Share any thoughts on
anything, including the exam!

ECE329: Tutorial Session 4

October 2nd, 2025

Bound charges

$$\nabla \cdot \vec{D} = \rho_{f,v} \longrightarrow \hat{n} \cdot (\vec{D}_1 - \vec{D}_2) = \rho_{f,s}$$

$$\nabla \cdot \vec{P} = -\rho_{b,v} \longrightarrow \hat{n} \cdot (\vec{P}_1 - \vec{P}_2) = -\rho_{b,s}$$

Poisson's Equation

$$\nabla^2 V = -\frac{\rho}{\varepsilon}$$

Laplace's Equation

$$\nabla^2 V = 0$$

$\rho_f = 0$

$$V(x) = Ax + B$$

$$Az + B$$

$V = a_1 x + a_2 y + a_3 z + c$

Capacitance

Capacitance: the ability of something to collect and store energy in the form of electrical charge.

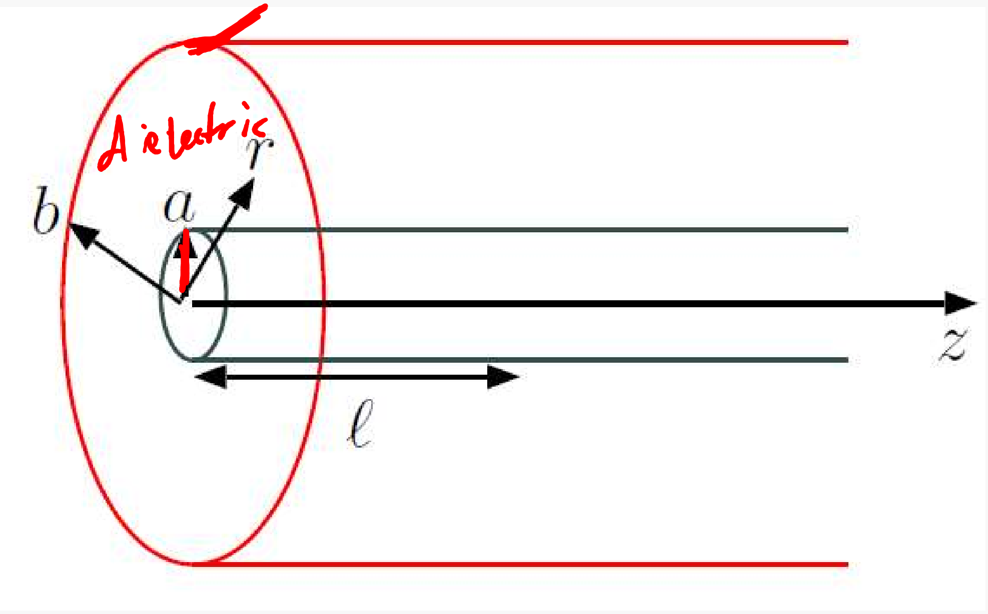
This energy is stored as opposite electric charges being held apart (and thus creating a difference in electric potential, aka a voltage drop).

[F] [Si] = [Siemens] [Ω]

$$Q = \underline{C}V \qquad G = \frac{\sigma}{\epsilon}C \qquad R = \frac{1}{G}$$

Problem 1

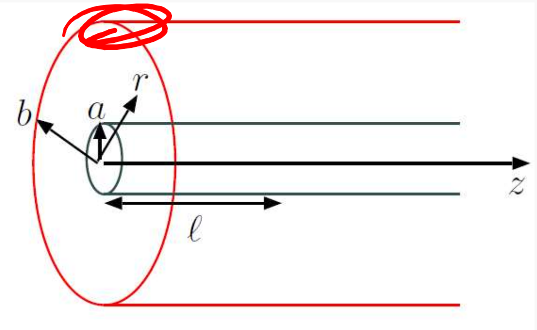
The central cylindrical volume with cross-sectional radius a is a conductor.
The pipe (drawn in red) is also a conductor and is grounded.
A dielectric with permittivity $\epsilon = 4\epsilon_0$ fills the space in between.



What is the capacitance, conductance, and resistance <u>per unit length</u> in the middle of the coaxial cable?

Problem 1: Blank slide

What is the capacitance, conductance, and resistance per unit length in the middle of the coaxial cable?



