ECE329: Tutorial Session 9

November 13th, 2024

Basic TL: Time Domain, Not Steady State

Generator injection coefficient: $\tau_g = \frac{Z_0}{R_g + Z_0} \neq 1 + \Gamma_g$

Load reflection coefficient: $\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$

Generator reflection coefficient: $\Gamma_g = \frac{R_g - Z_0}{R_g + Z_0}$

Basic TL: Time Domain, Not Steady State

Special cases:

- What if $Z_L = 0$?
- What if $Z_L = \infty$?
- What if $Z_L = Z_0$?

Problem 1: Bounce Diagrams

Input: $V_g = 5u(t)$ [V]

$Z_L = 50\Omega, R_g = 50\Omega, Z_0 = 100\Omega$

$v = c$, TL length = 3 [m].

Create a voltage bounce diagram for the first 45 nanoseconds.

Create a current bounce diagram for the first 45 nanoseconds.

Problem 2: Bounce Diagrams

Input: $V_g = 5u(t)$ [V]
$Z_L = 50\Omega, R_g = 50\Omega, Z_0 = 100\Omega$
$v = c$, TL length = 3 [m].

Plot $V(0.75\text{m}, t)$ for the first 45 nanoseconds.

Problem 2: Bounce Diagrams

Input: $V_g = 5u(t)$ [V]

$Z_L = 50\Omega, R_g = 50\Omega, Z_0 = 100\Omega$

$v = c$, TL length = 3 [m].

What is the steady-state voltage over the load?

What is the steady state current through the load?

Multiline TL Circuits Time Domain

When we travel FROM line j TO line k:

$$\Gamma_{jk} = \frac{Z_k - Z_j}{Z_k + Z_j}$$

$$\tau_{jk} = 1 + \Gamma_{jk}$$

Problem 3: Bounce Diagrams w/ Multiline

Input: $V_g = 5u(t)$ [V]

$Z_L = 50\Omega, R_g = 50\Omega$

$Z_0 = 100\Omega, Z_1 = 200\Omega$

$v = c$

First TL length = 3 [m]. Second TL length = 4.5 [m].

Create a voltage bounce diagram for the first 45 nanoseconds.

Create a current bounce diagram for the first 45 nanoseconds.

Bounce Diagrams: General Formulation

This is an LTI system!
Input: $\delta(t)$
Output (at position d): $V(t)$
Time to travel down the line: t_0.

$$V(d,t) = \tau_g \sum_{n=0}^{\infty} \left(\Gamma_L \Gamma_g\right)^n \delta(t + \frac{d}{v} - n t_0) + \tau_g \Gamma_L \sum_{n=0}^{\infty} \left(\Gamma_L \Gamma_g\right)^n \delta(t - \frac{d}{v} - (n+1) t_0)$$

For any general input: convolve!

Multiline TL Circuits

Series resistor (harder to deal with ☹):

Parallel resistor (easier to deal with ☺):

Multiline TL: Parallel Resistor

Basic TL: Phasor Domain, Steady State

On TL: Forward–going voltage wave + backward–going voltage wave

On TL: Forward–going current wave + backward–going current wave

Basic TL: Phasor Domain, Steady State

Impedance as a function of position?

Midterm 1 equations, in one place

$$\vec{F} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\vec{F} = q_1\vec{E} + q_1(\vec{v}_1 \times \vec{B})$$

$$\vec{E} = \frac{q_2}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\hat{n} \cdot (\vec{D}_1 - \vec{D}_2) = \rho_s$$

$$\hat{n} \times (\vec{E}_1 - \vec{E}_2) = 0$$

$$\hat{n} \cdot (\vec{P}_1 - \vec{P}_2) = -\rho_{b,s}$$

$$\epsilon \oiint \vec{E} \cdot d\vec{S} = Q_{\text{enclosed}}$$

$$\oiint \vec{D} \cdot d\vec{S} = Q_{\text{enclosed}}$$

$$\iiint \rho dV = Q_{\text{enclosed}}$$

$$\oiint \vec{B} \cdot d\vec{S} = 0$$

$$I = \oiint \vec{J} \cdot d\vec{S} = -\frac{\partial Q_{\text{enclosed}}}{\partial t}$$

$$\epsilon = \epsilon_0(1 + \chi_e)$$

$$\vec{P} = \epsilon_0\chi_e\vec{E}$$

$$\vec{D} = \epsilon_0\vec{E} + \vec{P} = \epsilon\vec{E}$$

$$\vec{J} = \sigma\vec{E}$$

$$\rho_b = -\nabla \cdot \vec{P}$$

$$\nabla \cdot \epsilon_0\vec{E} = \rho_f + \rho_b$$

$$\nabla \times \vec{E} = 0$$

$$\vec{E} = -\nabla V$$

$$\oint \vec{E} \cdot d\vec{l} = 0$$

$$V_{ab} = V(b) - V(a) = -\int_a^b \vec{E} \cdot d\vec{l}$$

$$\nabla \cdot \vec{D} = \rho$$

$$\nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$$

$$-\nabla^2 V = \frac{\rho}{\epsilon}$$

$$\oiint \vec{D} \cdot d\vec{S} = \iiint \nabla \cdot \vec{D} dV$$

$$\oint \vec{E} \cdot d\vec{l} = \iint (\nabla \times \vec{E}) \cdot d\vec{S}$$

$$\int_a^b \nabla V \cdot d\vec{l} = V(b) - V(a)$$

Midterm 2 equations, in one place

$$\vec{B} = \frac{\mu I}{2\pi r}\hat{\phi}$$

$$d\vec{B} = \frac{\mu I d\vec{\ell} \times \hat{r}}{4\pi r^2}$$

$$\oint_C \vec{H} \cdot d\vec{\ell} = \iint_S \vec{J} \cdot d\vec{S}$$

$$\oint_C \vec{B} \cdot d\vec{\ell} = \mu I_{\text{encl}}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0$$

