ECE329: Tutorial Session 8

November 6th, 2025

Wave Reflection & Transmission

TEM wave incident normally on a boundary

$$\tilde{E}_i(x) = -E_0 e^{-\alpha_1 x} e^{-j\beta_1 x}\hat{y}$$

$$\tilde{H}_i(x) = -\frac{E_0}{\eta_1} e^{-\alpha_1 x} e^{-j\beta_1 x}\hat{z}$$

$\sigma_1, \mu_1, \epsilon_1$
$x < 0$

$x = 0$

$\sigma_2, \mu_2, \epsilon_2$
$x > 0$

Reflection & Transmission Coefficients

$$\tilde{E}_i(x) = -E_0 e^{-\alpha_1 x} e^{-j\beta_1 x}\hat{y}$$

$$\tilde{H}_i(x) = -\frac{E_0}{\eta_1} e^{-\alpha_1 x} e^{-j\beta_1 x}\hat{z}$$

$$\tilde{E}_r(x) = -E_0 \Gamma e^{\alpha_1 x} e^{j\beta_1 x}\hat{y}$$

$$\tilde{H}_r(x) = \frac{E_0}{\eta_1} \Gamma e^{\alpha_1 x} e^{j\beta_1 x}\hat{z}$$

$$\tilde{E}_t(x) = -E_0 \tau e^{-\alpha_2 x} e^{-j\beta_2 x}\hat{y}$$

$$\tilde{H}_t(x) = -\frac{E_0}{\eta_2} \tau e^{-\alpha_2 x} e^{-j\beta_2 x}\hat{z}$$

$\sigma_1, \mu_1, \epsilon_1$
$x < 0$

$x = 0$

$\sigma_2, \mu_2, \epsilon_2$
$x > 0$

What to enforce?

$\hat{n} \times (\vec{E}_1 - \vec{E}_2) = 0$

$\cancel{\hat{n} \cdot (\vec{D}_1 - \vec{D}_2) = \rho_s}$ 0

$\hat{n} \times (\vec{H}_1 - \vec{H}_2) = \cancel{\vec{J}_s}$ 0

$\cancel{\hat{n} \cdot (\vec{B}_1 - \vec{B}_2) = 0}$

$$\tilde{E}_i(x) = -E_0 e^{-\alpha_1 x} e^{-j\beta_1 x} \hat{y}$$

$$\tilde{H}_i(x) = -\frac{E_0}{\eta_1} e^{-\alpha_1 x} e^{-j\beta_1 x} \hat{z}$$

$$\tilde{E}_r(x) = -E_0 \Gamma e^{\alpha_1 x} e^{j\beta_1 x} \hat{y}$$

$$\tilde{H}_r(x) = \frac{E_0}{\eta_1} \Gamma e^{\alpha_1 x} e^{j\beta_1 x} \hat{z}$$

$\sigma_1, \mu_1, \epsilon_1$
$x < 0$

$-\hat{x}$

1 +

2 −

$x = 0$

$-\hat{x} \times (\vec{E}_1 - \vec{E}_2) = 0$

$-\hat{x} \times \left((\tilde{E}_i + \tilde{E}_r) - \tilde{E}_t \right) = \vec{0}$

$$\tilde{E}_t(x) = -E_0 \tau e^{-\alpha_2 x} e^{-j\beta_2 x} \hat{y}$$

$$\tilde{H}_t(x) = -\frac{E_0}{\eta_2} \tau e^{-\alpha_2 x} e^{-j\beta_2 x} \hat{z}$$

$\tilde{E}_i(0) + \tilde{E}_r(0) = \tilde{E}_t(0)$

$\tilde{H}_i(0) + \tilde{H}_r(0) = \tilde{H}_t(0)$

$\sigma_2, \mu_2, \epsilon_2$
$x > 0$

Solve!

$$\tilde{E}_i(0) + \tilde{E}_r(0) = \tilde{E}_t(0)$$

$$\boxed{\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}}$$

$$\boxed{\tau = 1 + \Gamma}$$

$$\tilde{E}_i(0) = -E_0\hat{y}$$

$$\tilde{H}_i(0) = -\frac{E_0}{\eta_1}\hat{z}$$

$$\tilde{E}_t(0) = -E_0\tau\hat{y}$$

$$\tilde{H}_i(0) + \tilde{H}_r(0) = \tilde{H}_t(0)$$

$$\tilde{E}_r(0) = -E_0\Gamma\hat{y}$$

$$\tilde{H}_t(0) = -\frac{E_0}{\eta_2}\tau\hat{z}$$

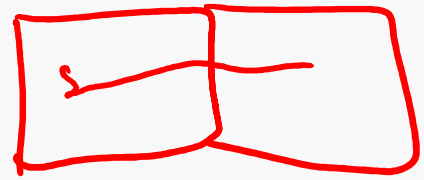
$$\tilde{H}_r(0) = \frac{E_0}{\eta_1}\Gamma\hat{z}$$

$\sigma_1, \mu_1, \epsilon_1$
$x < 0$

$x = 0$

$\sigma_2, \mu_2, \epsilon_2$
$x > 0$

Coefficients



$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \qquad \tau = \frac{2\eta_2}{\eta_2 + \eta_1} = 1 + \Gamma$$

Check our work:

