ECE329: Tutorial Session 5

October 9th, 2025

Last week: Statics

Electrostatics	Magnetostatics
$\oiint_S \vec{D} \cdot d\vec{S} = \iiint_V \rho dV$ $\nabla \cdot \vec{D} = \rho$	$\oiint_S \vec{B} \cdot d\vec{S} = 0$ $\nabla \cdot \vec{B} = 0$
$\oint_C \vec{E} \cdot d\vec{\ell} = 0$ $\nabla \times \vec{E} = 0$	$\oint_C \vec{H} \cdot d\vec{\ell} = \iint_S \vec{J} \cdot d\vec{S}$ $\nabla \times \vec{H} = \vec{J}$

This week: Dynamics!

$$-\frac{d\Psi}{dt} = -\frac{d}{dt}\iint_S \vec{B}\cdot d\vec{S} = \oint_C \vec{E}\cdot d\vec{l} = \varepsilon$$

Ψ is **magnetic flux** (notice that we integrated magnetic flux density $\vec{B}$ [Wb/m^2] to get magnetic flux). Units: [Wb]

In electrodynamics, $\vec{E}$ is no longer curl-free/path-independent/conservative; integration over a closed loop yields a nonzero number.
ε is **electromotive force** (or emf). Units: [V]

Electromotive ‘force’

$$-\frac{d\Psi}{dt} = -\frac{d}{dt}\iint_S \vec{B}\cdot d\vec{S} = \oint_C \vec{E}\cdot d\vec{l} = \varepsilon$$

ε is **electromotive force** (or emf). Units: [V]
It is not a force.

J

It is the electromagnetic work done to move a unit electric charge once around the closed loop.

