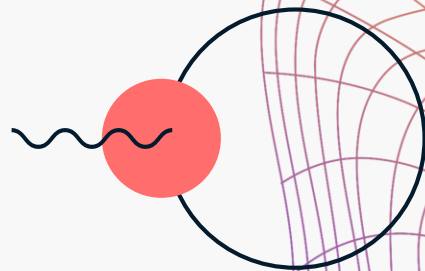




ECE329: Tutorial Session 5

October 9th, 2025



Last week: Statics

Electrostatics

$$\oiint_S \vec{D} \cdot d\vec{S} = \iiint_V \rho dV$$

$$\nabla \cdot \vec{D} = \rho$$

$$\oint_C \vec{E} \cdot d\vec{\ell} = 0$$

$$\nabla \times \vec{E} = 0$$

Magnetostatics

$$\oiint_S \vec{B} \cdot d\vec{S} = 0$$

$$\nabla \cdot \vec{B} = 0$$

$$\oint_C \vec{H} \cdot d\vec{\ell} = \iint_S \vec{J} \cdot d\vec{S}$$

$$\nabla \times \vec{H} = \vec{J}$$

This week: Dynamics!

$$-\frac{d\Psi}{dt} = -\frac{d}{dt} \iint_S \vec{B} \cdot d\vec{S} = \oint_C \vec{E} \cdot d\vec{l} = \varepsilon$$

Ψ is **magnetic flux** (notice that we integrated magnetic flux density \vec{B} [Wb/m²] to get magnetic flux). Units: [Wb]

In electrodynamics, \vec{E} is no longer curl-free/path-independent/conservative; integration over a closed loop yields a nonzero number.

ε is **electromotive force** (or emf). Units: [V]

Electromotive 'force'

$$-\frac{d\Psi}{dt} = -\frac{d}{dt} \iint_S \vec{B} \cdot d\vec{S} = \oint_C \vec{E} \cdot d\vec{l} = \varepsilon$$

ε is **electromotive force** (or emf). Units: [V]
It is not a force.

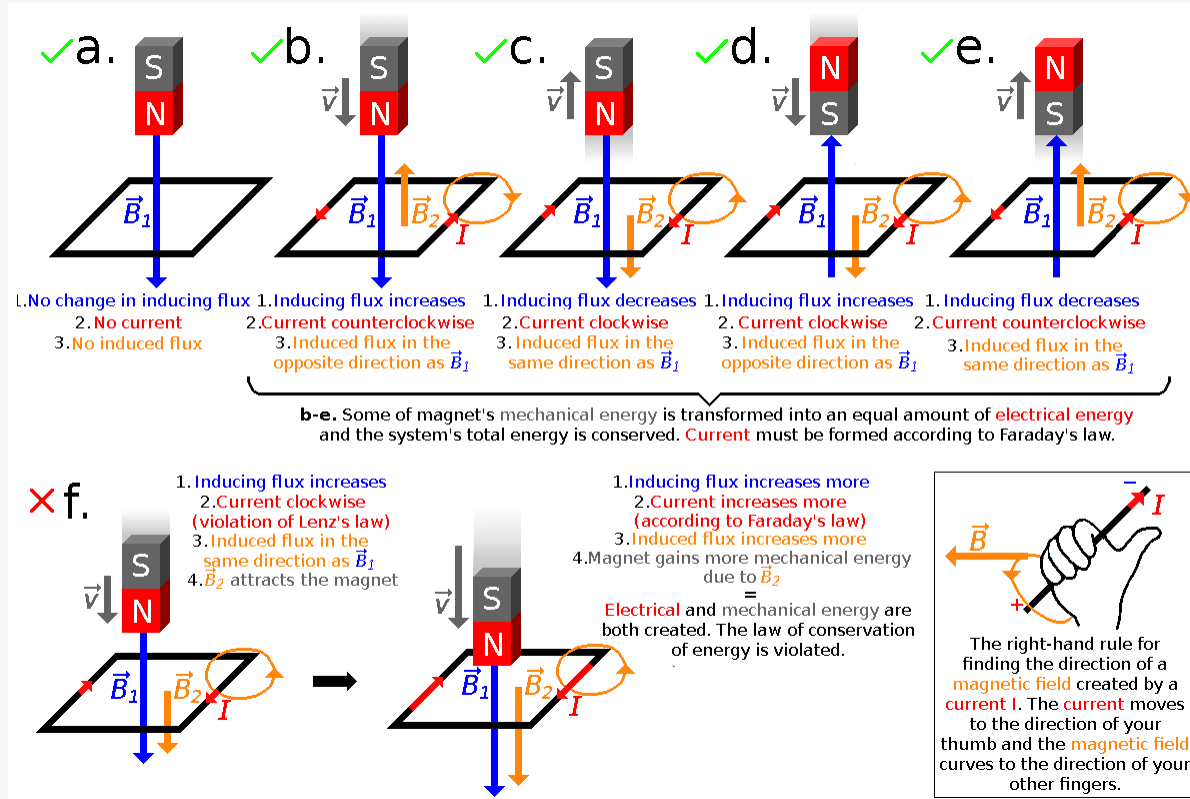
It is the electromagnetic work done to move a unit electric charge once around the closed loop.