C G R

$Q = CV \leftarrow V(0,0,0) = 0$

$V(x,y,z) = Ax + B$

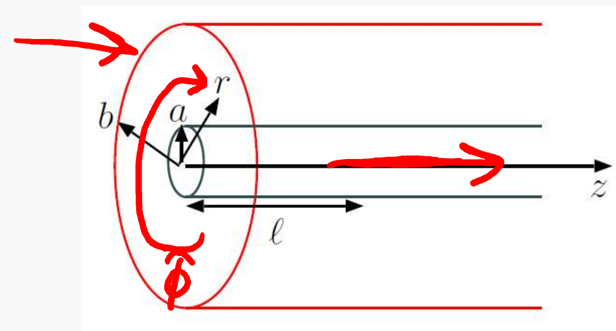
$\vec{E}$ $\vec{D}$

$V(l) = 0$

$V(0) = 0$

Problem 1: Blank slide

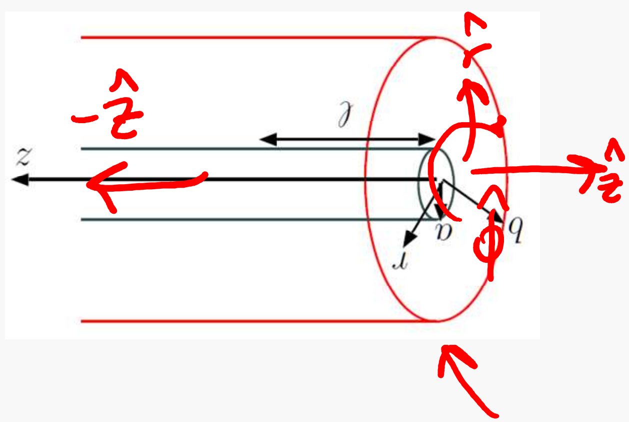
What is the capacitance, conductance, and resistance per unit length in the middle of the coaxial cable?



$\vec{E} =$

direction ✓ $\hat{r}$

magnitude



$\hat{r}$ yes

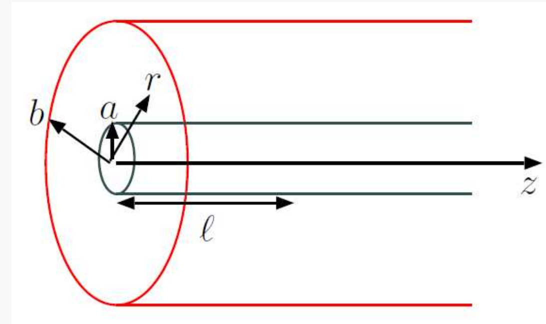
$\hat{z}$ ✗

$\hat{\phi}$ ✗

by symmetry

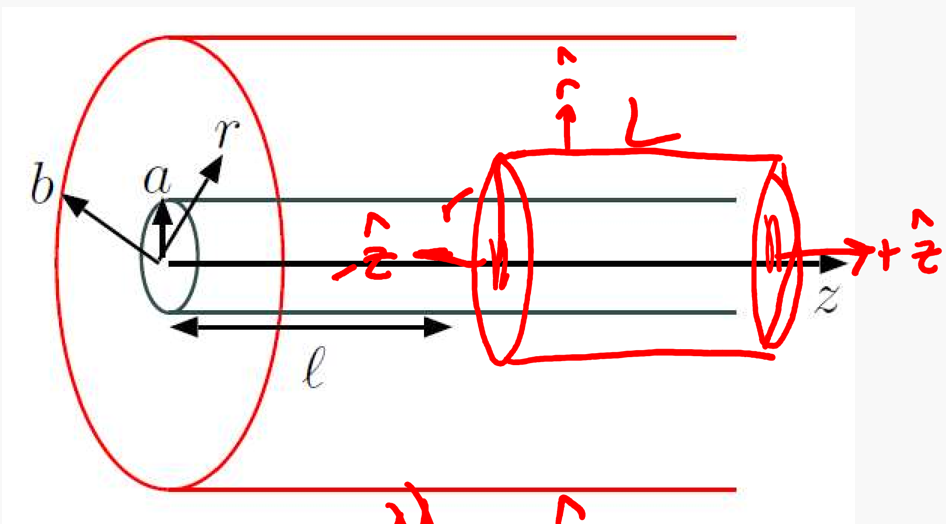
Problem 1: Blank slide

What is the capacitance, conductance, and resistance per unit length in the middle of the coaxial cable?



