VECTOR CALCULUS

$$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \qquad \nabla f = \hat{x}\frac{\partial f}{\partial x} + \hat{y}\frac{\partial f}{\partial y} + \hat{z}\frac{\partial f}{\partial z}$$

$$\nabla \times \mathbf{A} = \hat{x}(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}) + \hat{y}(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}) + \hat{z}(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y})$$

$$\nabla \cdot \nabla \times \mathbf{A} = 0 \qquad \nabla \times \nabla f = 0 \qquad \nabla \times \nabla \times \mathbf{A} = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$$

FIELD LAWS

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) = m\mathbf{a} = m\frac{\partial \mathbf{v}}{\partial t}$$

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} \qquad \oint_S \mathbf{D} \cdot \mathbf{dS} = \int_V \rho \, dV = Q_{encl} \qquad \nabla \cdot \mathbf{D} = \rho$$

$$\hat{n} \times (\mathbf{E}^+ - \mathbf{E}^-) = 0 \qquad \hat{n} \cdot (\mathbf{D}^+ - \mathbf{D}^-) = \rho_s$$

For static conditions: $\nabla \times \mathbf{E} = 0$ $\qquad \mathbf{E} = -\nabla V$ $\qquad V_A - V_B = \int_A^B \mathbf{E} \cdot \mathbf{dl}$

$$\oint_C \mathbf{E} \cdot \mathbf{dl} = 0 \qquad \nabla^2 V = -\rho/\epsilon$$

SOME STATIC E FIELDS (IN A VACUUM)

- point charge Q [C] at the origin: $\mathbf{E} = \hat{r}\frac{Q}{4\pi r^2 \epsilon_0}$
- line charge ρ_l [C/m] along the z-axis: $\mathbf{E} = \hat{r}\frac{\rho_l}{2\pi r \epsilon_0}$
- surface charge ρ_s [C/m^2] in the $z = 0$ plane: $\mathbf{E} = \pm\hat{z}\frac{\rho_s}{2\epsilon_0}$ (for $z \gtrless 0$)
- slab charge ρ [C/m^3] at $-\frac{W}{2} \leq z \leq \frac{W}{2}$: $\mathbf{E} = \begin{cases} -\hat{z}\frac{\rho W}{\epsilon_0 2} & z \leq -\frac{W}{2} \\ \hat{z}\frac{\rho}{\epsilon_0} z & -\frac{W}{2} \leq z \leq \frac{W}{2} \\ \hat{z}\frac{\rho W}{\epsilon_0 2} & z \geq \frac{W}{2} \end{cases}$
- cylindrical charge ρ [C/m^3] of radius R along the z-axis: $\mathbf{E} = \begin{cases} \hat{r}\frac{\rho R^2}{2\epsilon_0 r} & r \geq R \\ \hat{r}\frac{\rho}{2\epsilon_0} r & r \leq R \end{cases}$

E FIELDS IN MATERIAL MEDIA

$\mathbf{J}_{\mathbf{free}} = \sigma \mathbf{E}$ $\qquad \sigma = 0$ (perfect diel.) $\qquad \sigma = \infty$ (perfect cond.)

$\rho_{bound} = -\nabla \cdot \mathbf{P}$ $\qquad \mathbf{P} = \epsilon_0 \chi_e \mathbf{E}$ (for linear dielectrics) $\qquad \epsilon = \epsilon_0 \epsilon_r = \epsilon_0(1 + \chi_e)$

$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$ (always) $= \epsilon \mathbf{E}$ (for linear dielectrics)