1. What if $\eta_1 = \eta_2$?

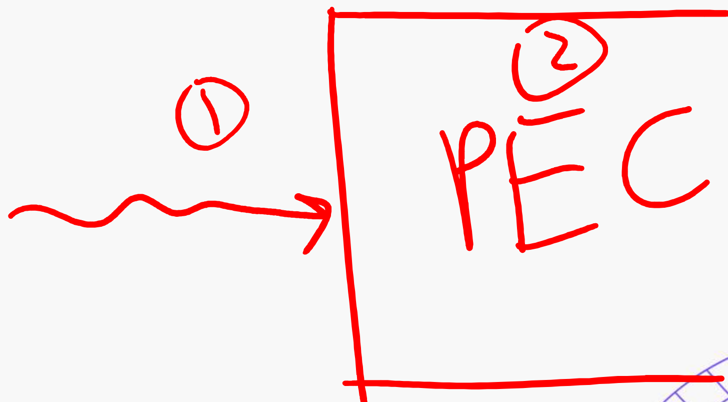
$\Gamma = 0$

$\tau = 1$

2. What if $\eta_2 = 0$?

$\Gamma = -1$

$\tau = 0$



$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \qquad \tau = \frac{2\eta_2}{\eta_2 + \eta_1} = 1 + \Gamma$$

Problem 1

A wave propagates through free space and is normally incident upon a perfect dielectric with $\epsilon = 16\epsilon_0$ and $\mu = \mu_0$. What are Γ and τ?

free

$120\pi = \eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}$

1

$\varepsilon_2 = 16\varepsilon_0$

$\mu_2 = \mu_0$

2

$\eta_2 = \sqrt{\frac{\mu}{\varepsilon}}$

$= \frac{1}{4}\eta_0$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\frac{1}{4}\cancel{\eta_0} - \cancel{\eta_0}}{\frac{1}{4}\cancel{\eta_0} + \cancel{\eta_0}} = \frac{-\frac{3}{4}}{\frac{5}{4}}$$

$$\Gamma = -\frac{3}{5}$$

$$\tau = \frac{2}{5}$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \qquad \tau = \frac{2\eta_2}{\eta_2 + \eta_1} = 1 + \Gamma$$

Problem 2

A single delta wave pops into existence at $t = 0$ halfway between the plates and moves to the right with amplitude 1000. Assume the area between the plates is free space. → $v = c \approx 3 \cdot 10^8$ m/s

Where is the wave at $t = 1$s?

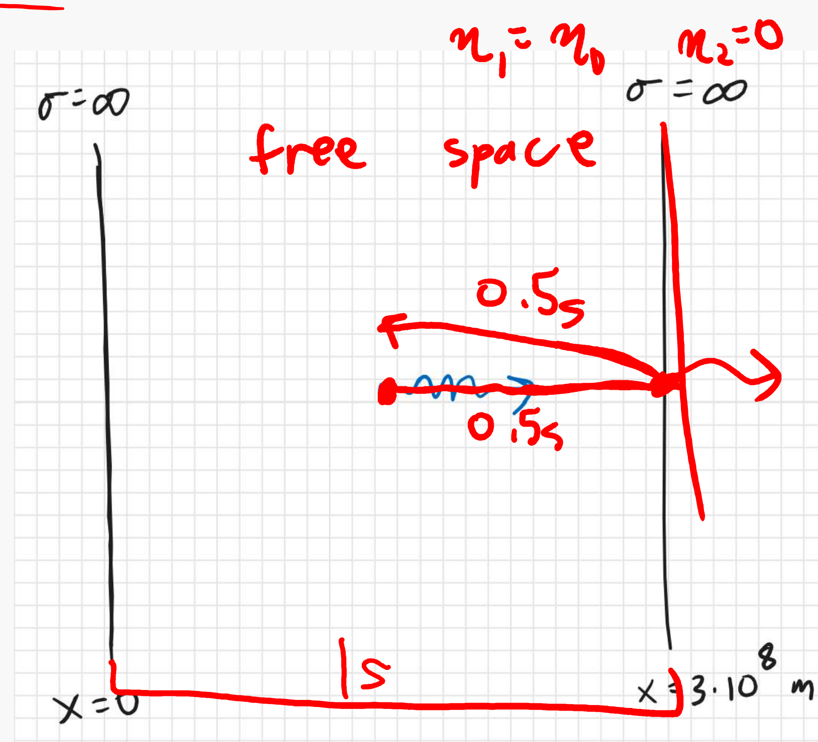
What will its amplitude be?

@ $x = 1.5 \cdot 10^8$

$\Gamma = -1$

$\tau = 0$

amp: -1000



Problem 2

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \qquad \tau = \frac{2\eta_2}{\eta_2 + \eta_1} = 1 + \Gamma$$

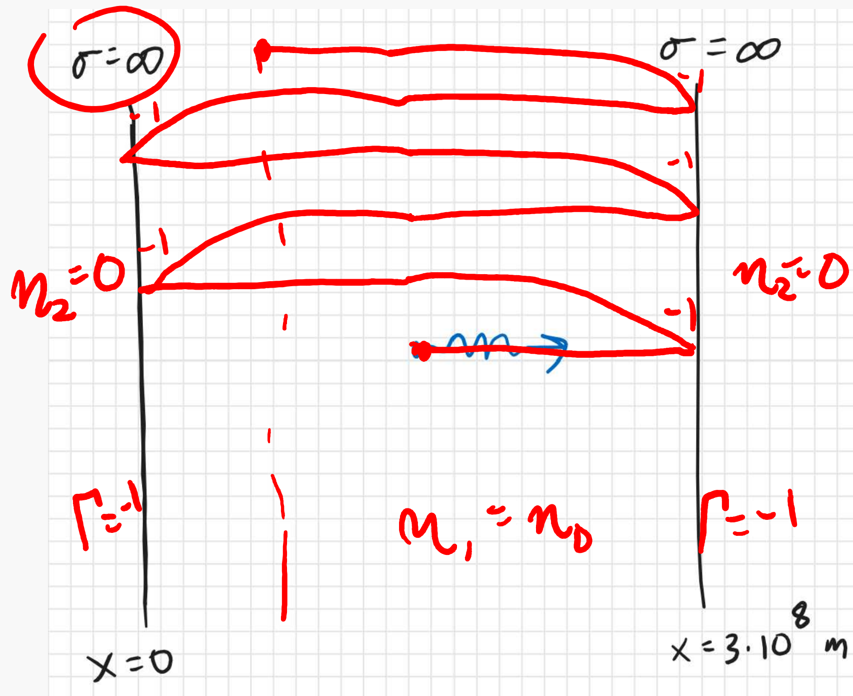
A single delta wave pops into existence at $t = 0$ halfway between the plates and moves to the right with amplitude 1000. Assume the area between the plates is free space.