$$\oiint \vec{D}\cdot d\vec{S} = Q_{encl}$$

$$\vec{D} = \varepsilon \vec{E}$$



$$\iint \varepsilon E_r \hat{r}\cdot\hat{r}\, dA = Q_{encl}$$

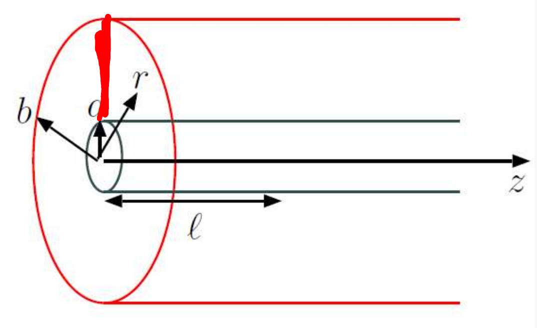
$$\int_{z=0}^{L}\int_{\phi=0}^{2\pi} \varepsilon E_r r\, d\phi\, dz = \lambda L$$

$$E_r 2\pi L \varepsilon r = \lambda L$$

$$\vec{E} = \frac{\lambda \cancel{L}}{2\pi \cancel{L} \varepsilon r}\hat{r}$$

Problem 1: Blank slide

What is the capacitance, conductance, and resistance per unit length in the middle of the coaxial cable?



$\vec{E} \rightarrow V$

$$V = -\int \vec{E} \cdot d\vec{l}$$

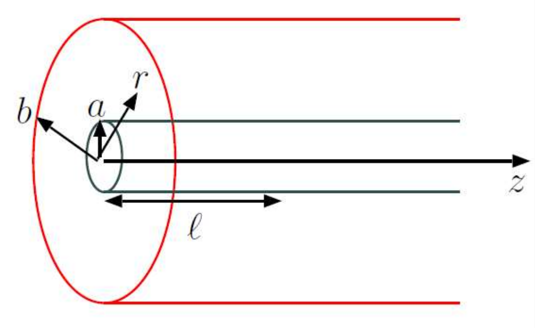
$C > 0$

$d\vec{l} = \hat{n}\, dl$

$$V = -\int_{r=b}^{r=a} \frac{\lambda}{2\pi\varepsilon r} \hat{r} \cdot \hat{r}\, dr = -\frac{\lambda}{2\pi\varepsilon} \ln(r) \Big|_{r=b}^{r=a} = -\frac{\lambda}{2\pi\varepsilon} \ln\left(\frac{a}{b}\right)$$

Problem 1: Blank slide

What is the capacitance, conductance, and resistance per unit length in the middle of the coaxial cable?



$$V = -\frac{\lambda}{2\pi\varepsilon}\ln\left(\frac{a}{b}\right) = \frac{\lambda}{2\pi\varepsilon}\ln\left(\frac{b}{a}\right)$$

$$Q = CV$$

$$C = \frac{Q}{V} = \frac{\lambda}{}$$

$$C = \frac{2\pi\varepsilon}{\ln\left(\frac{b}{a}\right)}$$

$$G = \frac{\sigma}{\varepsilon}C$$

$$G = \frac{2\pi\sigma}{\ln\left(\frac{b}{a}\right)}$$

$$R = \frac{\ln\left(\frac{b}{a}\right)}{2\pi\sigma}$$

Where is Laplace's equation satisfied?
What is the free charge density within the dielectric?
What is the free surface charge density at $r = a$ and $r = b$?
What is the bound charge density within the dielectric?
What is the bound surface charge density at $r = a$ and $r = b$?
Recall $\epsilon = 4\epsilon_0$ in the dielectric.

Problem 1 Extended

Where is Laplace's equation satisfied?
What is the free charge density within the dielectric? 0
What is the free surface charge density at $r = a$ and $r = b$? BC
What is the bound charge density within the dielectric? 0
What is the bound surface charge density at $r = a$ and $r = b$?
Recall $\epsilon = 4\epsilon_0$ in the dielectric.

$$\vec{D} = \begin{cases} 0 & 0<r<a \\ \dfrac{Q}{2\pi r} & a<r<b \\ 0 & b<r<\infty \end{cases} \qquad \vec{E} \ldots$$