$$\varepsilon = \frac{W}{q} = \oint_C \frac{\vec{F}}{q} \cdot d\vec{l}$$

$$\varepsilon = IR$$

Lenz's Law

the direction of the electric current induced in a conductor by a changing magnetic field is such that the magnetic field created by the induced current opposes changes in the initial magnetic field



Faraday's Law

$$\oint_c \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \iint_s \vec{B} \cdot d\vec{S}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Problem 1

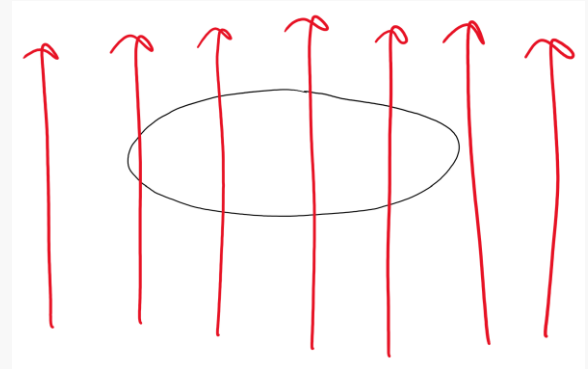
$$-\frac{d\Psi}{dt} = -\frac{d}{dt} \iint_s \vec{B} \cdot d\vec{S} = \oint_c \vec{E} \cdot d\vec{l} = \varepsilon$$

Express the unit [Wb] using [V] and other SI quantities.

Problem 2

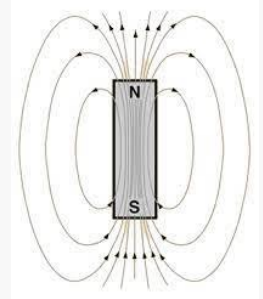
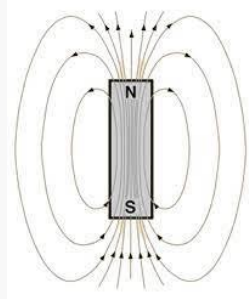
A circular loop is immersed in a uniform magnetic field such that the plane of the loop is perpendicular to the direction of \vec{B} . Which of the following will create a nonzero emf in the loop?

1. The coil is rotated about an axis perpendicular to \vec{B} with angular frequency ω .
2. The coil is moved upwards with velocity v .
3. The coil is moved to the left with velocity v .
4. The coil is deformed into a square.
5. The \vec{B} field intensity is increased.