Where is the wave at $t = 5.25\text{s}$?

What will its amplitude be?

@ $x = 0.75 \cdot 10^8$

amp: -1000



$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \qquad \tau = \frac{2\eta_2}{\eta_2 + \eta_1} = 1 + \Gamma$$

Problem 2

A single delta wave pops into existence at $t = 0$ halfway between the plates and moves to the right with amplitude 1000. Assume the area between the plates is free space.

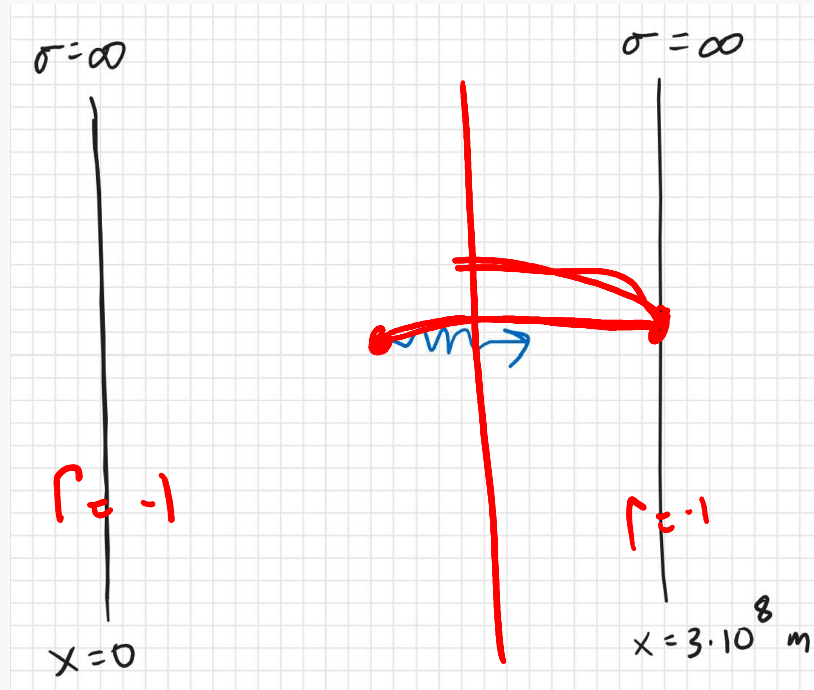
When will the wave be at $\mathrm{x} = 2.25 * 10^8\mathrm{m}$?

What will its amplitude be?

"n" $\in \mathbb{N}$

$t = 0.75 + 2n$, amp: -1000

and $0.25 + 2n$, amp: 1000



Problem 3

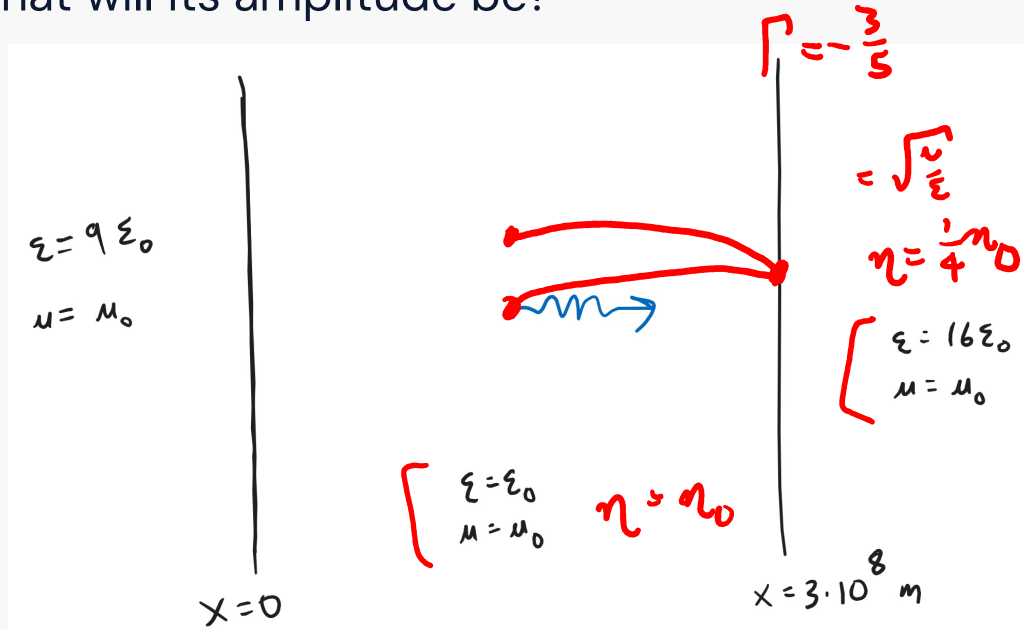
$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \qquad \tau = \frac{2\eta_2}{\eta_2 + \eta_1} = 1 + \Gamma$$

A single delta wave pops into existence at $t = 0$ in the middle and moves to the right with amplitude 1000. Assume the area between the plates is free space.

Where is the wave at $t = 1$s? What will its amplitude be?

@ $x = 1.5 \cdot 10^8$

amp: -600



Problem 3

$$\Gamma = \frac{\eta_2 - \overset{\eta_0}{\eta_1}}{\eta_2 + \eta_1} \qquad \tau = \frac{2\eta_2}{\eta_2 + \eta_1} = 1 + \Gamma$$

A single delta wave pops into existence at $t = 0$ in the middle and moves to the right with amplitude 1000. Assume the area between the plates is free space.