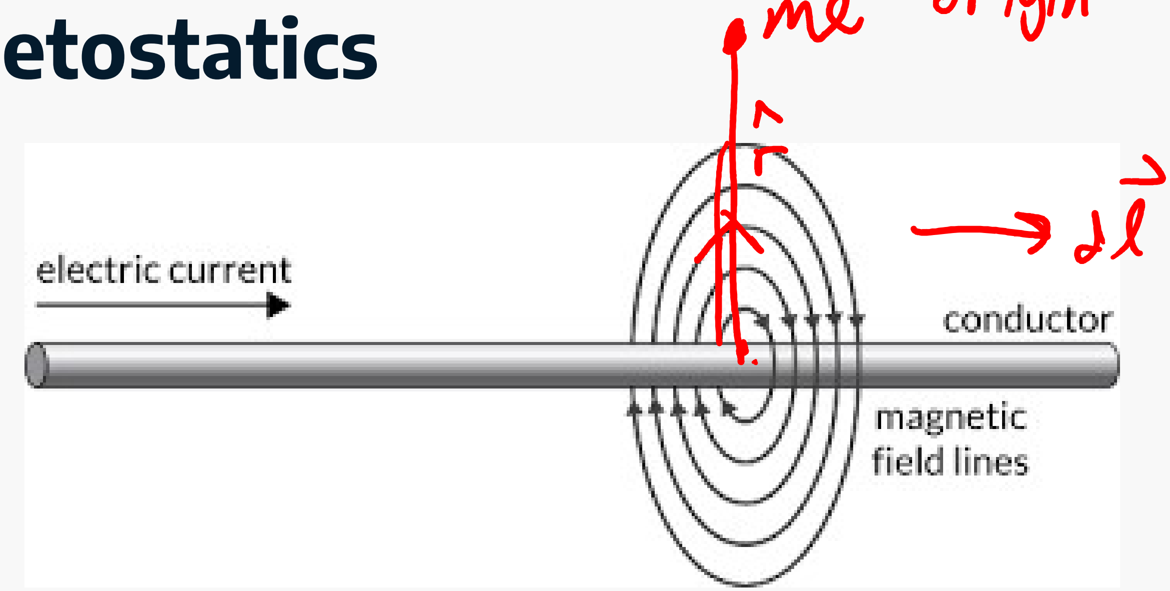
$$\nabla \cdot \vec{D} = \rho_f$$

$$\hat{n} \cdot (\vec{D}^{+} - \vec{D}^{-}) = \rho_{s,f}$$

$$\nabla \cdot \vec{P} = -\rho_b$$

$$\hat{n} \cdot (\vec{P}^{+} - \vec{P}^{-}) = -\rho_{s,b}$$

Magnetostatics



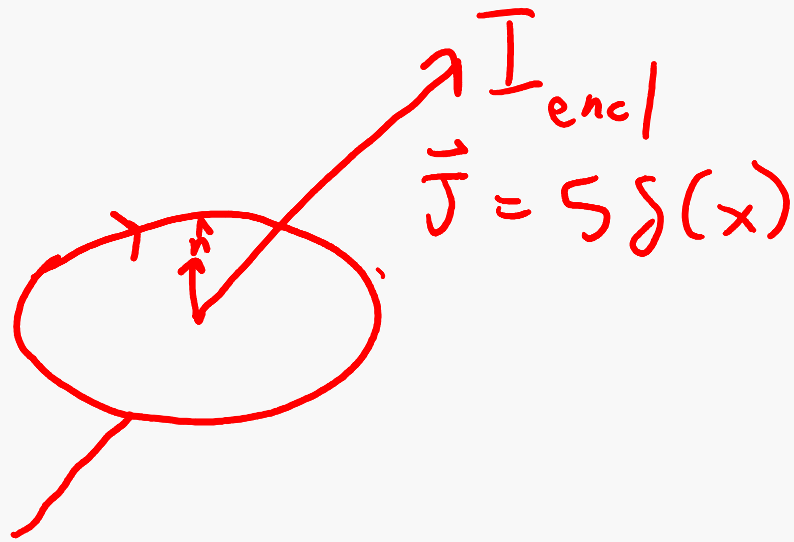
$$\vec{B} = \frac{\mu I}{2\pi r}\hat{\phi}$$

Biot-Savart Law

$$d\vec{B} = \frac{\mu I d\vec{\ell} \times \hat{r}}{4\pi r^2}$$

Ampere's Law

$$\oint_C \vec{B} \cdot d\vec{\ell} = \mu I_{\text{encl}} = \iint \mu \vec{J} \cdot d\vec{S}$$