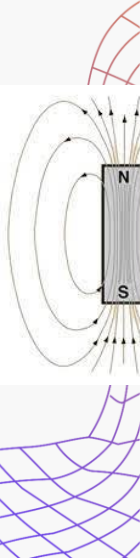
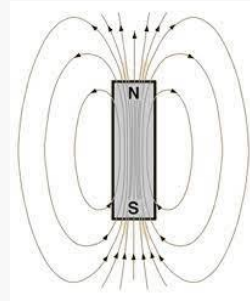
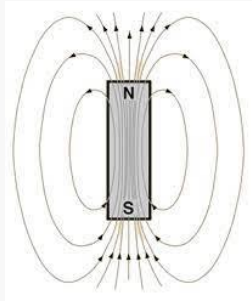
Problem 3, attempt 1

Assume the magnetic field at my location is negligible and the loop is parallel to the xy -plane. What is the direction of current generated in the conducting loop as it travels? Express your answer assuming you are looking down at the panels below.



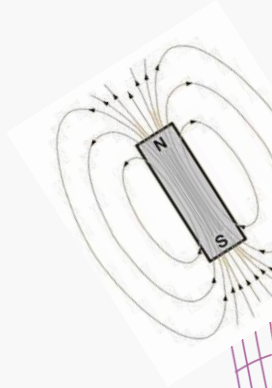
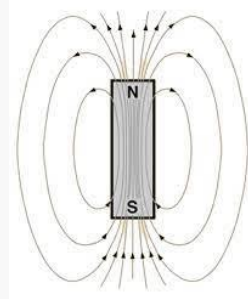
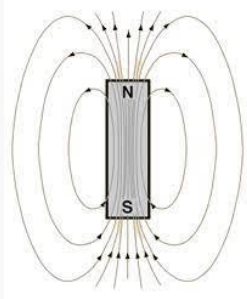
Problem 3, attempt 2

Assume the loop is parallel to the xy -plane. What is the direction of current generated in the conducting loop as it travels? Express your answer assuming you are looking down at the panels below.



Problem 3, attempt 3

Assume the loop is parallel to the xy -plane. What is the direction of current generated in the conducting loop as it travels? Express your answer assuming you are looking down at the panels below.



Problem 4

Consider two infinite line currents $I_x \hat{x}$ [A] and $I_y \hat{y}$ [A] along the x and y axis and crossing at the origin but not interfering with each other.

1. Find the \vec{B} field in the region $x > 0, y > 0$ on the xy -plane.
2. Find the emf generated in a square wire loop (sidelength ℓ) moving with a velocity $\vec{v} = v_x \hat{x} + v_y \hat{y}$ [m/s] in the region $x > 0, y > 0$ on the xy -plane. Assume the wire loop is very small such that \vec{B} can be assumed to be uniform across the loop.



Inductance

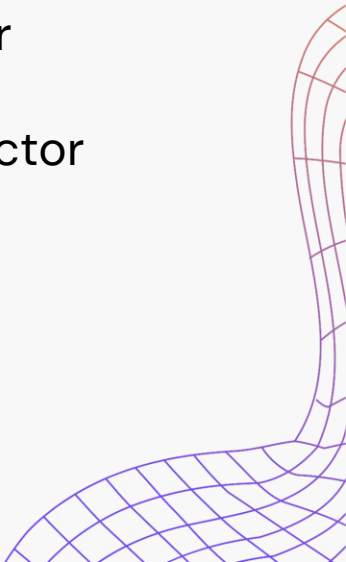
Inductance: the tendency of an electrical conductor to oppose a change in electric current flowing through it.

Inductance exists only in conductors!

Self inductance: change in current flowing through a conductor induces an emf in the conductor itself.