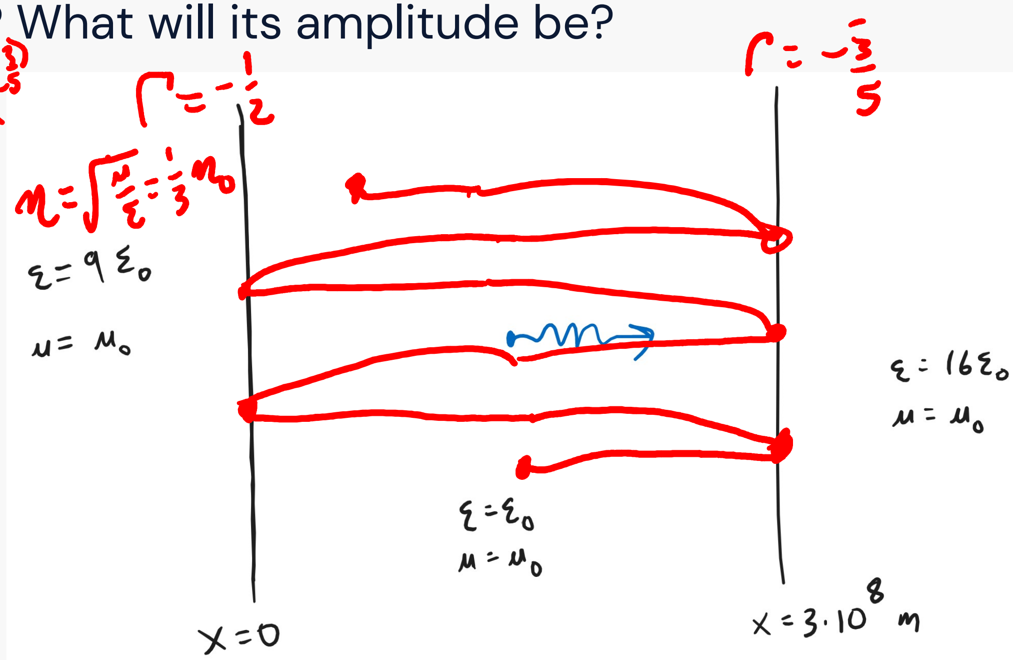
Where is the wave at $t = 5.25\text{s}$? What will its amplitude be?

$x = 0.75 \cdot 10^8$

amp: $1000\left(-\frac{3}{5}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{5}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{5}\right)$

$\frac{270\cancel{00}}{5 \cdot 1\cancel{00}}$

-54



Problem 3

$n \in \mathbb{N}$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \qquad \tau = \frac{2\eta_2}{\eta_2 + \eta_1} = 1 + \Gamma$$

A single delta wave pops into existence at $t = 0$ in the middle and moves to the right with amplitude 1000. Assume the area between the plates is free space.

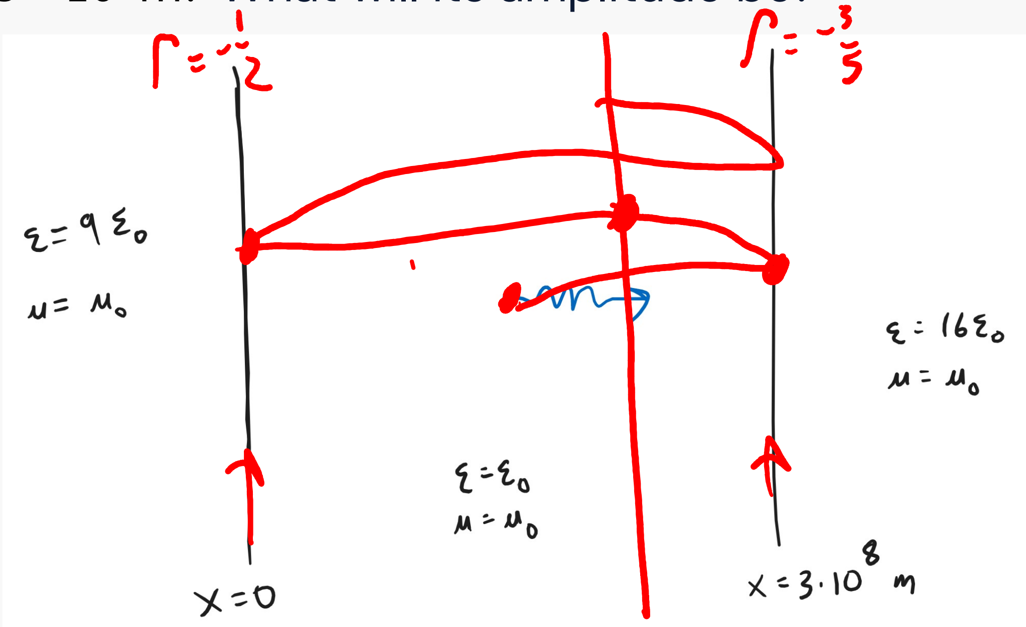
When will the wave be at $x = 2.25 * 10^8$m? What will its amplitude be?