Ampere's Law

$$\oint_C \vec{B} \cdot d\vec{\ell} = \mu I_{\text{encl}} = \iint \mu \vec{J} \cdot d\vec{S}$$

$$\cancel{\iint_S} (\nabla \times \vec{B}) \cdot \cancel{d\vec{S}} = \cancel{\iint_S} \mu \vec{J} \cdot \cancel{d\vec{S}}$$

$$\nabla \times \vec{B} = \mu \vec{J}$$

$$\underbrace{\mu \vec{H}}_{\text{strength}} = \underbrace{\vec{B}}_{\text{flux density}}$$

$$\nabla \times \vec{H} = \vec{J}$$

Deltas, Deltas, Deltas

$\int_{-\infty}^{\infty} \delta(x) = 1 \quad \infty$

What do the following represent physically?

1. $\rho = \delta(x-3)\delta(y)\delta(z)$

point @ (3,0,0)

$\rho = \delta(x-4)\delta(z-2)$

line @ $x=4$, $z=2$

$\rho = \delta(x-4)\delta(x-2)$

2 planes

no

$\rho = 0$

$\rho = \delta(y+5)$

$\rho = 1$

Gauss's Law

$$\nabla \cdot \vec{D} = \rho_e$$

$$\nabla \cdot \vec{B} = \rho_m = 0$$

Problem 2

Let $\vec{J} = 5\delta(x)(\hat{y} + \hat{z})$. Find the electric and magnetic field due to the given current density everywhere in free space.

$\vec{E} = 0$

$\vec{J} =$

$\vec{B}$

$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon} = 0$

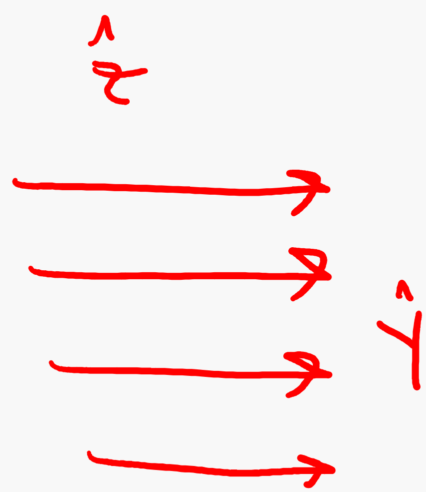
Problem 2: Blank Slide

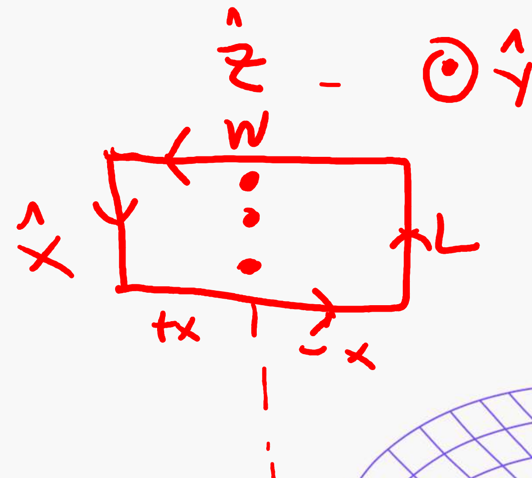
Ampere Law

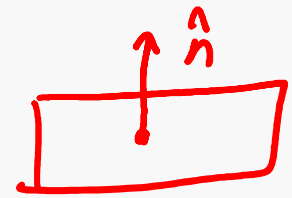
Let $\vec{J} = 5\delta(x)(\hat{y} + \hat{z})$. Find the electric and magnetic field due to the given current density everywhere in free space.

$$\vec{J} = \underbrace{5\delta(x)\hat{y}} + 5\delta(x)\hat{z}$$

$$\underbrace{\oint_C \vec{B}\cdot d\vec{l}} = \underbrace{\iint_S \mu\vec{J}\cdot d\vec{S}}$$

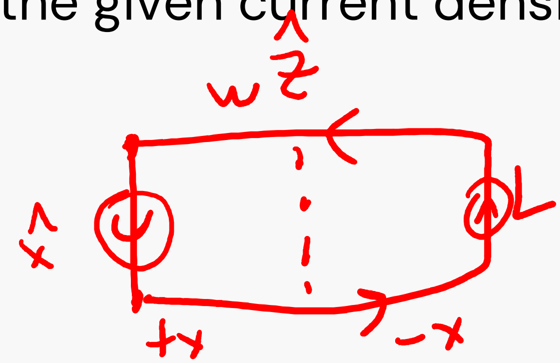






Problem 2: Blank Slide

Let $\vec{J} = 5\delta(x)(\hat{y} + \hat{z})$. Find the electric and magnetic field due to the given current density everywhere in free space.