$$\varepsilon = \frac{W}{q} = \oint_C \frac{\vec{F}}{q}\cdot d\vec{l}$$

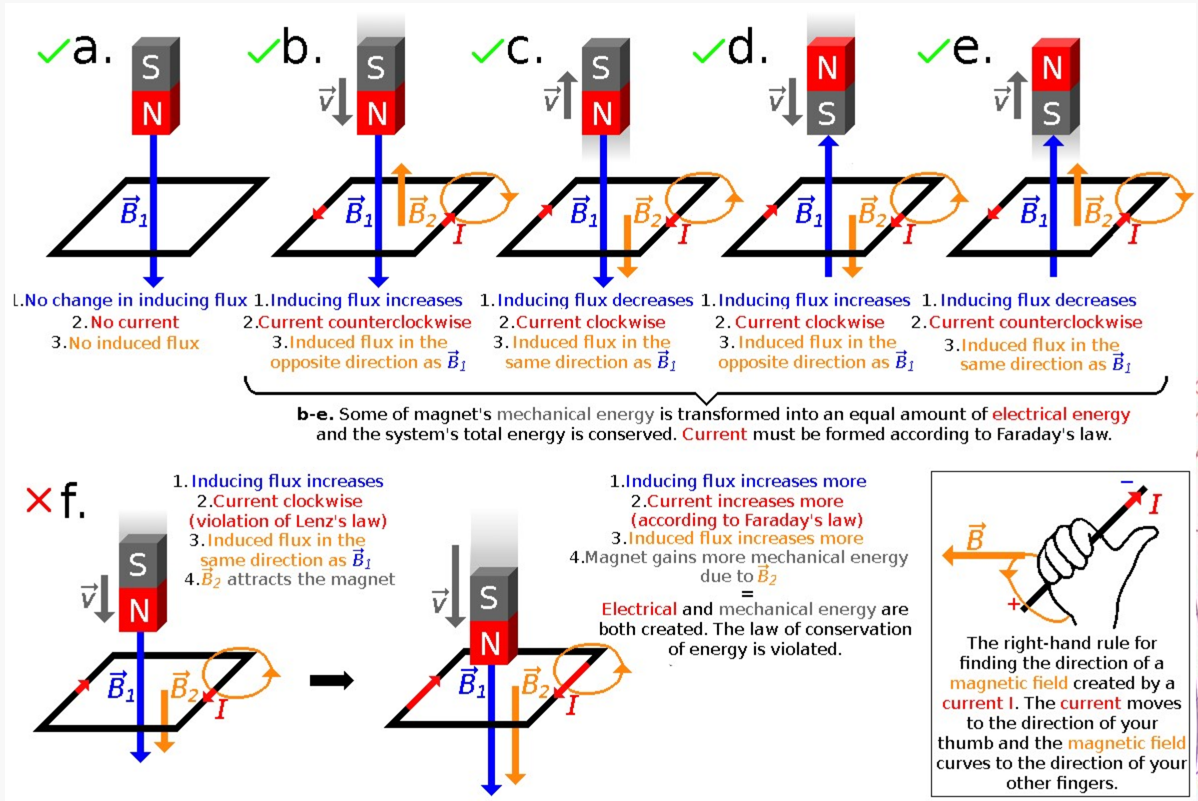
$W = Fd$

$V = \frac{J}{C} = \frac{Nm}{C}$

$$\varepsilon = IR$$

Lenz's Law

the direction of the electric current induced in a conductor by a changing magnetic field is such that the magnetic field created by the induced current opposes changes in the initial magnetic field



https://en.wikipedia.org/wiki/Lenz%27s_law

Faraday's Law

$$\oint_c \vec{E} \cdot d\vec{l} = -\frac{d}{dt}\iint_s \vec{B} \cdot d\vec{S}$$

stoke

$$\iint_S (\nabla \times \vec{E}) \cdot d\vec{S}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Problem 1

$$-\frac{d\Psi}{dt} = -\frac{d}{dt}\iint_S \vec{B}\cdot d\vec{S} = \oint_C \vec{E}\cdot d\vec{l} = \varepsilon$$

Express the unit [Wb] using [V] and other SI quantities.

s m C

$$\frac{Wb}{s} = V$$

$$W = \frac{J}{s}$$

$$\frac{C}{s}$$

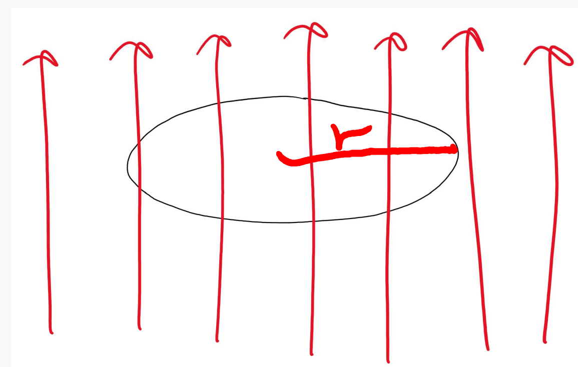
$$[Wb] = [Vs]$$

A

Problem 2

A circular loop is immersed in a uniform magnetic field such that the plane of the loop is perpendicular to the direction of $\vec{B}$. Which of the following will create a nonzero emf in the loop?

1. The coil is rotated about an axis perpendicular to $\vec{B}$ with angular frequency ω.
2. The coil is moved upwards with velocity v.
3. The coil is moved to the left with velocity v.
4. The coil is deformed into a square.
5. The $\vec{B}$ field intensity is increased.



$$\mathcal{E} = -\frac{d}{dt}\Phi = -\frac{d}{dt}\iint_S \vec{B}\cdot d\vec{S}$$

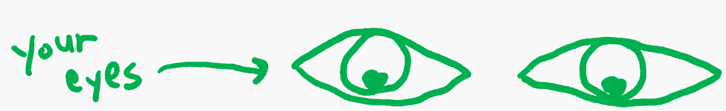
$B = 5t\,\hat{z}$

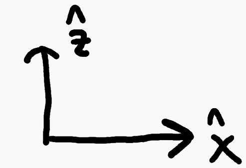
start: πr^2

end:

