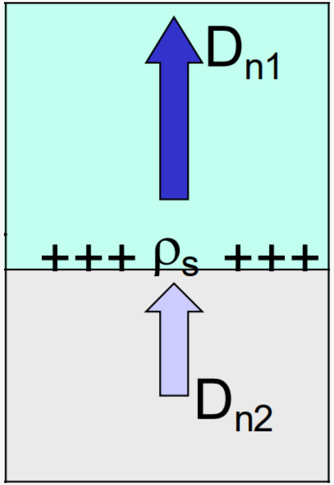
Share any thoughts on anything

ECE329: Tutorial Session 3

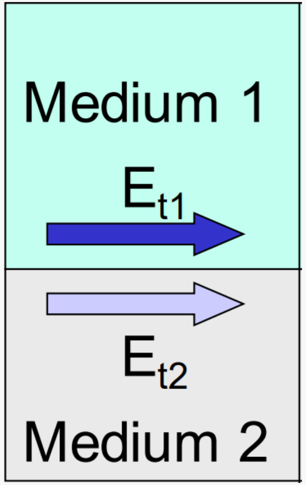
September 18th, 2025

Boundary Conditions

$$\hat{n} \cdot (\vec{D}_1 - \vec{D}_2) = \rho_s$$



$$\hat{n} \times (\vec{E}_1 - \vec{E}_2) = 0$$



Problem 1

Suppose the yz–plane in free space holds a surface charge density of $\rho_s = 5$ C/m². The electric displacement field on the $-x$ side is given as $\vec{D} = \hat{x} + \hat{y} + \hat{z}$. Find the electric displacement field on the $+x$ side using boundary conditions.

Conductors: The intuition

Conductors: The math

Described by σ, aka conductivity (units: Siemens/meter)

What to know:

- $\vec{J} = \sigma\vec{E}$ (Ohm's Law)

Assumption: We are dealing with electrostatics.

- If $\sigma \neq 0$, material is an equipotential with zero internal fields and finite surface charge densities.

P fields: The intuition

P fields: The math

$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$: The defining equation.

Assumption: Dielectric is 'isotropic', so $\vec{P}$ is collinear to $\vec{E}$. Then:

- $\vec{P} = \epsilon_0 \chi_e \vec{E}$ with electric susceptibility $\chi_e \geq 0$ nearly always in this class.
- Let electric permittivity be $\epsilon = \epsilon_0(1 + \chi_e)$.
- Let relative electric permittivity be $\epsilon_r = 1 + \chi_e$
- $\vec{D} = \epsilon \vec{E}$

P fields: The math

Assumption: Dielectric is ‘isotropic’, so $\vec{P}$ is collinear to $\vec{E}$. Then:

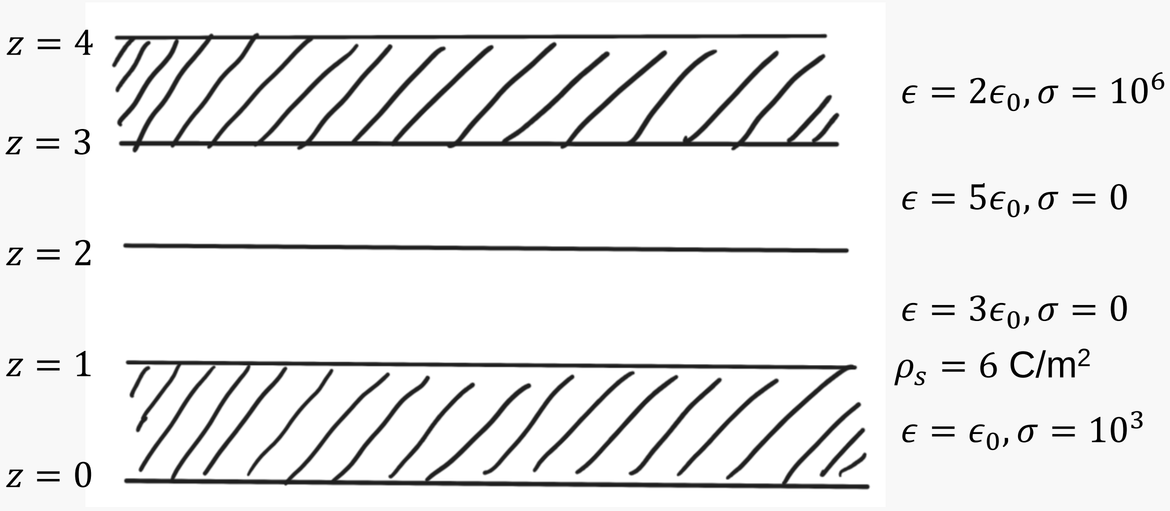
Divergences:

- Gauss’s Law: $\nabla \cdot \epsilon_0 \vec{E} = \rho_f + \rho_b$
- Gauss’s Law: $\nabla \cdot \vec{D} = \rho_f = \rho$

- Therefore, $\rho_b = -\nabla \cdot \vec{P}$

Problem 2

Find $\vec{D}$, $\vec{E}$, and $\vec{P}$, in all volumes. Find ρ_s at each material boundary. What is the voltage drop from $z = 0$ to $z = 4$?