$t = 0.75 + 2n$

$-600\left(-\frac{1}{2}\right)^n\left(-\frac{3}{5}\right)^n$

$t = 0.25 + 2n$

$1000\left(-\frac{3}{5}\right)^n\left(-\frac{1}{2}\right)^n$



Standing Waves (dielectric to PEC)

$\tilde{E}_i(y) = -E_0 e^{-j\beta_1 y}\hat{x}$

$\tilde{E}_r(y) = E_0 e^{j\beta_1 y}\hat{x}$

$$\left(-E_0 e^{-j\beta_1 y} + E_0 e^{j\beta_1 y}\right)\hat{x}$$

$$E_0\left(e^{j\beta_1 y} - e^{-j\beta_1 y}\right)\hat{x}$$

$$\tilde{E}_1 = E_0\, 2j \sin(\beta_1 y)\,\hat{x}$$

$$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$\vec{E} = \mathrm{Re}\{\tilde{E}_1 e^{j\omega t}\} = \mathrm{Re}\{2j E_0 \sin(\beta_1 y)\underbrace{e^{j\omega t}}\hat{x}\} = 2E_0 \sin(\beta_1 y)\cos\left(\omega t + \frac{\pi}{2}\right)$$

Problem 1: Standing Waves

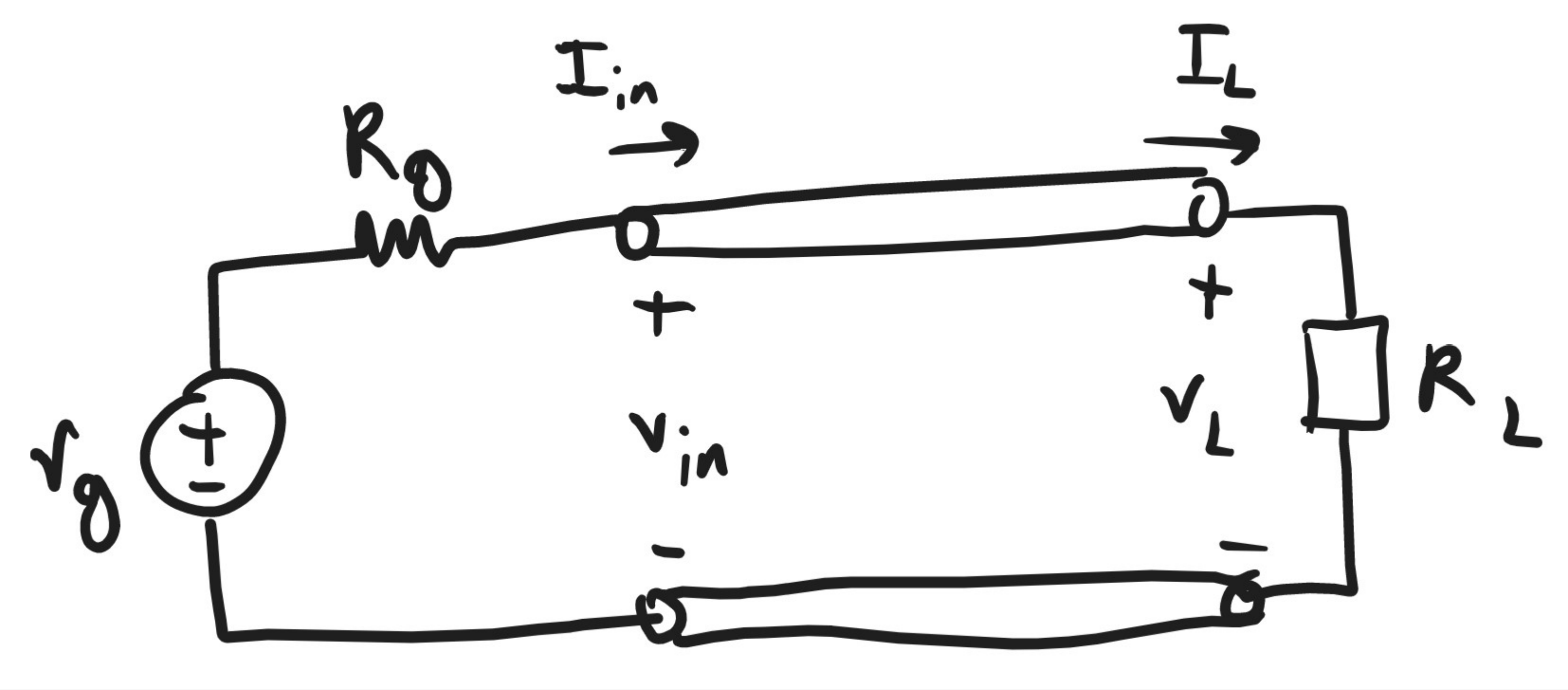
A wave propagates through an imperfect dielectric and is normally incident upon a perfect electrical conductor. Is a standing wave created in the imperfect dielectric?

$$\tilde{E}_i(y) = -E_0 e^{-\alpha_1 y} e^{-j\beta_1 y} \hat{x} \qquad \tilde{E}_r(y) = -E_0 \Gamma e^{\alpha_1 y} e^{j\beta_1 y} \hat{x}$$