$$\left(\frac{2\pi r}{4}\right)^2$$

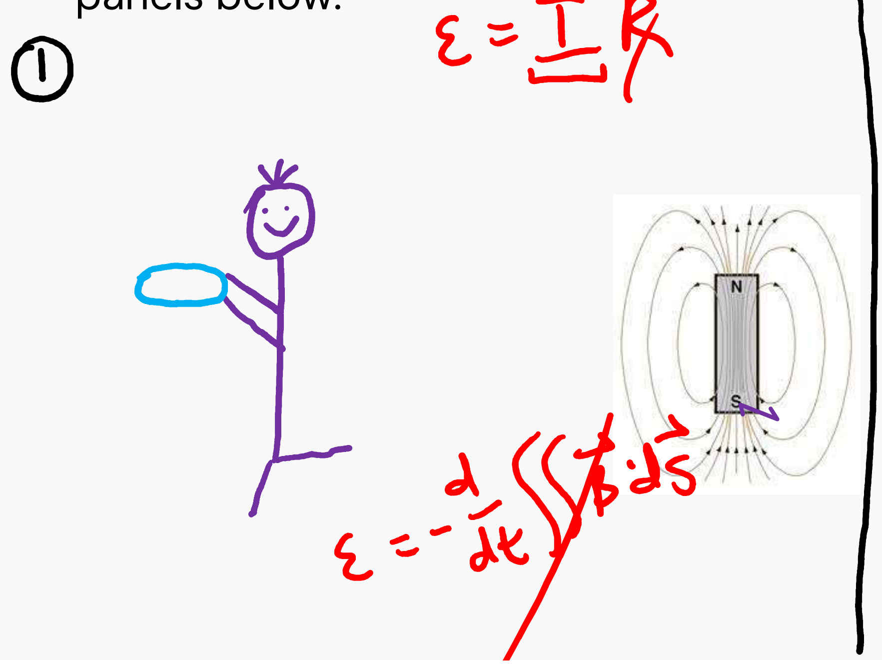
$\frac{1}{4}\pi^2 r^2 \neq \pi r^2$

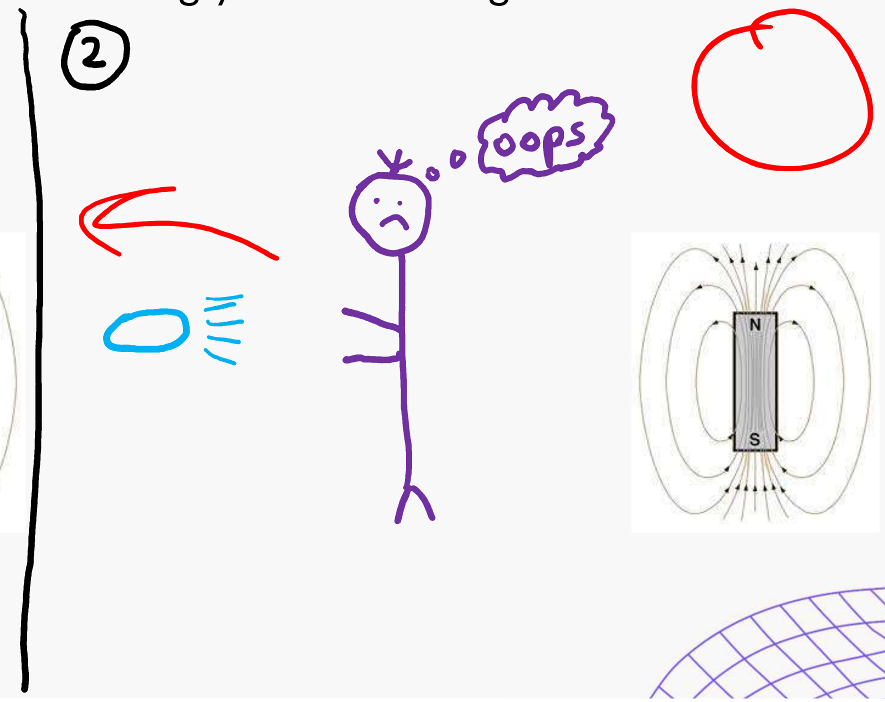




Problem 3, attempt 1

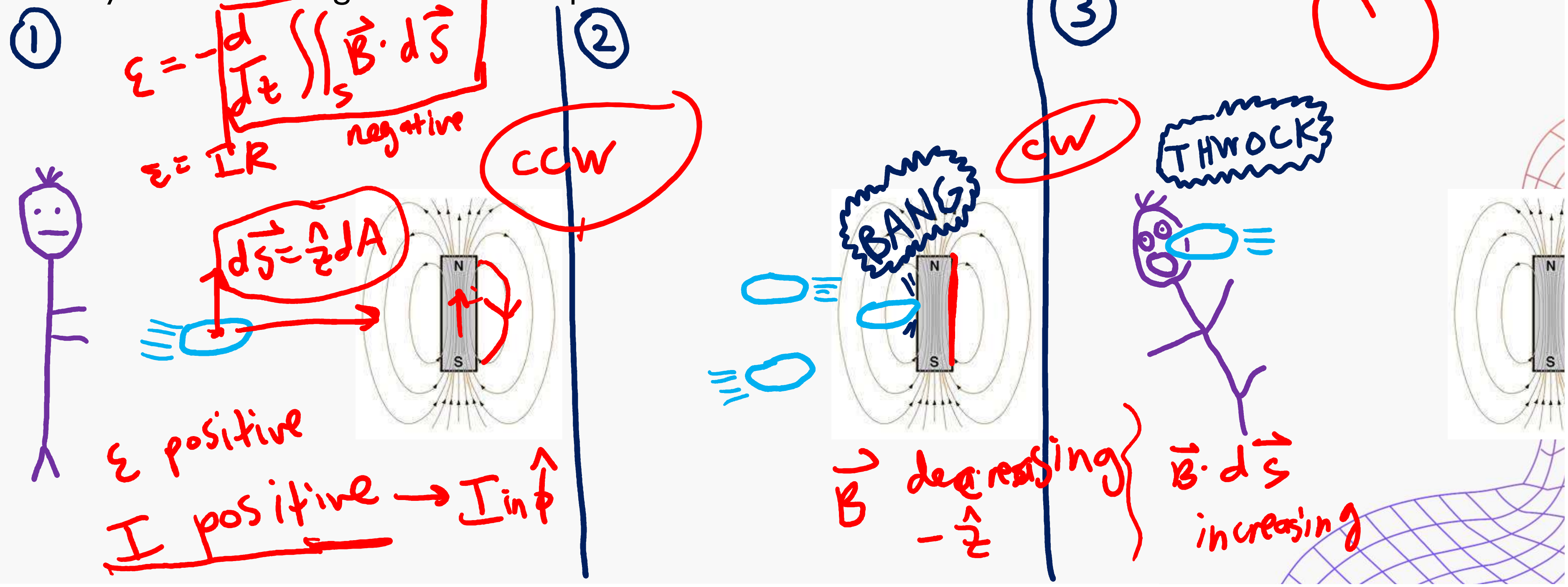
Assume the magnetic field at my location is negligible and the loop is parallel to the xy-plane. What is the direction of current generated in the conducting loop as it travels? Express your answer assuming you are looking down at the panels below.



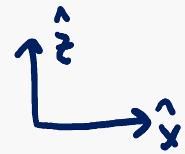


Problem 3, attempt 2

Assume the loop is parallel to the xy-plane. What is the direction of current generated in the conducting loop as it travels? Express your answer assuming you are looking down at the panels below.