Mutual inductance: change in current flowing through a conductor induces an emf in any nearby conductors.


$$\Psi = LI$$

$$Q = CV$$




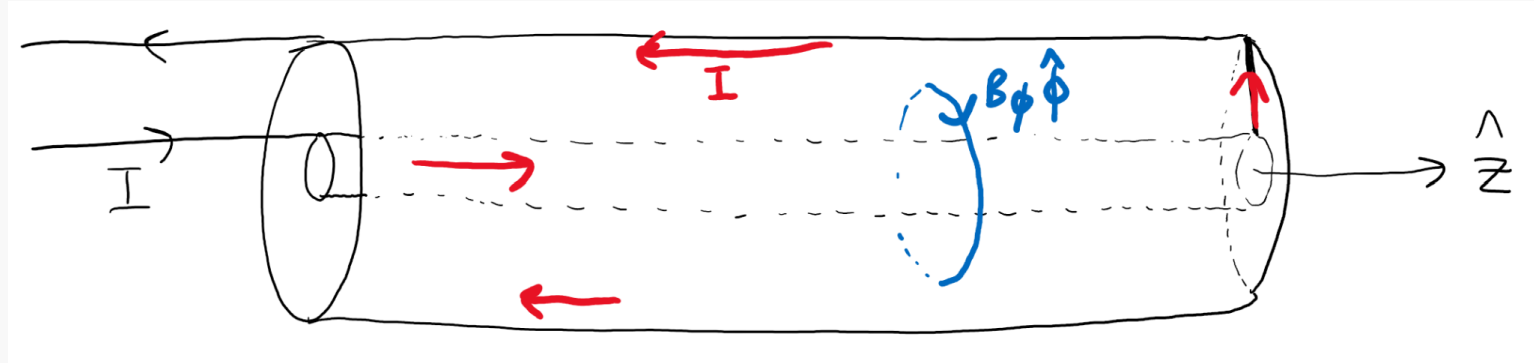
Problem 5: Solenoid

Given n , the number of turns in a solenoid per unit length, d , the length of the solenoid, R , the radius of the solenoid, and I , the current each turn carries, find inductance L of the solenoid.



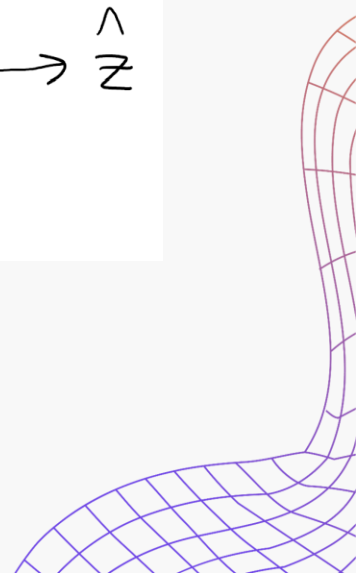
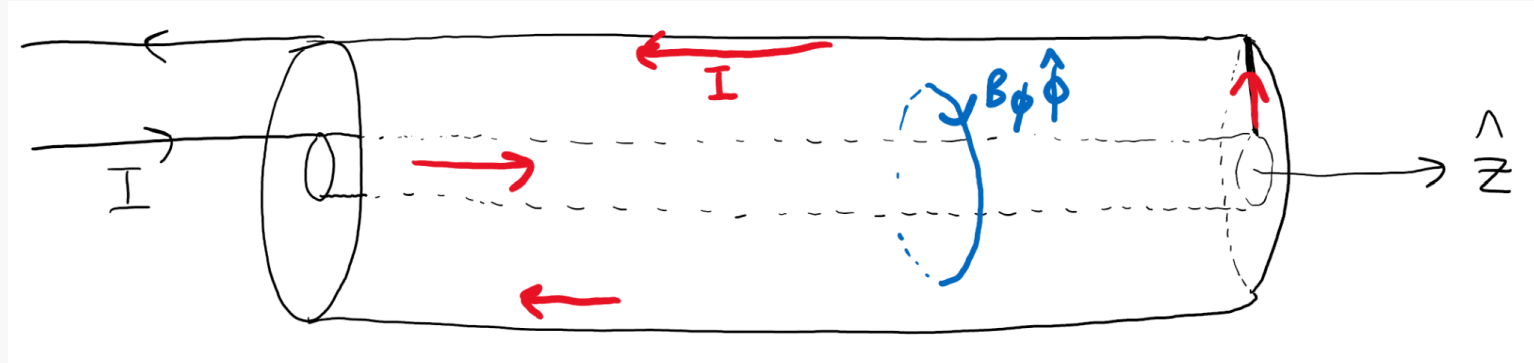
Problem 1

