac{\lambda'}{2\pi\varepsilon_o r} \hat{r} \approx qv\frac{\mu_0 I}{2\pi r} \hat{r}$$