$$\Psi = \iint_s \vec{B} \cdot d\vec{S}$$

$$-\frac{d}{dt}\iint_s \vec{B} \cdot d\vec{S} = \oint_c \vec{E} \cdot d\vec{l}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\oint_c \vec{E} \cdot d\vec{l} = \varepsilon$$

$$\varepsilon = \frac{W}{q} = \oint_C \frac{\vec{F}}{q} \cdot d\vec{l}$$

$$\Psi = LI$$

$$\varepsilon = IR$$

$$v = \frac{\omega}{\beta} = \lambda f = \frac{1}{\sqrt{\mu\epsilon}}$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$\eta = \frac{\sqrt{\mu}}{\sqrt{\epsilon}}$$

$$\beta = \omega\sqrt{\mu\epsilon}$$

$$\nabla^2 \vec{E} = \mu\epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\vec{S} = \vec{E} \times \vec{H}$$

$$\tilde{S} = \tilde{E} \times \tilde{H}^*$$

$$< \vec{S} > = \frac{1}{2}\text{Re}\{\tilde{E} \times \tilde{H}^*\}$$

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\epsilon\vec{E} \cdot \vec{E} + \frac{1}{2}\mu\vec{H} \cdot \vec{H}\right) + \nabla \cdot \vec{S} + \vec{J} \cdot \vec{E} = 0$$

$$Q = CV$$

$$G = \frac{\sigma}{\epsilon}C \qquad R = \frac{1}{G}$$

$$\vec{J}_b = \frac{\partial \vec{P}}{\partial t} + \nabla \times \vec{M}$$

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

$$\vec{M} = \chi_m \vec{H}$$

$$\vec{B} = \mu_0 \mu_r \vec{H} = \mu \vec{H}$$

$$A\cos(\omega t - \beta x)\hat{z} \leftrightarrow Ae^{-j\beta x}\hat{z}$$

$$\hat{n} \cdot (\vec{B}_1 - \vec{B}_2) = 0$$

$$\hat{n} \times (\vec{H}_1 - \vec{H}_2) = \vec{J}_s$$

$$\hat{n} \times (\vec{M}_1 - \vec{M}_2) = \vec{J}_{b,s}$$

Midterm 3 equations, in one place

Waves:

	Condition	β	α	$\lvert\eta\rvert$	τ	$\lambda = \frac{2\pi}{\beta}$	$\delta = \frac{1}{\alpha}$
Perfect dielectric	$\sigma = 0$	$\omega\sqrt{\epsilon\mu}$	0	$\sqrt{\frac{\mu}{\epsilon}}$	0	$\frac{2\pi}{\omega\sqrt{\epsilon\mu}}$	∞
Imperfect dielectric	$\frac{\sigma}{\omega\epsilon} \ll 1$	$\sim \omega\sqrt{\epsilon\mu}$	$\beta\frac{1}{2}\frac{\sigma}{\omega\epsilon} = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}$	$\sim \sqrt{\frac{\mu}{\epsilon}}$	$\sim \frac{\sigma}{2\omega\epsilon}$	$\sim \frac{2\pi}{\omega\sqrt{\epsilon\mu}}$	$\frac{2}{\sigma}\sqrt{\frac{\epsilon}{\mu}}$
Good conductor	$\frac{\sigma}{\omega\epsilon} \gg 1$	$\sim \sqrt{\pi f\mu\sigma}$	$\sim \sqrt{\pi f\mu\sigma}$	$\sqrt{\frac{\omega\mu}{\sigma}}$	$45°$	$\sim \frac{2\pi}{\sqrt{\pi f\mu\sigma}}$	$\sim \frac{1}{\sqrt{\pi f\mu\sigma}}$
Perfect conductor	$\sigma = \infty$	∞	∞	0	-	0	0

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \qquad \tau = \frac{2\eta_2}{\eta_2 + \eta_1} = 1 + \Gamma$$

$$v = \frac{\omega}{\beta} = \lambda f$$

$$\nabla^2 \tilde{E} = (j\omega\mu)(\sigma + j\omega\epsilon)\tilde{E}$$

TLs:

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\tau_g = \frac{Z_0}{R_g + Z_0}$$

$$\Gamma_g = \frac{R_g - Z_0}{R_g + Z_0}$$

$$\Gamma_{jk} = \frac{Z_k - Z_j}{Z_k + Z_j}$$

$$\tau_{jk} = 1 + \Gamma_{jk}$$

Half-wave:

$$V_{in} = -V_{out}$$
$$I_{in} = -I_{out}$$
$$Z_{in} = Z_{out}$$

Quarter-wave:

$$V_{in} = jI_{out}Z_0$$
$$I_{in} = \frac{jV_{out}}{Z_0}$$
$$Z_{in} = \frac{Z_0^2}{Z_{out}}$$

Units

Charge Q: C
Current I: A
Electric field strength $\vec{E}$: N/C or V/m
Electric flux density $\vec{D}$: C/m^2
Polarization field $\vec{P}$: C/m^2
Electric potential V: V
Capacitance C: F
Magnetic flux density $\vec{B}$: T or Wb/m^2
Magnetic field strength $\vec{H}$: A/m
Magnetic flux Ψ: Wb
Electromotive force ε: V
Inductance L: H

Electric permittivity ϵ: F/m
Magnetic permeability μ: H/m
Conductivity σ: Si/m

Charge density ρ: C/m^3
Surface charge density ρ_S: C/m^2
Current density $\vec{J}$: A/m^2

Intrinsic impedance η: Ohm
Wave number β: rad/m

Characteristic impedance Z: Ohm