ECE 329 Fields and Waves I
Homework 8

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Due October 17, 2023, 11:59 PM

Homework Policy:

- Write your name and NetID on top of every page. This habit will help you in exams in the event of having loose page(s).
- Tag all the questions in Gradescope. Failure to do so results in a 5 points deduction.
- Cheating results in ZERO and 50% reduction in HW average on first offense. A 100% reduction in HW average on second offense.
- Please show detailed process for each problem instead of just an answer. No partial credits would be given otherwise. All answers should include units wherever appropriate.
- No late HW is accepted.
- Regrade requests are available one week following grade release.

You are allowed to work with anyone else, but the work you submit should only belong to you. Note that if you have knowledge of a violation of the Honor Code, then you are obligated to report it. By submitting this homework, you are agreeing to the Honor Code: "I have neither given nor received unauthorized aid on this homework, nor have I concealed any violations of the Honor Code."

Question	Points	Score
1	10	
2	10	
3	14	
4	12	
5	21	
6	10	
Total:	77	

1. A z-polarized plane TEM wave is propagating in vacuum (i.e., $v = c \approx 3 \times 10^8$ m/s and $\eta = \eta_o \approx 120\pi\,\Omega$) in the $-\hat{y}$ direction. If the wave field at $y = 0$ varies with time according to $E_z(0,t) = 80\triangle(\frac{t}{\tau})$ V/m, where $\tau = 50\,\text{ns}$ and $\triangle(\frac{t}{\tau})$ is the unit triangle function with width τ,
   (a) (1 point) Determine the vector wave field $\mathbf{E}(y,t)$ written explicitly in terms of all space and time variables y and t,
   (b) (1 point) Determine the accompanying wave field $\mathbf{H}(y,t)$ in A/m units,
   (c) (2 points) Determine the maximum value of *Poynting vector* $\mathbf{E} \times \mathbf{H}$.
   (d) (2 points) What trajectory function $y = y(t)$ describes instantaneous locations of the peak of $\mathbf{E} \times \mathbf{H}$.
   (e) (2 points) Plot $E_z(y,t)$ vs t at $y = 1000$ m.
   (f) (2 points) Plot $H_x(y,t)$ vs y at $t = -200\,\text{ns}$.

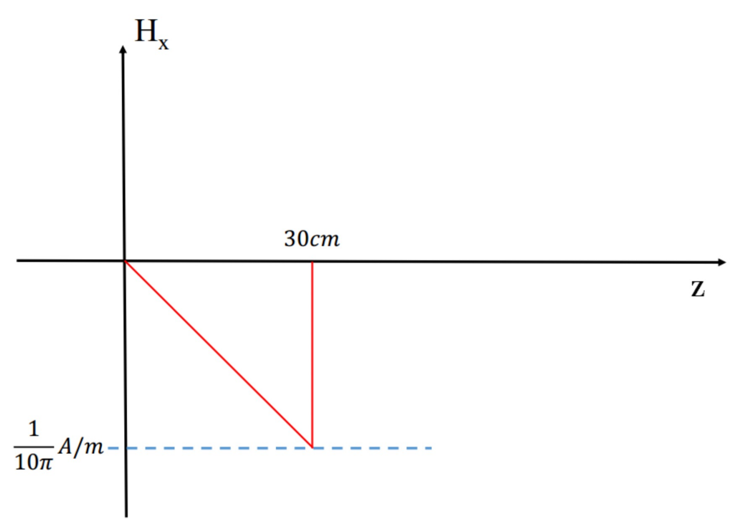
2. In a homogeneous lossless dielectric with $\epsilon = \epsilon_r\epsilon_o$, $\mu = \mu_r\mu_o$ and $\eta = \eta_o/4$, a plane TEM wave with the following components is observed:

$$\mathbf{E} = \hat{x}u\left(t - \frac{z}{c/4}\right) + \hat{y}g\left(t - \frac{z}{c/4}\right)\frac{\text{V}}{\text{m}}$$

and

$$\mathbf{H} = \hat{x}H_x(z,t) + \hat{y}H_{yo}u\left(t - \frac{4z}{c}\right)\frac{\text{A}}{\text{m}},$$

where $u(t)$ denotes the unit-step function and c is the speed of light in free space. Using the above information,
   (a) (2 points) Determine the propagation velocity v.
   (b) (4 points) Determine ϵ_r and μ_r.
   (c) (2 points) Determine the value of H_{yo}.
   (d) (2 points) If $H_x(z,t)$ at $t = 0$ is plotted below, determine $g(t)$ in terms of fundamental signal waveforms.