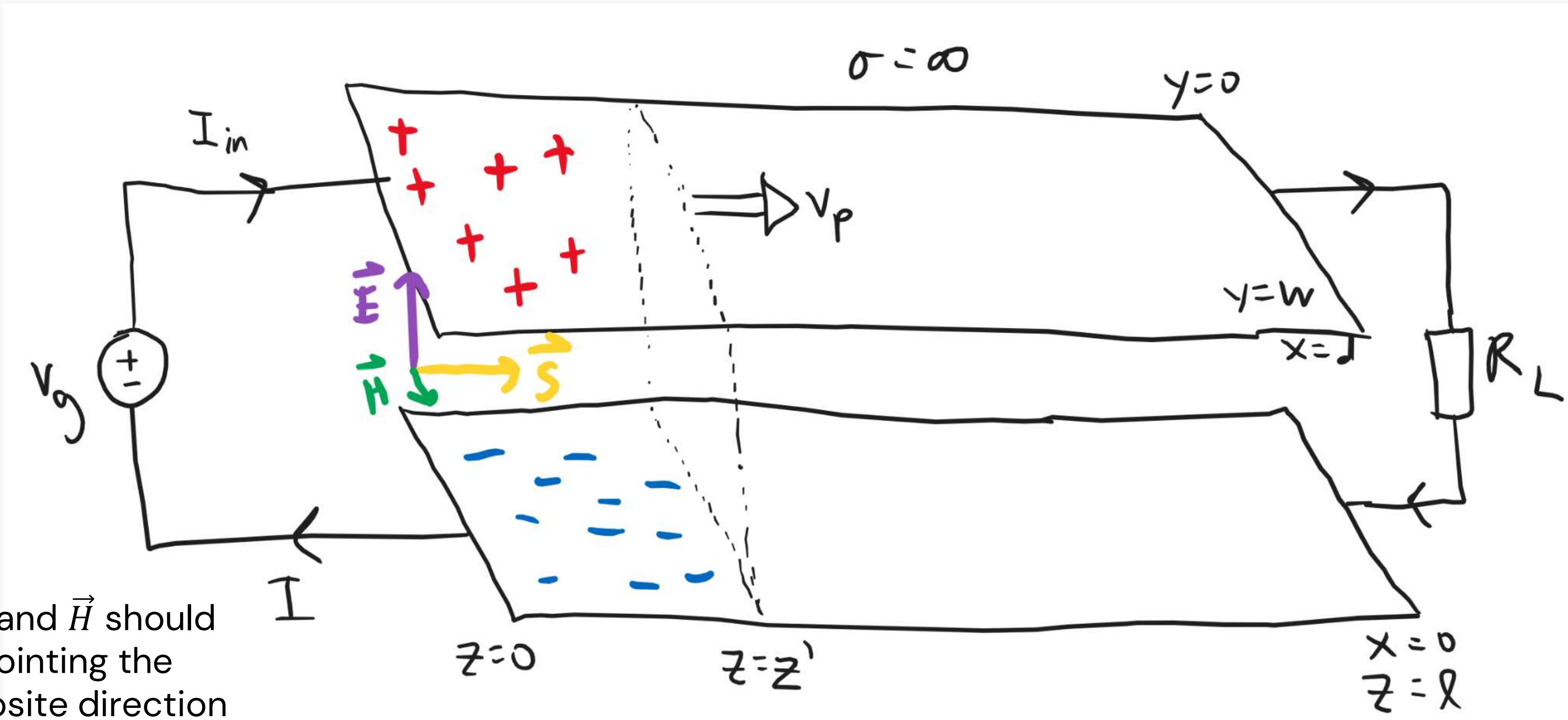
Transmission Lines!

Why do we care?