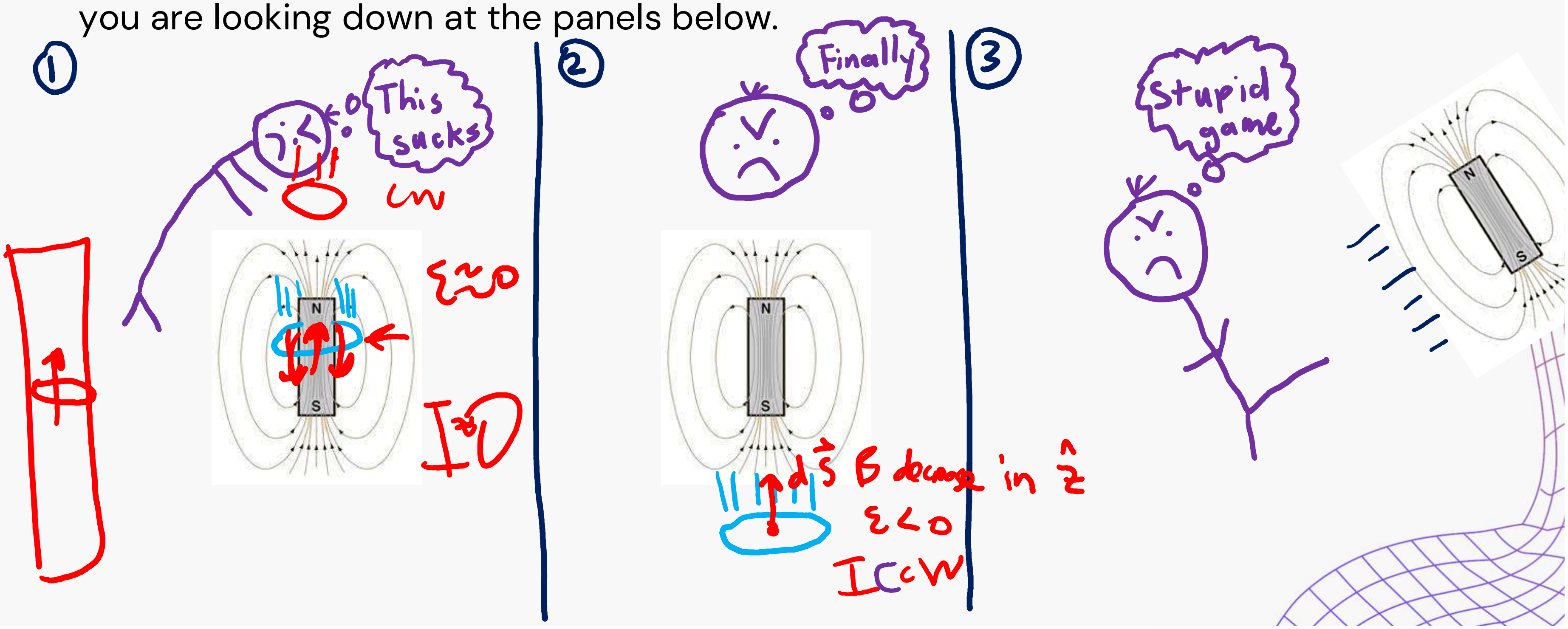


Problem 3, attempt 3



$$\mathcal{E} = -\frac{d}{dt}\iint \vec{B}\cdot d\vec{S}$$

Assume the loop is parallel to the xy-plane. What is the direction of current generated in the conducting loop as it travels? Express your answer assuming you are looking down at the panels below.

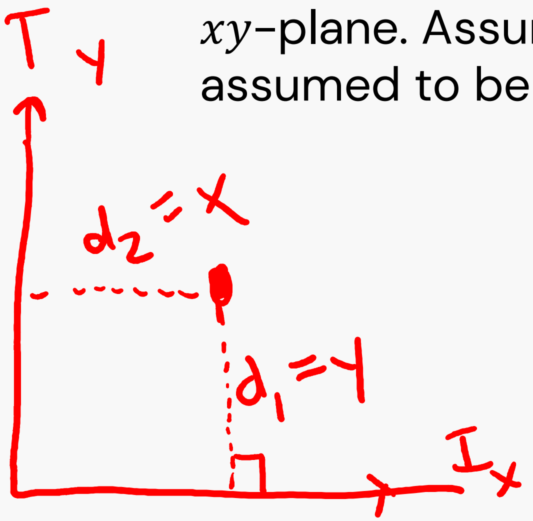


$$I = I_z \hat{z} \longrightarrow \vec{B} = \frac{\mu_0 I_z}{2\pi r}\hat{\phi}$$

Problem 4

Consider two infinite line currents $I_x\hat{x}$ [A] and $I_y\hat{y}$ [A] along the x and y axis and crossing at the origin but not interfering with each other.

1. Find the $\vec{B}$ field in the region $x > 0$, $y > 0$ on the xy-plane.
2. Find the emf generated in a square wire loop (sidelength ℓ) moving with a velocity $\vec{v} = v_x\hat{x} + v_y\hat{y}$ [m/s] in the region $x > 0$, $y > 0$ on the xy-plane. Assume the wire loop is very small such that $\vec{B}$ can be assumed to be uniform across the loop.



$$\vec{B} = \left(\frac{\mu_0 I_x}{2\pi y} - \frac{\mu_0 I_y}{2\pi x}\right)\hat{z}$$

Problem 4: Blank slide

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Consider two infinite line currents $I_x\hat{x}$ [A] and $I_y\hat{y}$ [A] along the x and y axis and crossing at the origin but not interfering with each other.

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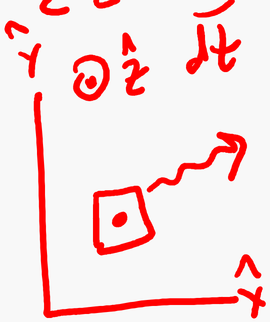
$$d\vec{S} = \hat{z}\, dx\, dy$$

$$\mathcal{E} = -\frac{d}{dt}\iint \vec{B}\cdot d\vec{S} = -\frac{d}{dt}\int_{x=}\int_{y=}\left(\frac{\mu_0 I_x}{2\pi y} - \frac{\mu_0 I_y}{2\pi x}\right)dx\,dy$$

$$= -\frac{d}{dt}\left(\frac{\mu_0 I_x}{2\pi y} - \frac{\mu_0 I_y}{2\pi x}\right)\ell^2$$



Problem 4: Blank slide

$$\mathcal{E} = -\frac{d}{dt}\,\ell^2\left(\frac{\mu_0 I_x}{2\pi y} - \frac{\mu_0 I_y}{2\pi x}\right)$$

Consider two infinite line currents $I_x\hat{x}$ [A] and $I_y\hat{y}$ [A] along the x and y axis and crossing at the origin but not interfering with each other.

