ECE 329 Fields and Waves I
Homework 1

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Homework Policy:

- Write your name and NetID on top of every page. This habit will help you in exams in the event of having loose page(s).
- Tag all the questions in Gradescope. Failure to do so results in a 5 points deduction.
- Cheating results in ZERO and 50% reduction in HW average on first offense. A 100% reduction in HW average on second offense.
- Please show detailed process for each problem instead of just an answer. No partial credits would be given otherwise. All answers should include units wherever appropriate.
- No late HW is accepted.
- Regrade requests are available one week following grade release.

You are allowed to work with anyone else, but the work you submit should only belong to you. Note that if you have knowledge of a violation of the Honor Code, then you are obligated to report it. By submitting this homework, you are agreeing to the Honor Code: "I have neither given nor received unauthorized aid on this homework, nor have I concealed any violations of the Honor Code."

Question	Points	Score
1	10	
2	7	
3	13	
4	10	
5	10	
6	10	
Total:	60	

1. **Review exercises on vectors:** Consider the 3D vectors

$$\begin{aligned}\mathbf{A} &= 3\hat{x} + \hat{y} - 2\hat{z},\\ \mathbf{B} &= \hat{x} + \hat{y} - \hat{z},\\ \mathbf{C} &= \hat{x} - 2\hat{y} + 3\hat{z},\end{aligned}$$

where $\hat{x} \equiv (1,0,0)$, $\hat{y} \equiv (0,1,0)$, and $\hat{z} \equiv (0,0,1)$ constitute an orthogonal set of unit vectors directed along the principal axes of a right-handed Cartesian coordinate system.

Vectors can also be represented in component form — e.g., $\mathbf{A} = (3, 1, -2)$, which is equivalent to $3\hat{x} + \hat{y} - 2\hat{z}$. Determine the following.

(a) (1 point) The vector $\mathbf{D} \equiv \mathbf{A} + \mathbf{B}$,

(b) (1 point) The vector $\mathbf{A} + \mathbf{B} - 4\mathbf{C}$,

(c) (2 points) The vector *magnitude* $|\mathbf{A} + \mathbf{B} - 4\mathbf{C}|$.

(d) (2 points) The unit vector $\hat{u}$ along vector $\mathbf{A} + 2\mathbf{B} - \mathbf{C}$.

(e) (2 points) The *dot product* $\mathbf{A} \cdot \mathbf{B}$.

(f) (2 points) The *cross product* $\mathbf{B} \times \mathbf{C}$.

2. (7 points) A particle with charge $q = 1$ C passing through the origin $\mathbf{r} = (x, y, z) = \mathbf{0}$ of the lab frame is observed to accelerate with forces

$$\mathbf{F}_1 = 3\hat{z}, \quad \mathbf{F}_2 = \hat{z}, \quad \mathbf{F}_3 = 3\hat{z} + 4\hat{y}\,\mathrm{N}$$

when the velocity of the particle is

$$\mathbf{v}_1 = 0, \quad \mathbf{v}_2 = 1\hat{y}, \quad \mathbf{v}_3 = 2\hat{z}\,\frac{\mathrm{m}}{\mathrm{s}},$$

in the same unknown fields $\mathbf{E}$ and $\mathbf{B}$.

Use the Lorentz force equation $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$ to determine the fields $\mathbf{E}$ and $\mathbf{B}$ at the origin.

3. (13 points) Let $\mathbf{J} = z^2(\hat{x}+\hat{y}+\hat{z})$ A/m^2 denote the electrical *current density* field — i.e., current flux per unit area — in a region of space represented in Cartesian coordinates. A current density of $\mathbf{J} = z^2(\hat{x} + \hat{y} + \hat{z})$ A/m^2 implies the flow of electrical current in direction $\frac{\mathbf{J}}{|\mathbf{J}|} = \frac{\hat{x}+\hat{y}+\hat{z}}{\sqrt{3}}$ with a magnitude of $|\mathbf{J}| = z^2\sqrt{3}$ amperes (A) per unit area. Calculate the total **current flux** $\oint_S \mathbf{J} \cdot d\mathbf{S}$ out of a closed surface S enclosing a cubic volume $V = 1\,\mathrm{m}^3$ with vertices at $(x, y, z) = (0, 0, 0)$ and $(1, 1, 1)$ m.