3. We have on $z = 0$ plane a pulse of sheet current $\mathbf{J}_s(t) = \hat{y}\, 8t\, \mathrm{rect}(\frac{t}{\tau})$ A/m, where $\tau = 80\,\mathrm{ns}$. Regions adjacent to the current sheet are vacuum.
   (a) (8 points) Determine and plot $E_y(z,t)$ and $H_x(z,t)$ vs t for $z = -600$ m and $z = +600$ m, respectively.
   (b) (4 points) Determine and plot $E_y(z,t)$ and $H_x(z,t)$ vs z for $t = 120\,\mathrm{ns}$.
   (c) (2 points) Determine the TEM wave energy radiated per unit area (in J/m^2 units) by the current pulse $\mathbf{J}_s(t)$.
   Hint: integrate the power injected per unit area, $-\mathbf{J}_s \cdot \mathbf{E}$, over the duration of pulse $\mathbf{J}_s(t)$.

4. For each of the four plane waves (in free space) described by
   (a) (3 points) $\mathbf{E}_1 = 3\cos(\omega t - \beta z)\hat{y}\,\mathrm{V/m}$
   (b) (3 points) $\mathbf{E}_2 = E_o \cos(\omega t - \beta z)\hat{x} + E_o \sin(\omega t - \beta z)\hat{y}\,\mathrm{V/m}$
   (c) (3 points) $\mathbf{H}_3 = \cos(\omega t + \beta z + \frac{\pi}{3})\hat{x} - \sin(\omega t + \beta z - \frac{\pi}{6})\hat{y}\,\mathrm{A/m}$
   (d) (3 points) $\mathbf{H}_4 = \cos(\omega t - \beta x)\hat{z} + \sin(\omega t - \beta x)\hat{y}\,\mathrm{A/m}$:
      i. Determine the expression for **H** or **E** that accompanies the given wave field.
      ii. Find the expression for instantaneous power that crosses a 1 m^2 area in the xy-plane from $-z$ to $+z$.
      iii. Find time averaged power that crosses a 1 m^2 area in the xy-plane from $-z$ to $+z$.

5. (10 points) **Conceptual Questions:**
   (a) (1 point) True or false? If $\mathbf{E_1}$ and $\mathbf{E_2}$ represent two plane waves and they are orthogonal, then their directions of propagation are opposite.
   (b) (1 point) True or false? The **E** and **H** of a given TEM wave in free space are always orthogonal
   (c) (1 point) True or false? Given **E** of a TEM wave in free space, we only know the direction but not magnitude of the corresponding to **H**.
   (d) (1 point) True or false? The propagation speed of a TEM wave in vacuum is c.
   (e) (1 point) True or false? $\mathbf{E} \times \mathbf{H}$ will always point in the direction of propogation for TEM waves.
   (f) (3 points) A wave moves in free space in the +x direction. If the magnitude and direction of the electric field is known at $x = 200m$, $t = 1\mu s$, which of the following position and time combinations is the magnitude and direction of the magnetic field known (approximately)? (choose all that apply):
   (a) $x = 100m, t = 0\mu s$
   (b) $x = 800m, t = 3\mu s$
   (c) $x = -400m, t = -1\mu s$
   (d) $x = 500m, t = 0\mu s$
   (e) none of the above

(g) (3 points) If the current sheet that produces a wave is originally $J = J_0$ in the +x direction on the y = 0 plane is changed to $J = J_0$ in the +z direction on the y = 0 plane, how does the direction of the magnetic field for $y > 0$ change?
(a) It goes from +z to -x.
(b) It goes from -x to y.
(c) It goes from -x to +z.
(d) It goes from -z to +x.
(e) none of the above

6. (10 points) **BONUS QUESTION**
Two plane waves propagate in free space and are described by

$$\mathbf{E}_1 = E_0 \cos(\omega_1 t - \beta_1 z)\hat{x}\,\mathrm{V/m}$$

$$\mathbf{E}_2 = E_0 \cos(\omega_2 t - \beta_2 z)\hat{x}\,\mathrm{V/m},$$

where ω_1, ω_2 are at optical frequencies. The two waves simultaneously illuminate an optical detector at some position along $\hat{z}$. An optical detector converts power into current and the current produced on that detector is therefore proportional to $|\ \mathbf{E}_1 + \mathbf{E}_2\ |^2$. Determine the frequencies of the currents that are produced at the detector.