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$$y = v_y t + y_0$$

$$x = v_x t + x_0$$

$$-\frac{d}{dt} = -\frac{\partial}{\partial y}\underbrace{\frac{\partial y}{\partial t}}_{v_y} - \frac{\partial}{\partial x}\underbrace{\frac{\partial x}{\partial t}}_{v_x}$$

$$\mathcal{E} = \ell^2\frac{\mu_0 I_x}{2\pi y^2}v_y - \ell^2\frac{\mu_0 I_y}{2\pi x^2}v_x$$

[V]

Inductance

$$\frac{d}{dt}\theta(t) = \frac{d}{d\theta}\frac{d\theta}{dt}$$

Inductance: the tendency of an electrical conductor to oppose a change in electric current flowing through it.

Inductance exists only in conductors!

Self inductance: change in current flowing through a conductor induces an emf in the conductor itself.
Mutual inductance: change in current flowing through a conductor induces an emf in any nearby conductors.

$$\Psi = LI \qquad\qquad Q = CV$$

Problem 5: Solenoid

Given n, the number of turns in a solenoid per unit length, d, the length of the solenoid, R, the radius of the solenoid, and I, the current each turn carries, find inductance L of the solenoid.

$$I = \frac{\varepsilon_{induced}}{R}$$ ← fish

$$\varepsilon_{total} = \varepsilon_{induced} + \varepsilon_{self}$$

$\varepsilon_{induced}$: IR

$$\varepsilon_{total} = -\frac{d}{dt}\Phi_{total}$$

m

$$\varepsilon = L\frac{dI}{dt}$$ → ε_{self}

Problem 5: Solenoid

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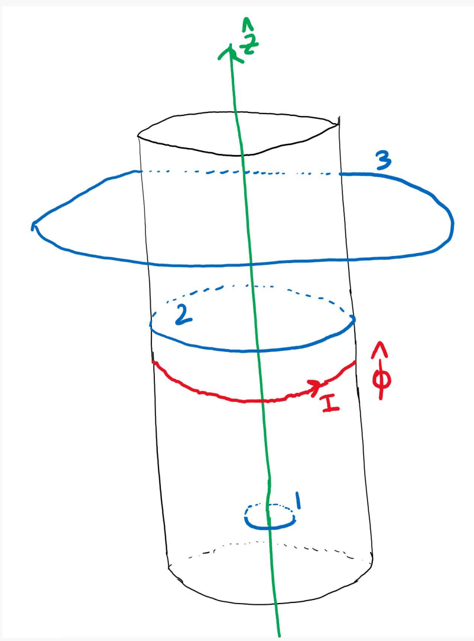
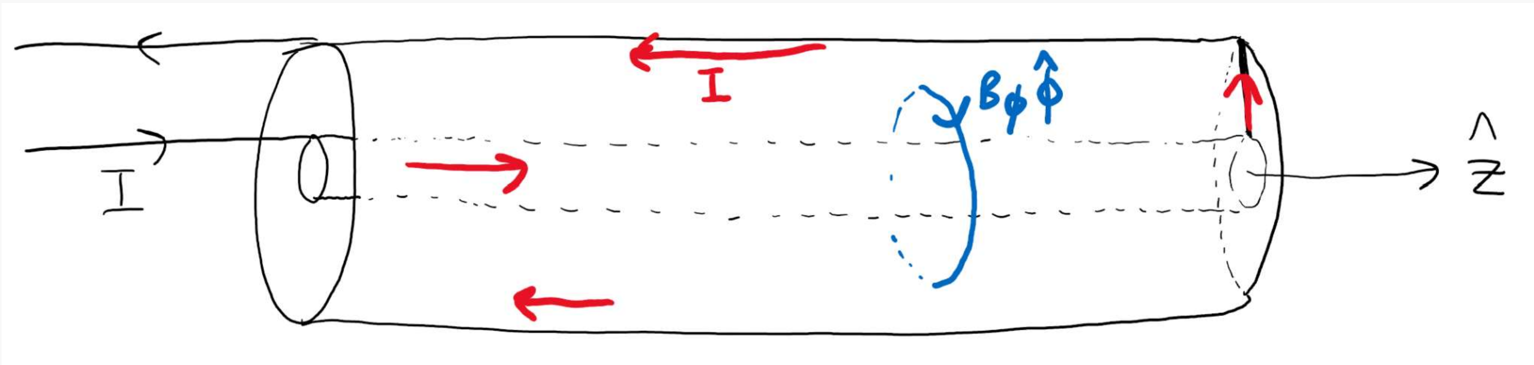