Hint: Surface S of cube V consists of six surfaces of square shapes having equal areas $S_i = 1\,\mathrm{m}^2$, $i = 1, 2, \cdots, 6$. The flux $\oint_S \mathbf{J} \cdot d\mathbf{S}$ is therefore the sum of six *surface integrals* $\int_{S_i} \mathbf{J} \cdot d\mathbf{S}$ taken over surfaces S_i, where the infinitesimal area vectors $d\mathbf{S}$ are, in turn, $\pm\hat{z}dxdy$, $\pm\hat{x}dydz$, and $\pm\hat{y}dzdx$ — by convention $d\mathbf{S}_i$ are taken as vectors pointing away from volume V (at each subsurface S_i) in flux calculations.

4. (10 points) Charges $Q_1 = 8\pi\epsilon_o$ C and $Q_2 = -Q_1/2$ are located at points P_1 and P_2 having the position vectors $\mathbf{r}_1 = -\hat{x} = (-1, 0, 0)$ m and $\mathbf{r}_2 = \hat{x} = (1, 0, 0)$ m, respectively. Determine the electric field vector $\mathbf{E}$ at points P_3 and P_4 having the position vectors $\mathbf{r}_3 = \hat{y} = (0, 1, 0)$ m and $\mathbf{r}_4 = \hat{z} = (0, 0, 1)$ m, respectively. Make sketches showing the charge locations and the resulting electric field vectors (drawn coming out of points P_3 and P_4, respectively) in each case.

5. (10 points) **Conceptual Question**: Every HW will have three multiple choice conceptual questions. These are not meant to be difficult or time-consuming if you have been following along during lecture.

   (i) A positive charge moving with velocity $< v_x, v_y, v_z >$ feels a net force in the y direction. The electric field is non-zero and points along the z direction. The magnetic field is also non-zero and points along x direction. Which of the following scenarios could make this possible?
   (a) this is not possible
   (b) the charge moves along x: $v_x \neq 0,\ v_y = v_z = 0$
   (c) the charge moves along y: $v_y \neq 0, v_x = v_z = 0$
   (d) the charge moves along z: $v_z \neq 0, v_x = v_y = 0$
   (e) the charge moves at an angle along y and z: $v_y \neq 0, v_z \neq 0, v_x = 0$

   (ii) You learned about the 3D dot product. Consider its extension into 4D. Now consider the vectors $\mathbf{A} = a_1\mathbf{e_1} + a_2\mathbf{e_2} + a_3\mathbf{e_3} + a_4\mathbf{e_4}$ and $\mathbf{B} = b_1\mathbf{e_1} + b_2\mathbf{e_2} + b_3\mathbf{b_3} + b_4\mathbf{e_4}$. Here, $\mathbf{e_i}$ are the bases vectors operating in the same capacity as $\hat{x}, \hat{y}, \hat{z}$ in 3 dimensions. Which of the following is the numerical expression for $\mathbf{A} \cdot \mathbf{B}$?
   (a) $a_1b_1 + a_2b_2 + a_3b_3 + a_4b_4$
   (b) $a_1b_2 - a_2b_1 + a_3b_4 - a_4b_3$
   (c) $a_1b_3 - a_2b_4 + a_3b_1 - a_4b_2$
   (d) None of the above

   (iii) Suppose charge $q_1 = 1C$ is located at $(-1, 0, 0)$ and $q_2 = 2C$ is located at $(2, 0, 0)$. Charge $q_3 = 12384.234247C$ is located at the origin $(0, 0, 0)$. Then q_3 experiences a net force
   (a) Towards q_1
   (b) Towards q_2
   (c) Net force is zero

6. (10 points) **Bonus Question**: During the semester, some HWs will have an **optional** bonus question on an advanced concept that can help you to increase your HW average for the semester up to a maximum of 100%. For this week, derive Green's first identity: $\int_{\partial V} \psi(\nabla\phi \cdot \boldsymbol{n})\, dS = \int_V (\psi\nabla^2\phi + \nabla\phi \cdot \nabla\psi)\, dV$. (**Hint**: apply the divergence theorem.)