Share any thoughts on anything, including the exam!

ECE329: Tutorial Session 4

October 2nd, 2025

Capacitance

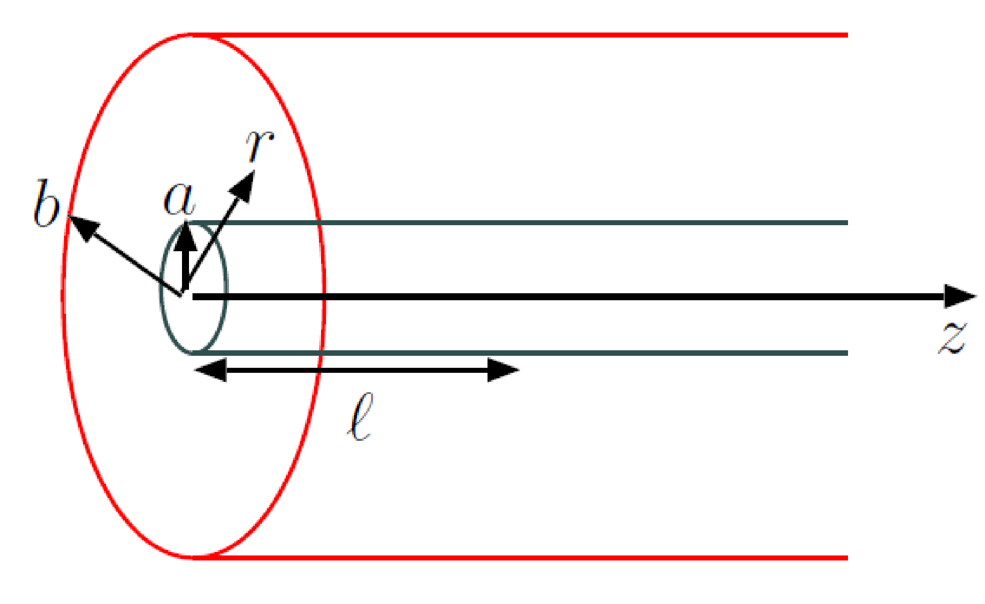
Capacitance: the ability of something to collect and store energy in the form of electrical charge.

This energy is stored as opposite electric charges being held apart (and thus creating a difference in electric potential, aka a voltage drop).

$$Q = CV \qquad G = \frac{\sigma}{\epsilon} C \qquad R = \frac{1}{G}$$

Problem 1

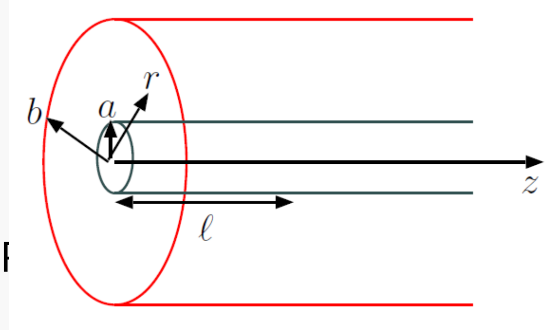
The central cylindrical volume with cross-sectional radius a is a conductor.
The pipe (drawn in red) is also a conductor and is grounded.
A dielectric with permittivity $\epsilon = 4\epsilon_0$ fills the space in between.



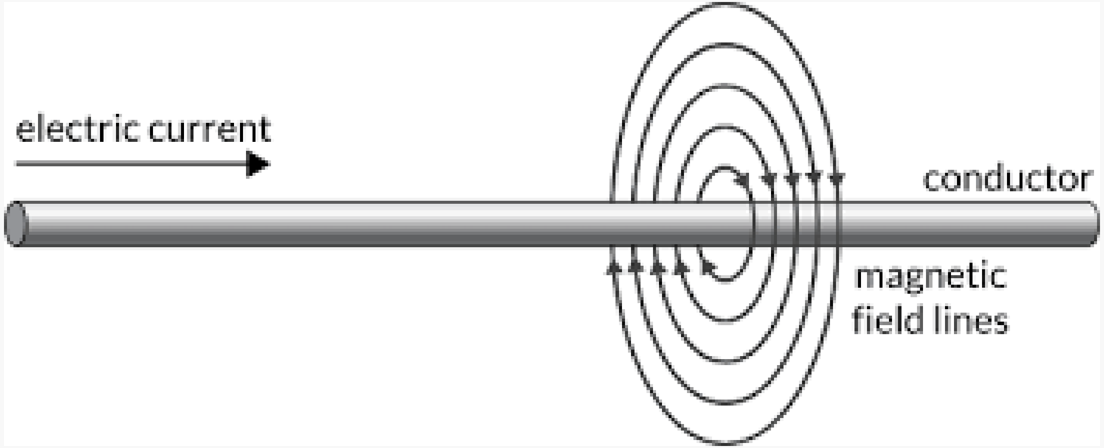
What is the capacitance, conductance, and resistance per unit length in the middle of the coaxial cable?

Problem 1 Extended

Where is Laplace's equation satisfied?
What is the free charge density within the dielectric?
What is the free surface charge density at $r = a$ and $r = b$? F
$4\epsilon_0$ in the dielectric.



Magnetostatics



$$\vec{B} = \frac{\mu I}{2\pi r}\hat{\phi}$$

Biot-Savart Law

$$d\vec{B} = \frac{\mu I d\vec{\ell} \times \hat{r}}{4\pi r^2}$$

Ampere's Law

$$\oint_C \vec{B} \cdot d\vec{\ell} = \mu I_{\mathrm{encl}}$$

$$\nabla \times \vec{H} = \vec{J}$$

Deltas, Deltas, Deltas

What do the following represent physically?