$I \to \vec{B} \to \Phi \to L$

Problem 6: Shorted coaxial cable

Given a shorted coaxial cable with inner radius a, outer radius b and length ℓ, carrying current I on the inner conductor, find the inductance L of the shorted coax cable.



Problem 7

$$\Psi = LI$$

Express the unit [H] using the SI quantities [kg], [m], [s], and [C].

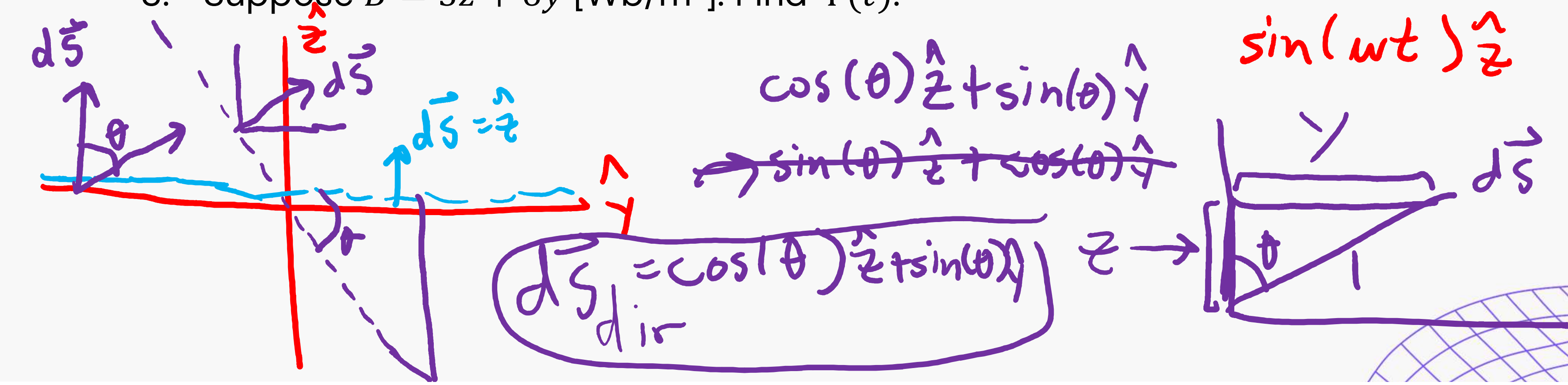
Problem 8: Rotations!

A square loop of sidelength 2m sits on the xy-plane at $t = 0$ and begins to rotate about the x-axis clockwise with angular frequency ω when viewed at $x = 10$ looking at the origin.

1. Express the angle between the plane of the current loop at the xy-plane as a function of t.

   $\theta = \omega t$

2. Suppose $d\vec{S}$ points in the $+\hat{z}$ direction at $t = 0$. Find the direction of $d\vec{S}$ as a function of t.
3. Suppose $\vec{B} = 3\hat{z} + 6\hat{y}$ [Wb/m²]. Find $\Psi(t)$.



Problem: Rotations!

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1. Express the angle between the plane of the current loop at the xy-plane as a function of t.
   $\theta = \omega t$
2. Suppose $d\vec{S}$ points in the $+\hat{z}$ direction at $t = 0$. Find the direction of $d\vec{S}$ as a function of t.
   $\cos(\omega t)\,\hat{z} + \sin(\omega t)\,\hat{y}$
3. Suppose $\vec{B} = 3\hat{z} + 6\hat{y}$ [Wb/m²]. Find $\Psi(t)$.

$$\Psi = \iint \vec{B} \cdot d\vec{S}$$

$$\Psi(t) = (3\hat{z} + 6\hat{y}) \cdot (\cos(\omega t)\,\hat{z} + \sin(\omega t)\,\hat{y}) \quad \Psi = \iint \vec{B} \cdot d\vec{S}$$