Given a shorted coaxial cable with inner radius a , outer radius b and length ℓ , carrying current I on the inner conductor, find the inductance L of the shorted coax cable.



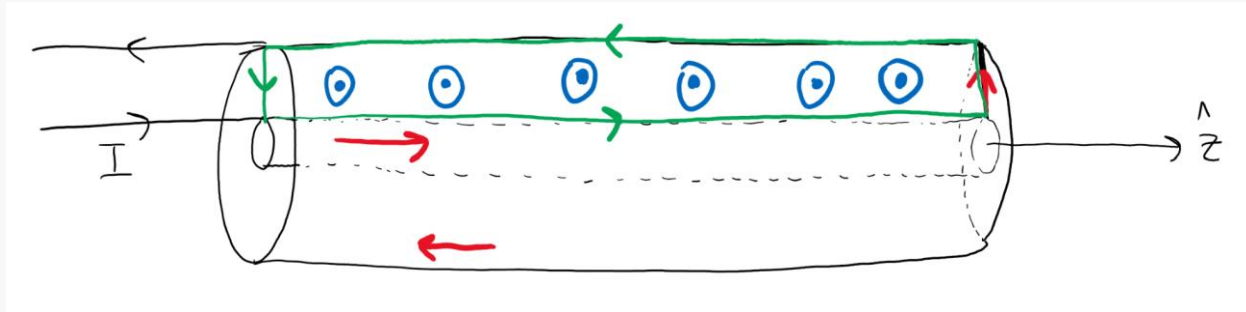
Problem 1

Given a shorted coaxial cable with inner radius a , outer radius b and length ℓ , carrying current I on the inner conductor, find the inductance L of the shorted coax cable.



Problem 6: Shorted coaxial cable


Given a shorted coaxial cable with inner radius a , outer radius b and length ℓ , carrying current I on the inner conductor, find the inductance L of the shorted coax cable.





Problem 6: Shorted coaxial cable


Given a shorted coaxial cable with inner radius a , outer radius b and length ℓ , carrying current I on the inner conductor, find the inductance L of the shorted coax cable.





Problem 6: Shorted coaxial cable

Given a shorted coaxial cable with inner radius a , outer radius b and length ℓ , carrying current I on the inner conductor, find the inductance L of the shorted coax cable.



Problem 7

$$\Psi = LI$$

Express the unit [H] using the SI quantities [kg], [m], [s], and [C].

Problem 8: Rotations!

A square loop of sidelength $2m$ sits on the xy -plane at $t = 0$ and begins to rotate about the x -axis clockwise with angular frequency ω when viewed at $x = 10$ looking at the origin.

1. Express the angle between the plane of the current loop at the xy -plane as a function of t .
2. Suppose $d\vec{S}$ points in the $+\hat{z}$ direction at $t = 0$. Find the direction of $d\vec{S}$ as a function of t .
3. Suppose $\vec{B} = 3\hat{z} + 6\hat{y}$ [Wb/m²]. Find $\Psi(t)$.