$\rho = \delta(x-3)\delta(y)\delta(z)$

$\rho = \delta(x-4)\delta(z-2)$

$\rho = \delta(x-4)\delta(x-2)$

$\rho = \delta(y+5)$

$\rho = 1$

Deltas, Deltas, Deltas

What do the following represent physically?

$$\vec{J} = \delta(x-3)\delta(y)\delta(z)\hat{x}$$

$$\vec{J} = \delta(x-4)\delta(z-2)\hat{y}$$

$$\vec{J} = \delta(x-4)\delta(x-2)\hat{y}$$

$$\vec{J} = \delta(y+5)\hat{z}$$

$$\vec{J} = 1\hat{z}$$

Gauss's Law

$$\nabla \cdot \vec{D} = \rho_e$$

$$\nabla \cdot \vec{B} = \rho_m = 0$$

Problem 2

Let $\vec{I} = 5\delta(x)(\hat{y} + \hat{z})$. Find the magnetic field due to the given current density everywhere in free space.

Problem 3

Let our point of view be from above the xy-plane looking down. Suppose we have some current distribution $I_1 = I(x, y, z)$ that results in a magnetic field $\vec{B} = g(x, y, z)\hat{y} + h(x, y, z)\hat{z}$.

Suppose now that we have another current distribution I_2, which is just the current distribution I_1 rotated 90 degrees clockwise and shifted such that the center is now at $(3, -4,5)$. What is the magnetic field at the origin?

Summary of the Statics

Electrostatics	Magnetostatics
$\oiint_S \vec{D} \cdot d\vec{S} = \iiint_V \rho dV$ $\nabla \cdot \vec{D} = \rho$	$\oiint_S \vec{B} \cdot d\vec{S} = 0$ $\nabla \cdot \vec{B} = 0$
$\oint_C \vec{E} \cdot d\vec{\ell} = 0$ $\nabla \times \vec{E} = 0$	$\oint_C \vec{H} \cdot d\vec{\ell} = \iint_S \vec{J} \cdot d\vec{S}$ $\nabla \times \vec{H} = \vec{\mathrm{J}}$

Midterm 1 equations, in one place

$$\vec{F} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\vec{F} = q_1\vec{E} + q_1(\vec{v}_1 \times \vec{B})$$

$$\vec{E} = \frac{q_2}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\epsilon \oiint \vec{E}\cdot d\vec{S} = Q_{\text{enclosed}}$$

$$\oiint \vec{D}\cdot d\vec{S} = Q_{\text{enclosed}}$$

$$\iiint \rho dV = Q_{\text{enclosed}}$$

$$\oiint \vec{B}\cdot d\vec{S} = 0$$

$$I = \oiint \vec{J}\cdot d\vec{S} = -\frac{\partial Q_{\text{enclosed}}}{\partial t}$$

$$\nabla \times \vec{E} = 0$$

$$\vec{E} = -\nabla V$$

$$\oint \vec{E}\cdot d\vec{l} = 0$$

$$V_{ab} = V(b) - V(a) = -\int_a^b \vec{E}\cdot d\vec{l}$$

$$\hat{n}\cdot(\vec{D}_1 - \vec{D}_2) = \rho_s$$

$$\hat{n}\times(\vec{E}_1 - \vec{E}_2) = 0$$

$$\hat{n}\cdot(\vec{P}_1 - \vec{P}_2) = -\rho_{b,s}$$

$$\epsilon = \epsilon_0(1 + \chi_e)$$

$$\vec{P} = \epsilon_0\chi_e\vec{E}$$

$$\vec{D} = \epsilon_0\vec{E} + \vec{P} = \epsilon\vec{E}$$

$$\vec{J} = \sigma\vec{E}$$

$$\rho_b = -\nabla\cdot\vec{P}$$

$$\nabla\cdot\epsilon_0\vec{E} = \rho_f + \rho_b$$

$$\nabla\cdot\vec{D} = \rho$$

$$\nabla\cdot\vec{J} = -\frac{\partial\rho}{\partial t}$$

$$-\nabla^2 V = \frac{\rho}{\epsilon}$$

$$\oiint \vec{D}\cdot d\vec{S} = \iiint \nabla\cdot\vec{D}dV$$

$$\oint \vec{E}\cdot d\vec{l} = \iint (\nabla\times\vec{E})\cdot d\vec{S}$$

$$\int_a^b \nabla V\cdot d\vec{l} = V(b) - V(a)$$

Midterm 2 equations, in one place

$$\vec{B} = \frac{\mu I}{2\pi r}\hat{\phi}$$

$$d\vec{B} = \frac{\mu I d\vec{\ell} \times \hat{r}}{4\pi r^2}$$

$$\oint_C \vec{H} \cdot d\vec{\ell} = \iint_S \vec{J} \cdot d\vec{S}$$

$$\oint_C \vec{B} \cdot d\vec{\ell} = \mu I_{\text{encl}}$$

$$\nabla \times \vec{H} = \vec{J}$$

$$\nabla \cdot \vec{B} = 0$$

$$Q = CV$$

$$G = \frac{\sigma}{\epsilon} C \qquad R = \frac{1}{G}$$

Units

Charge Q: C
Electric field strength $\vec{E}$: N/C or V/m
Electric flux density $\vec{D}$: C/m^2
Polarization field $\vec{P}$: C/m^2
Electric potential V: V
Capacitance C: F
Magnetic flux density $\vec{B}$: T or Wb/m^2
Magnetic field strength $\vec{H}$: A/m

Charge density ρ: C/m^3
Surface charge density ρ_s: C/m^2
Current density $\vec{J}$: A/m^2

Electric permittivity ϵ: F/m
Magnetic permeability μ: H/m
Conductivity σ: Si/m