Transmission Line

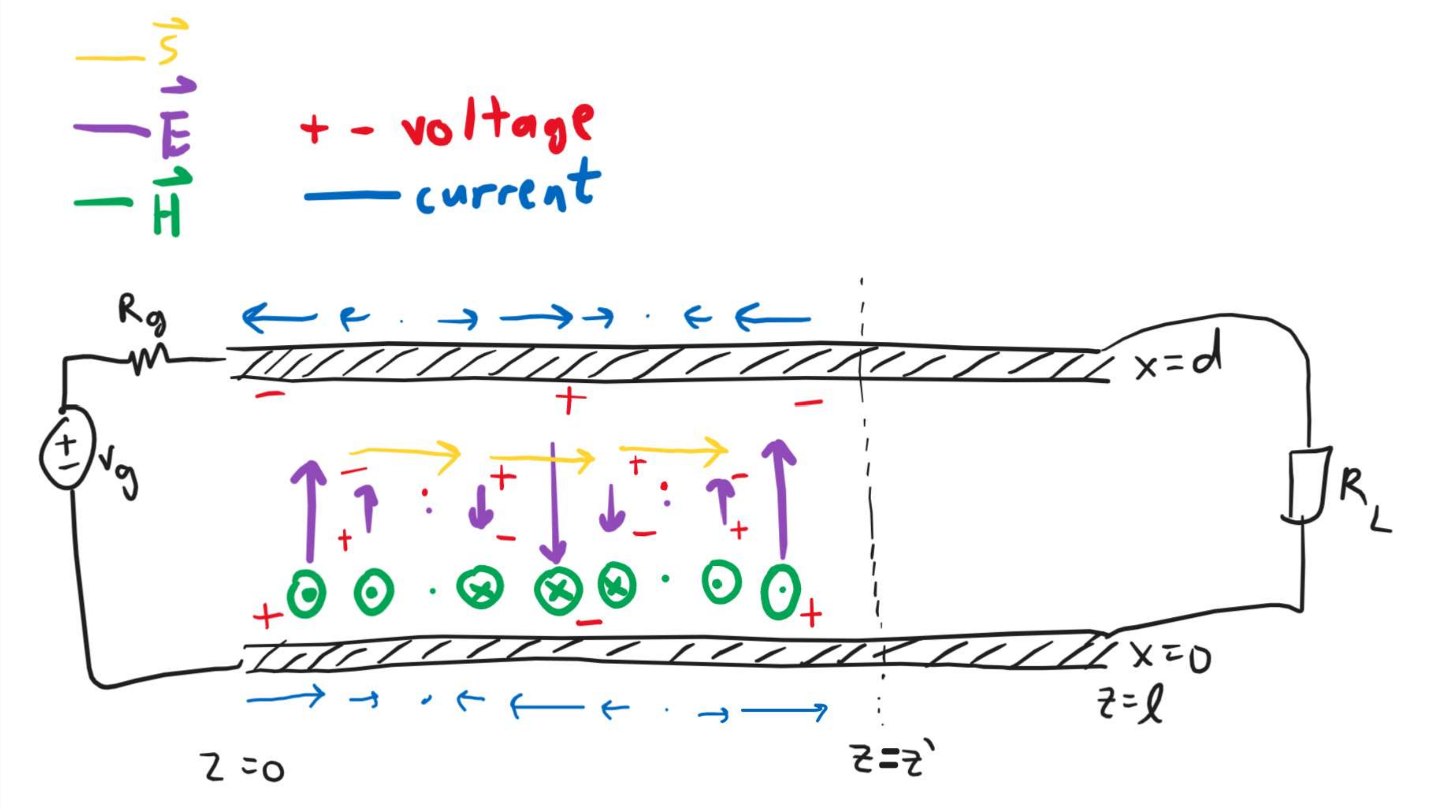


Transmission Line: Parallel Plate Version

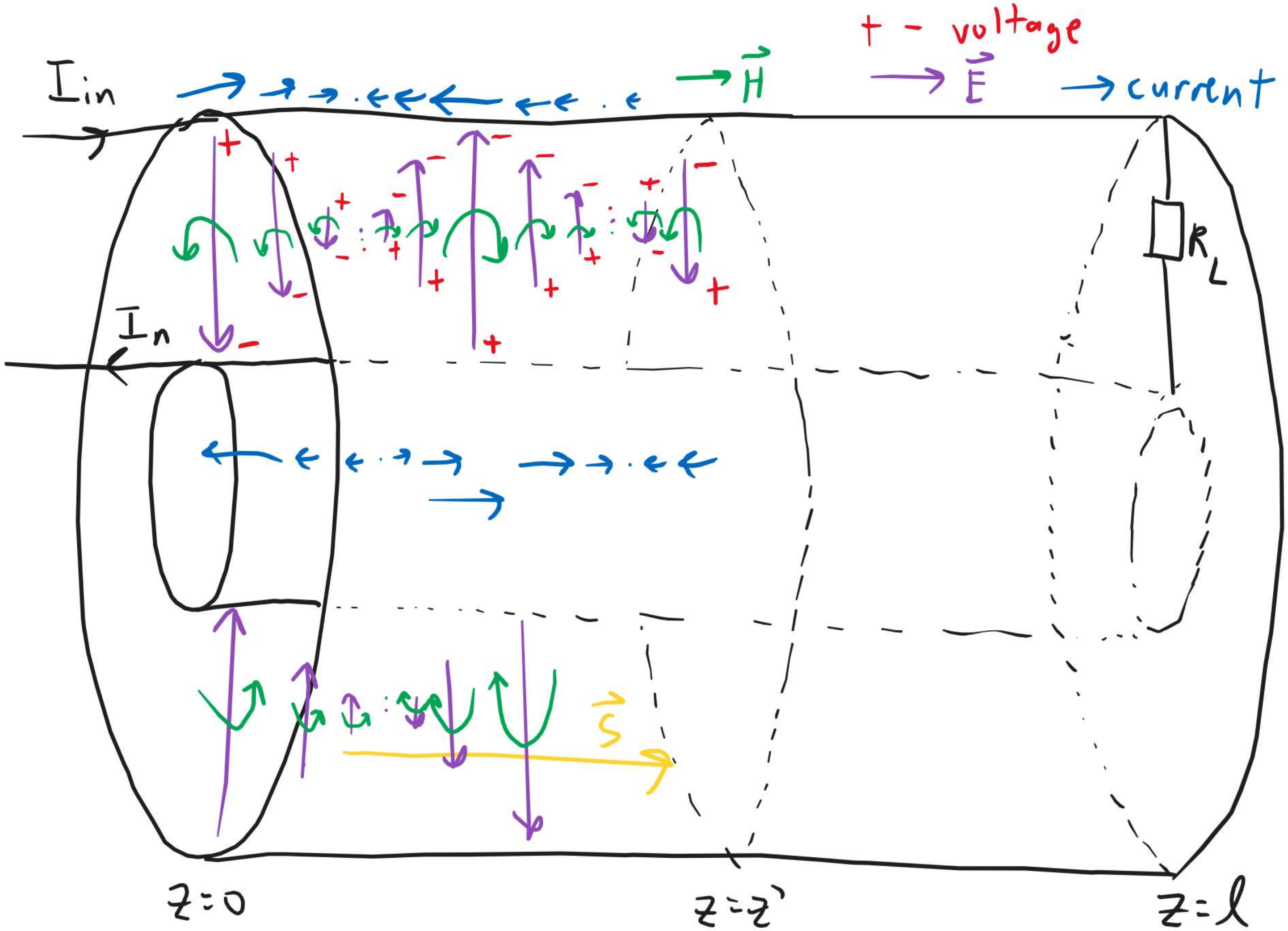


***$\vec{E}$ and $\vec{H}$ should be pointing the opposite direction oops

Transmission Line: Parallel Plate Version



Coax Version:



Midterm 1 equations, in one place

$$\vec{F} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\vec{F} = q_1\vec{E} + q_1(\vec{v}_1 \times \vec{B})$$

$$\vec{E} = \frac{q_2}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\epsilon \oiint \vec{E} \cdot d\vec{S} = Q_{\text{enclosed}}$$

$$\oiint \vec{D} \cdot d\vec{S} = Q_{\text{enclosed}}$$

$$\iiint \rho dV = Q_{\text{enclosed}}$$

$$\oiint \vec{B} \cdot d\vec{S} = 0$$

$$I = \oiint \vec{J} \cdot d\vec{S} = -\frac{\partial Q_{\text{enclosed}}}{\partial t}$$