$-B(x) + B(-x) = 5\mu$

$-B(x) = B(-x)$

$\oint \vec{B}\cdot d\vec{l} =$

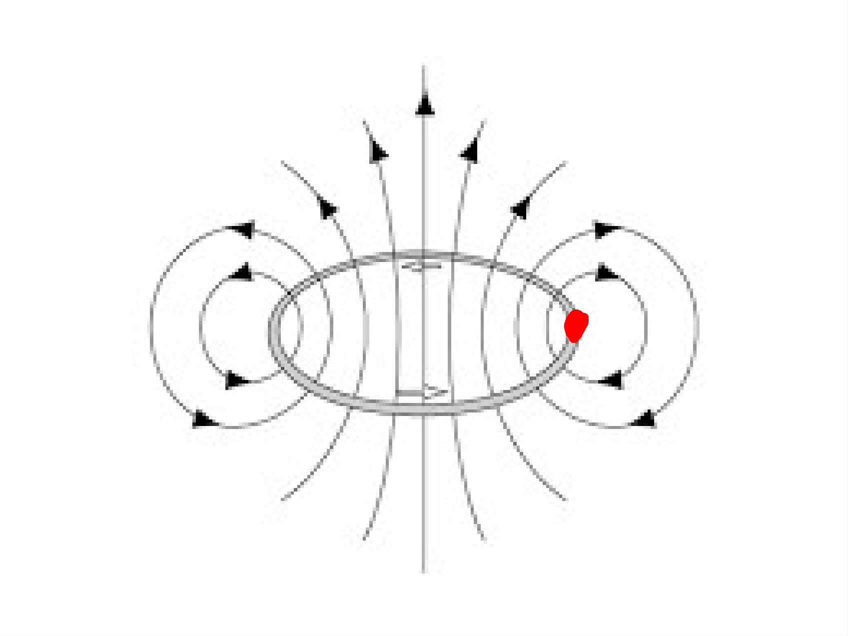
$\int_{L/2}^{-L/2} \vec{B}\cdot\hat{z} + \int_{-L/2}^{L/2} \vec{B}\cdot\hat{z} = \int^{w/2} \mu \int^{L/2} 5\delta(x)\hat{y}$ ~~$\hat{x}$~~ $\,dz$ ~~dx~~

+x　　-x　　$x = -\frac{w}{2}$　　$z = -\frac{L}{2}$

$-\cancel{L}B(+x) + LB(-x) = 5\cancel{L}\mu$

$\vec{B} = \begin{cases} -\frac{5\mu}{2}\hat{z} & x > 0 \\ 5\mu/2\,\hat{z} & x < 0 \end{cases}$

Current Loops



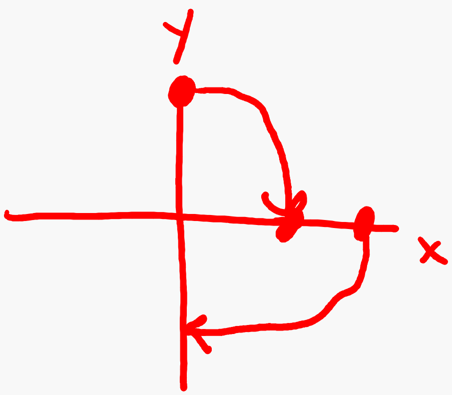
$\nabla \times \vec{E} = 0$

$\nabla \times \vec{H} = \vec{J}$

Problem 3

Let our point of view be from above the xy-plane looking down. Suppose we have some current distribution $I_1 = I(x, y, z)$ that results in a magnetic field $\vec{B} = g(x, y, z)\hat{y} + h(x, y, z)\hat{z}$.

Suppose now that we have another current distribution I_2, which is just the current distribution I_1 rotated 90 degrees clockwise and shifted such that the center is now at $(3, -4, 5)$. What is the magnetic field at the origin?