Midterm 1 equations, in one place

$$\vec{F} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2} \hat{r}$$

$$\vec{F} = q_1 \vec{E} + q_1 (\vec{v}_1 \times \vec{B})$$

$$\vec{E} = \frac{q_2}{4\pi\epsilon_0 r^2} \hat{r}$$

$$\hat{n} \cdot (\vec{D}_1 - \vec{D}_2) = \rho_s$$

$$\hat{n} \times (\vec{E}_1 - \vec{E}_2) = 0$$

$$\hat{n} \cdot (\vec{P}_1 - \vec{P}_2) = -\rho_{b,s}$$

$$\epsilon \oint \vec{E} \cdot d\vec{S} = Q_{\text{enclosed}}$$

$$\oint \vec{D} \cdot d\vec{S} = Q_{\text{enclosed}}$$

$$\iiint \rho dV = Q_{\text{enclosed}}$$

$$\oint \vec{B} \cdot d\vec{S} = 0$$

$$I = \oint \vec{J} \cdot d\vec{S} = -\frac{\partial Q_{\text{enclosed}}}{\partial t}$$

$$\epsilon = \epsilon_0(1 + \chi_e)$$

$$\vec{P} = \epsilon_0 \chi_e \vec{E}$$

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P} = \epsilon \vec{E}$$

$$\vec{J} = \sigma \vec{E}$$

$$\rho_b = -\nabla \cdot \vec{P}$$

$$\nabla \cdot \epsilon_0 \vec{E} = \rho_f + \rho_b$$

$$\nabla \cdot \vec{D} = \rho$$

$$\nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$$

$$-\nabla^2 V = \frac{\rho}{\epsilon}$$

$$\nabla \times \vec{E} = 0$$

$$\vec{E} = -\nabla V$$

$$\oint \vec{E} \cdot d\vec{l} = 0$$

$$V_{ab} = V(b) - V(a) = -\int_a^b \vec{E} \cdot d\vec{l}$$

$$\oint \vec{D} \cdot d\vec{S} = \iiint \nabla \cdot \vec{D} dV$$

$$\oint \vec{E} \cdot d\vec{l} = \iint (\nabla \times \vec{E}) \cdot d\vec{S}$$

$$\int_a^b \nabla V \cdot d\vec{l} = V(b) - V(a)$$



Midterm 2 equations, in one place

$$\vec{B} = \frac{\mu I}{2\pi r} \hat{\phi}$$

$$d\vec{B} = \frac{\mu I d\vec{\ell} \times \hat{r}}{4\pi r^2}$$

$$\oint_C \vec{H} \cdot d\vec{\ell} = \iint_S \vec{J} \cdot d\vec{S}$$

$$\oint_C \vec{B} \cdot d\vec{\ell} = \mu I_{\text{encl}}$$

$$\nabla \times \vec{H} = \vec{J}$$

$$\nabla \cdot \vec{B} = 0$$

$$\Psi = \iint_S \vec{B} \cdot d\vec{S}$$

$$-\frac{d}{dt} \iint_S \vec{B} \cdot d\vec{S} = \oint_C \vec{E} \cdot d\vec{\ell}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\oint_C \vec{E} \cdot d\vec{\ell} = \varepsilon$$

$$\varepsilon = \frac{W}{q} = \oint_C \frac{\vec{F}}{q} \cdot d\vec{\ell}$$

$$\Psi = LI$$

$$\varepsilon = IR$$

$$Q = CV$$
$$G = \frac{\sigma}{\epsilon} C \quad R = \frac{1}{G}$$



Units

Charge Q : C

Current I : A

Electric field strength \vec{E} : N/C or V/m

Electric flux density \vec{D} : C/m²

Polarization field \vec{P} : C/m²

Electric potential V : V

Capacitance C : F

Magnetic flux density \vec{B} : T or Wb/m²

Magnetic field strength \vec{H} : A/m

Magnetic flux Ψ : Wb

Electromotive force ε : V

Inductance L : H

Electric permittivity ϵ : F/m

Magnetic permeability μ : H/m

Conductivity σ : Si/m

Charge density ρ : C/m³

Surface charge density ρ_s : C/m²

Current density \vec{j} : A/m²