ECE 329 Fields and Waves I
Homework 12

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Due November 14, 2023, 11:59 PM

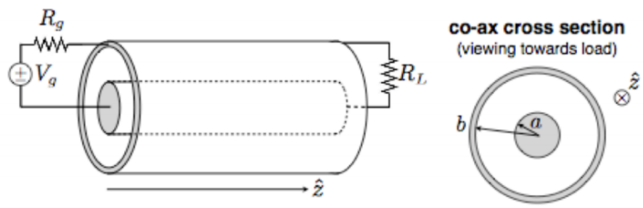
Homework Policy:

- Write your name and NetID on top of every page. This habit will help you in exams in the event of having loose page(s).
- Tag all the questions in Gradescope. Failure to do so results in a 5 points deduction.
- Cheating results in ZERO and 50% reduction in HW average on first offense. A 100% reduction in HW average on second offense.
- Please show detailed process for each problem instead of just an answer. No partial credits would be given otherwise. All answers should include units wherever appropriate.
- No late HW is accepted.
- Regrade requests are available one week following grade release.

You are allowed to work with anyone else, but the work you submit should only belong to you. Note that if you have knowledge of a violation of the Honor Code, then you are obligated to report it. By submitting this homework, you are agreeing to the Honor Code: "I have neither given nor received unauthorized aid on this homework, nor have I concealed any violations of the Honor Code."

Question	Points	Score
1	30	
2	6	
3	14	
4	15	
5	15	
6	10	
7	10	
Total:	100	

1. Co-axial (co-ax) cables are the most common kind of transmission line, whereby propagating transverse electromagnetic fields are confined within a tubular dielectric material that lies between two conducting cylinders which share the same geometric axis (see the figure below). Consider a lossless co-ax cable having characteristic impedance $Z_0 = 50\ \Omega$, inner radius $a = 1.2$ mm, and length $l = 15$ cm that is filled with a polyethelene dielectric ($\epsilon_r = 2.25,\ \mu_r = 1$). At $z = 0$, the co-ax cable is connected to a Thevenin voltage source $V_g = 10u(t)$ V with internal resistance $R_g = Z_0$, while at $z = l$, it is connected to a resistive load $R_L = 25\ \Omega$.



(a) (3 points) What is the radius of the outer conductor?

(b) (4 points) How long will it take the transmission line to reach steady state equilibrium?

(c) (5 points) Write expressions for the time-dependent voltage and current on the line at $z = \frac{l}{2}$.

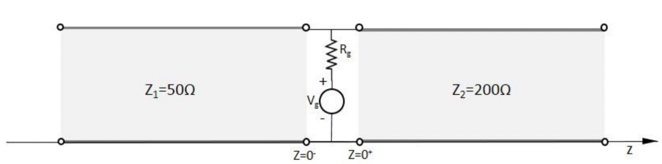
(d) (10 points) Write expressions for the associated time-dependent electric and magnetic fields inside the dielectric (i.e., for $a < r < b$) at $z = \frac{l}{2}$.

(e) (4 points) Write expressions for the associated time-dependent surface charge density ρ_s and surface current density $\mathbf{J_s}$ on the inner conductor at $z = \frac{l}{2}$.

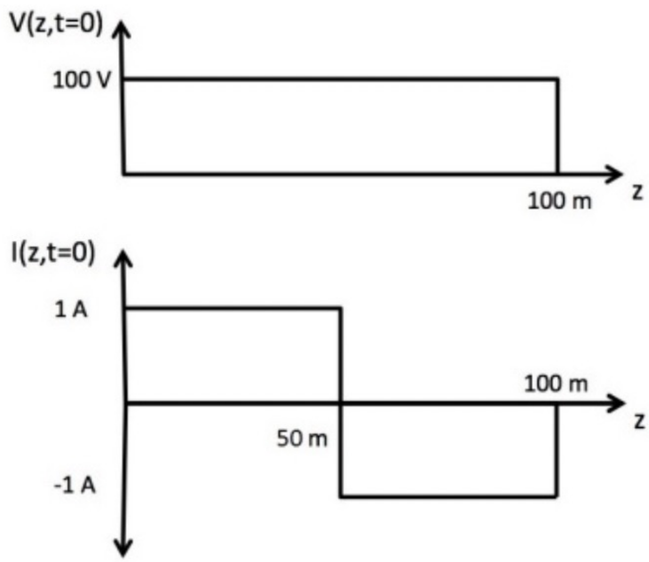
(f) (2 points) **TRUE** or **FALSE**: The surface charge and current densities at $z = \frac{l}{2}$ on the outer conductor are the same as in part (e), but with the opposite algebraic sign. Explain.