$y \to x$

$x \to -y$

$z \to z$

$x \to x-3$

$y \to y+4$

$z \to z-5$

$I_1' = I(-y, x, z)$

$\vec{B}_1' = g(-y, x, z)\hat{x} + h(-y, x, z)\hat{z}$

Summary of the Statics

Electrostatics	Magnetostatics
$\oiint_S \vec{D} \cdot d\vec{S} = \iiint_V \rho dV$ $\nabla \cdot \vec{D} = \rho$	$\oiint_S \vec{B} \cdot d\vec{S} = 0$ $\nabla \cdot \vec{B} = 0$
$\oint_C \vec{E} \cdot d\vec{\ell} = 0$ $\nabla \times \vec{E} = 0$	$\oint_C \vec{H} \cdot d\vec{\ell} = \iint_S \vec{J} \cdot d\vec{S}$ $\nabla \times \vec{H} = \vec{J}$

Midterm 1 equations, in one place

$$\vec{F} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}\hat{r}$$
$$\vec{F} = q_1\vec{E} + q_1(\vec{v}_1 \times \vec{B})$$
$$\vec{E} = \frac{q_2}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\epsilon \oiint \vec{E}\cdot d\vec{S} = Q_{\text{enclosed}}$$
$$\oiint \vec{D}\cdot d\vec{S} = Q_{\text{enclosed}}$$
$$\iiint \rho dV = Q_{\text{enclosed}}$$
$$\oiint \vec{B}\cdot d\vec{S} = 0$$
$$I = \oiint \vec{J}\cdot d\vec{S} = -\frac{\partial Q_{\text{enclosed}}}{\partial t}$$

$$\nabla \times \vec{E} = 0$$
$$\vec{E} = -\nabla V$$
$$\oint \vec{E}\cdot d\vec{l} = 0$$
$$V_{ab} = V(b) - V(a) = -\int_a^b \vec{E}\cdot d\vec{l}$$

$$\hat{n}\cdot\left(\vec{D}_1 - \vec{D}_2\right) = \rho_s$$
$$\hat{n}\times\left(\vec{E}_1 - \vec{E}_2\right) = 0$$
$$\hat{n}\cdot\left(\vec{P}_1 - \vec{P}_2\right) = -\rho_{b,s}$$

$$\epsilon = \epsilon_0(1 + \chi_e)$$
$$\vec{P} = \epsilon_0\chi_e\vec{E}$$
$$\vec{D} = \epsilon_0\vec{E} + \vec{P} = \epsilon\vec{E}$$
$$\vec{J} = \sigma\vec{E}$$
$$\rho_b = -\nabla\cdot\vec{P}$$
$$\nabla\cdot\epsilon_0\vec{E} = \rho_f + \rho_b$$

$$\nabla\cdot\vec{D} = \rho$$
$$\nabla\cdot\vec{J} = -\frac{\partial\rho}{\partial t}$$
$$-\nabla^2 V = \frac{\rho}{\epsilon}$$

$$\oiint \vec{D}\cdot d\vec{S} = \iiint \nabla\cdot\vec{D}dV$$
$$\oint \vec{E}\cdot d\vec{l} = \iint \left(\nabla\times\vec{E}\right)\cdot d\vec{S}$$
$$\int_a^b \nabla V\cdot d\vec{l} = V(b) - V(a)$$

Midterm 2 equations, in one place

$$\vec{B} = \frac{\mu I}{2\pi r}\hat{\phi}$$

$$d\vec{B} = \frac{\mu I d\vec{\ell} \times \hat{r}}{4\pi r^2}$$

$$\oint_C \vec{H} \cdot d\vec{\ell} = \iint_S \vec{J} \cdot d\vec{S}$$

$$\oint_C \vec{B} \cdot d\vec{\ell} = \mu I_{\text{encl}}$$

$$\nabla \times \vec{H} = \vec{J}$$

$$\nabla \cdot \vec{B} = 0$$

$$Q = CV$$

$$G = \frac{\sigma}{\epsilon}C \qquad R = \frac{1}{G}$$

Units

Charge Q: C
Electric field strength $\vec{E}$: N/C or V/m
Electric flux density $\vec{D}$: C/m^2
Polarization field $\vec{P}$: C/m^2
Electric potential V: V
Capacitance C: F
Magnetic flux density $\vec{B}$: T or Wb/m^2
Magnetic field strength $\vec{H}$: A/m

Charge density ρ: C/m^3
Surface charge density ρ_s: C/m^2
Current density $\vec{J}$: A/m^2

Electric permittivity ϵ: F/m
Magnetic permeability μ: H/m
Conductivity σ: Si/m