Midterm 1 equations, in one place

$$\vec{F} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\vec{F} = q_1\vec{E} + q_1(\vec{v}_1 \times \vec{B})$$

$$\vec{E} = \frac{q_2}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\epsilon \oiint \vec{E} \cdot d\vec{S} = Q_{\text{enclosed}}$$

$$\oiint \vec{D} \cdot d\vec{S} = Q_{\text{enclosed}}$$

$$\iiint \rho dV = Q_{\text{enclosed}}$$

$$\oiint \vec{B} \cdot d\vec{S} = 0$$

$$I = \oiint \vec{J} \cdot d\vec{S} = -\frac{\partial Q_{\text{enclosed}}}{\partial t}$$

$$\nabla \times \vec{E} = 0$$

$$\vec{E} = -\nabla V$$

$$\oint \vec{E} \cdot d\vec{l} = 0$$

$$V_{ab} = V(b) - V(a) = -\int_a^b \vec{E} \cdot d\vec{l}$$

$$\hat{n} \cdot (\vec{D}_1 - \vec{D}_2) = \rho_s$$

$$\hat{n} \times (\vec{E}_1 - \vec{E}_2) = 0$$

$$\hat{n} \cdot (\vec{P}_1 - \vec{P}_2) = -\rho_{b,s}$$

$$\epsilon = \epsilon_0(1 + \chi_e)$$

$$\vec{P} = \epsilon_0 \chi_e \vec{E}$$

$$\vec{D} = \epsilon_0\vec{E} + \vec{P} = \epsilon\vec{E}$$

$$\vec{J} = \sigma\vec{E}$$

$$\rho_b = -\nabla \cdot \vec{P}$$

$$\nabla \cdot \epsilon_0\vec{E} = \rho_f + \rho_b$$

$$\nabla \cdot \vec{D} = \rho$$

$$\nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$$

$$-\nabla^2 V = \frac{\rho}{\epsilon}$$

$$\oiint \vec{D} \cdot d\vec{S} = \iiint \nabla \cdot \vec{D} dV$$

$$\oint \vec{E} \cdot d\vec{l} = \iint (\nabla \times \vec{E}) \cdot d\vec{S}$$

$$\int_a^b \nabla V \cdot d\vec{l} = V(b) - V(a)$$

Midterm 2 equations, in one place

$$\vec{B} = \frac{\mu I}{2\pi r}\hat{\phi}$$

$$d\vec{B} = \frac{\mu I d\vec{\ell} \times \hat{r}}{4\pi r^2}$$

$$\oint_C \vec{H} \cdot d\vec{\ell} = \iint_S \vec{J} \cdot d\vec{S}$$

$$\oint_C \vec{B} \cdot d\vec{\ell} = \mu I_{\text{encl}}$$

$$\nabla \times \vec{H} = \vec{J}$$

$$\nabla \cdot \vec{B} = 0$$

$$\Psi = \iint_S \vec{B} \cdot d\vec{S}$$

$$-\frac{d}{dt}\iint_S \vec{B} \cdot d\vec{S} = \oint_c \vec{E} \cdot d\vec{l}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\oint_c \vec{E} \cdot d\vec{l} = \varepsilon$$

$$\varepsilon = \frac{W}{q} = \oint_C \frac{\vec{F}}{q} \cdot d\vec{l}$$

$$\Psi = LI$$

$$\varepsilon = IR$$

$$Q = CV$$

$$G = \frac{\sigma}{\epsilon}C \qquad R = \frac{1}{G}$$

$$I = \frac{\varepsilon_{\text{induced}}}{R}$$

$$\varepsilon_{\text{total}} = -\frac{d}{dt}\Psi_{\text{total}}$$

$$\varepsilon_{\text{total}} = \varepsilon_{\text{induced}} + \underbrace{\varepsilon_{\text{self}}}$$

Units

Charge Q: C
Current I: A
Electric field strength $\vec{E}$: N/C or V/m
Electric flux density $\vec{D}$: C/m^2
Polarization field $\vec{P}$: C/m^2
Electric potential V: V
Capacitance C: F
Magnetic flux density $\vec{B}$: T or Wb/m^2
Magnetic field strength $\vec{H}$: A/m
Magnetic flux Ψ: Wb
Electromotive force ε: V
Inductance L: H

Electric permittivity ϵ: F/m
Magnetic permeability μ: H/m
Conductivity σ: Si/m

Charge density ρ: C/m^3
Surface charge density ρ_s: C/m^2
Current density $\vec{J}$: A/m^2