(g) (2 points) Once it reaches steady state equilibrium, what is the power delivered to the load?

2. (6 points) At the center of two infinitely long transmission lines aligned along the z-direction is a voltage source and source resistance connected in parallel to both lines. The impedance in each line is different. If $Z_1 = 50\Omega$ ($z < 0$) and $Z_2 = 200\Omega$ ($z > 0$), the source voltage $V_g = 10u(t)\,$V, where $u(t)$ is the unit step function, and the source resistance $R_g = 150\Omega$ find $V(t = 0^+, z = 0^+)$, $I(t = 0^+, z = 0^+)$, and $V(t = 0^+, z = 0^-)$, $I(t = 0^+, z = 0^-)$ i.e. the voltage and current injected into each tranmission line just as the voltage source turns on. The figure below shows the geometry.



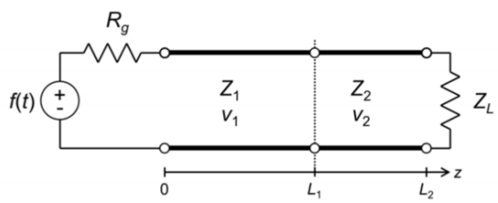
3. (14 points) A transmission line of length $L = 100$m, impedance $Z_0 = 100\Omega$, and propagation speed $v_p = 100[\frac{m}{\mu s}]$ with matched loads at both ends is initially charged such that voltage $V(z, t = 0)$ and $I(z,\ t = 0)$ are given in the figure below. Decompose these initial conditions into the forward and backward moving voltage waves and then find the voltage and current along the line at all time and plot $V(z,\ t = 0.5)$ and $I(z,\ t = 0.5)$. Note all times are in microseconds.

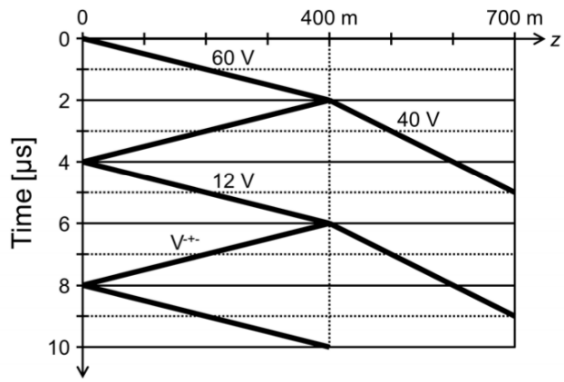


4. A generator with internal resistance $R_g = 50\,\Omega$ and an open circuit output voltage $f(t) = 90\delta(t)$ feeds a T.L. that has an unknown characteristic impedance Z_o and an unknown resistive load termination R_L at an unknown distance, L, from the generator. At a distance $z_0 = 300$ m from the generator, smaller than L, the voltage waveform as a function of time for $0 < t < 9$ μs is found to be $V(z_0, t) = 60\delta(t - 2) - 20\delta(t - 4) + \frac{20}{3}\delta(t - 8)$ V
   (a) (1 point) Determine the injection coefficient τ_g and the impedance of the transmission line, Z_o.
   (b) (2 points) Determine the load reflection coefficient Γ_L and the load resistance, R_L.
   (c) (1 point) Determine the transmission time, $T = L/v_p$, of the impulse propagating on the transmission line.
   (d) (2 points) Determine the speed of wave propagation on the line, v_p, in m/s and the length of the transmission line, L in meters.
   (e) (3 points) Determine the next two voltage impulses (magnitudes and time delays) that will be measured at z_0 on the line for $t > 9$ μs.
   (f) (4 points) Sketch a bounce diagram for the *current* waveform $I(z, t)$ — not the voltage waveform as we have often done — for $0 < t < 12\,\mu$s. Be sure to mark the numerical values for the amplitude of the current in the diagram.