$$\nabla \times \vec{E} = 0$$

$$\vec{E} = -\nabla V$$

$$\oint \vec{E} \cdot d\vec{l} = 0$$

$$V_{ab} = V(b) - V(a) = -\int_a^b \vec{E} \cdot d\vec{l}$$

$$\hat{n} \cdot (\vec{D}_1 - \vec{D}_2) = \rho_s$$

$$\hat{n} \times (\vec{E}_1 - \vec{E}_2) = 0$$

$$\hat{n} \cdot (\vec{P}_1 - \vec{P}_2) = -\rho_{b,s}$$

$$\epsilon = \epsilon_0(1 + \chi_e)$$

$$\vec{P} = \epsilon_0 \chi_e \vec{E}$$

$$\vec{D} = \epsilon_0\vec{E} + \vec{P} = \epsilon\vec{E}$$

$$\vec{J} = \sigma\vec{E}$$

$$\rho_b = -\nabla \cdot \vec{P}$$

$$\nabla \cdot \epsilon_0\vec{E} = \rho_f + \rho_b$$

$$\nabla \cdot \vec{D} = \rho$$

$$\nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$$

$$-\nabla^2 V = \frac{\rho}{\epsilon}$$

$$\oiint \vec{D} \cdot d\vec{S} = \iiint \nabla \cdot \vec{D} dV$$

$$\oint \vec{E} \cdot d\vec{l} = \iint (\nabla \times \vec{E}) \cdot d\vec{S}$$

$$\int_a^b \nabla V \cdot d\vec{l} = V(b) - V(a)$$

Midterm 2 equations, in one place

$$\vec{B} = \frac{\mu I}{2\pi r}\hat{\phi}$$

$$d\vec{B} = \frac{\mu I d\vec{\ell} \times \hat{r}}{4\pi r^2}$$

$$\oint_C \vec{H} \cdot d\vec{\ell} = \iint_S \vec{J} \cdot d\vec{S}$$

$$\oint_C \vec{B} \cdot d\vec{\ell} = \mu I_{\text{encl}}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0$$

$$\Psi = \iint_s \vec{B} \cdot d\vec{S}$$

$$-\frac{d}{dt}\iint_s \vec{B} \cdot d\vec{S} = \oint_c \vec{E} \cdot d\vec{l}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\oint_c \vec{E} \cdot d\vec{l} = \varepsilon$$

$$\varepsilon = \frac{W}{q} = \oint_C \frac{\vec{F}}{q} \cdot d\vec{l}$$

$$\Psi = LI$$

$$\varepsilon = IR$$

$$v = \frac{\omega}{\beta} = \lambda f = \frac{1}{\sqrt{\mu\epsilon}}$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$\eta = \frac{\sqrt{\mu}}{\sqrt{\epsilon}}$$

$$\beta = \omega\sqrt{\mu\epsilon}$$

$$\nabla^2 \vec{E} = \mu\epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\vec{S} = \vec{E} \times \vec{H}$$

$$\tilde{S} = \tilde{E} \times \tilde{H}^*$$

$$< \vec{S} > = \frac{1}{2}\text{Re}\{\tilde{E} \times \tilde{H}^*\}$$

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\epsilon\vec{E} \cdot \vec{E} + \frac{1}{2}\mu\vec{H} \cdot \vec{H}\right) + \nabla \cdot \vec{S} + \vec{J} \cdot \vec{E} = 0$$

$$Q = CV$$

$$G = \frac{\sigma}{\epsilon}C \qquad R = \frac{1}{G}$$

$$\vec{J}_b = \frac{\partial \vec{P}}{\partial t} + \nabla \times \vec{M}$$

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

$$\vec{M} = \chi_m \vec{H}$$

$$\vec{B} = \mu_0 \mu_r \vec{H} = \mu \vec{H}$$

$$A\cos(\omega t - \beta x)\hat{z} \leftrightarrow Ae^{-j\beta x}$$

$$\hat{n} \cdot (\vec{B}_1 - \vec{B}_2) = 0$$

$$\hat{n} \times (\vec{H}_1 - \vec{H}_2) = \vec{J}_s$$

$$\hat{n} \times (\vec{M}_1 - \vec{M}_2) = \vec{J}_{b,s}$$

Midterm 3 equations, in one place

	Condition	β	α	$\lvert\eta\rvert$	τ	$\lambda = \frac{2\pi}{\beta}$	$\delta = \frac{1}{\alpha}$
Perfect dielectric	$\sigma = 0$	$\omega\sqrt{\epsilon\mu}$	0	$\sqrt{\frac{\mu}{\epsilon}}$	0	$\frac{2\pi}{\omega\sqrt{\epsilon\mu}}$	∞
Imperfect dielectric	$\frac{\sigma}{\omega\epsilon} \ll 1$	$\sim \omega\sqrt{\epsilon\mu}$	$\beta\frac{1}{2}\frac{\sigma}{\omega\epsilon} = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}$	$\sim \sqrt{\frac{\mu}{\epsilon}}$	$\sim \frac{\sigma}{2\omega\epsilon}$	$\sim \frac{2\pi}{\omega\sqrt{\epsilon\mu}}$	$\frac{2}{\sigma}\sqrt{\frac{\epsilon}{\mu}}$
Good conductor	$\frac{\sigma}{\omega\epsilon} \gg 1$	$\sim \sqrt{\pi f \mu\sigma}$	$\sim \sqrt{\pi f \mu\sigma}$	$\sqrt{\frac{\omega\mu}{\sigma}}$	$45°$	$\sim \frac{2\pi}{\sqrt{\pi f \mu\sigma}}$	$\sim \frac{1}{\sqrt{\pi f \mu\sigma}}$
Perfect conductor	$\sigma = \infty$	∞	∞	0	-	0	0

$$v = \frac{\omega}{\beta} = \lambda f \qquad \nabla^2 \tilde{E} = (j\omega\mu)(\sigma + j\omega\epsilon)\tilde{E}$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \qquad \tau = \frac{2\eta_2}{\eta_2 + \eta_1} = 1 + \Gamma$$

Units

Charge Q: C
Current I: A
Electric field strength $\vec{E}$: N/C or V/m
Electric flux density $\vec{D}$: C/m^2
Polarization field $\vec{P}$: C/m^2
Electric potential V: V
Capacitance C: F
Magnetic flux density $\vec{B}$: T or Wb/m^2
Magnetic field strength $\vec{H}$: A/m
Magnetic flux Ψ: Wb
Electromotive force ε: V
Inductance L: H

Electric permittivity ϵ: F/m
Magnetic permeability μ: H/m
Conductivity σ: Si/m

Charge density ρ: C/m^3
Surface charge density ρ_s: C/m^2
Current density $\vec{J}$: A/m^2

Intrinsic impedance η: Ohm
Wave number β: rad/m