(g) (2 points) What is the algebraic expression for the current waveform as a function of (z, t) for the domain $0 < z < L$ and $0 < t < 12\,\mu$s?

5. A system of two transmission lines connected in series are driven by a voltage source $f_i(t) = V_0 u(t)$ and terminated by a resistive load of $60\,\Omega$ as shown in the figure below. A switch is closed at $t = 0$ and the positive voltages are measured for 5 μs giving the bounce diagram shown in the figure — the voltage values indicated in the diagram correspond to delta function weights times the source voltage V_o, products such as $V_o\tau_g$, $V_o\tau_g(1+\Gamma_{12})$, etc.





The impedance and transmission time of line 1 are Z_1 and T_1 and those of line 2 are Z_2 and T_2. Using the figures, identify the following parameters in appropriate units:

(a) (2 points) Transmission times T_1 and T_2

(b) (2 points) Propagation velocity v_{p1} on line 1, and propagation velocity v_{p2} on line 2

(c) (1 point) Impedance Z_2

(d) (2 points) Reflection coefficient Γ_{12} between lines 1 and 2

(e) (2 points) Impedance Z_1

(f) (2 points) Source resistance R_g

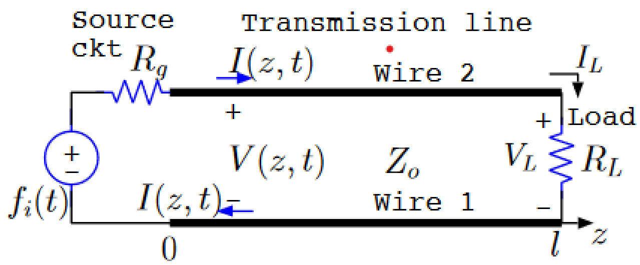
(g) (1 point) Source voltage V_o

(h) (1 point) Reflected voltage V^{-+-} on line 1 (see the diagram for this notation)

(i) (1 point) Steady state voltage V_1 on line 1 as $t \to \infty$

(j) (1 point) Steady state voltage V_2 on line 2 as $t \to \infty$

6. **Conceptual questions:** For the following questions, use the diagram below:



(a) (4 points) Suppose we increase R_g and hold R_L and Z_0 constant. Which of the following holds?
(a) τ_g increases
(b) τ_g decreases
(c) τ_g stays the same
(d) Not enough information to tell

(b) (3 points) Suppose we increase R_L and hold R_g and Z_0 constant. Which of the following holds?
(a) Γ_g increases
(b) Γ_g decreases
(c) Γ_g stays the same
(d) Not enough information to tell

(c) (3 points) Suppose we increase Z_0 and hold R_L and R_g constant. Which of the following holds?
(a) Γ_L increases
(b) Γ_L decreases
(c) Γ_L stays the same
(d) Not enough information to tell

7. **Bonus Problem:** The figure below shows a graph of the reflection coefficient Γ, the transmission coefficient τ, and $1-\Gamma^2$ coefficient for the incident+reflected power in terms of a parameter designated α when a wave $V(t-\frac{z}{v_p})$ encounters a change in impedance at an arbitrary z. This parameter α is defined so that the impedance of the transmission line into which the wave is transmitted Z_2 can be written in terms of Z_1 by assuming that $Z_2 = \alpha Z1$. This translates the reflection coefficient from $\Gamma = \frac{Z_2 - Z_1}{Z_2 + Z1}$ to $\Gamma = \frac{\alpha - 1}{\alpha + 1}$.

(a) (5 points) Derive the formula for the voltage and current in the incident, reflected, and transmitted waves in terms of α, and Z_1.

(b) (2 points) Show that conservation of energy holds at the interface between the two materials.

(c) (3 points) There are 3 special values $\alpha = 0$, $\alpha = 1$, and $\alpha = \infty$. Explain physically the special significance of each in terms of the relative impedances of the two regions, including explanations of the value of each of the parameters in the figure at these special points. Hint: Look at the relative values of the reflected voltage and current as compared to the relative values of the transmitted voltage and current for each case.