Problem 3

Let $\rho = 6\epsilon_0\delta(z) + \rho_s\delta(z-4)$ C/m³. The displacement field in the $0 < z < 4$ region is given as $\vec{D} = \epsilon_0\hat{x} + 3\epsilon_0\hat{z}$ and the electric permittivity is known to be $\epsilon_2 = 4\epsilon_0$. It is known that $D_z = 2\epsilon_0$ and $\epsilon_3 = 2\epsilon_0$ for the $z > 4$ region, while $\epsilon_1 = \epsilon_3$ for the $z < 0$ region.

Find ρ_s. Find $\vec{D}$ and $\vec{E}$, in all volumes. Is the plane at $z = 4$ an equipotential surface?

Poisson’s Equation

Laplace's Equation

Problem 4

PEC

$z = 1$

$\vec{E}_2 = -\frac{3\epsilon_1}{8\epsilon_0}\hat{z}$ ϵ_2

$z = d$

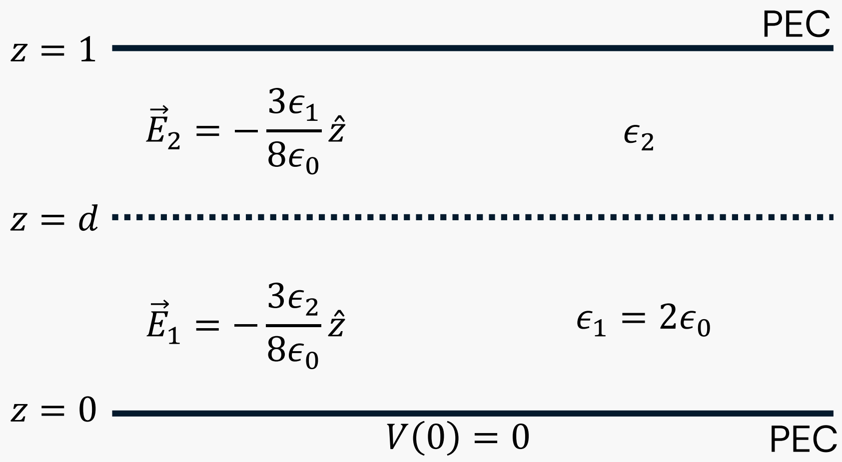
$\vec{E}_1 = -\frac{3\epsilon_2}{8\epsilon_0}\hat{z}$ $\epsilon_1 = 2\epsilon_0$

$z = 0$

$V(0) = 0$ PEC

Verify that the fields given satisfies Maxwell's boundary condition regarding $\vec{D}$ at the boundary between the two dielectric slabs.

Problem 4



Write the expression for the electrostatic potential V (z) for $0 < z < 1$ in terms of ϵ_1, ϵ_2, and d.

Determine ϵ_2 if the surface charge density on the top plate $\rho_s = 3\epsilon_0$ C/m^2.

Jumping Between Quantities

Q

$\rho_f = \rho$

$\vec{D}$

V

$\vec{E}$

$\vec{P}$

ρ_b

Week 3 equations, in one place

$$\vec{F} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\vec{F} = q_1\vec{E} + q_1(\vec{v}_1 \times \vec{B})$$

$$\vec{E} = \frac{q_2}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\hat{n}\cdot\left(\vec{D}_1 - \vec{D}_2\right) = \rho_s$$

$$\hat{n}\times\left(\vec{E}_1 - \vec{E}_2\right) = 0$$

$$\nabla\cdot\vec{D} = \rho$$

$$\nabla\cdot\vec{J} = -\frac{\partial\rho}{\partial t}$$

$$-\nabla^2 V = \frac{\rho}{\epsilon}$$

$$\epsilon \oiint \vec{E}\cdot d\vec{S} = Q_{\text{enclosed}}$$

$$\oiint \vec{D}\cdot d\vec{S} = Q_{\text{enclosed}}$$

$$\iiint \rho dV = Q_{\text{enclosed}}$$

$$\oiint \vec{B}\cdot d\vec{S} = 0$$

$$I = \oiint \vec{J}\cdot d\vec{S} = -\frac{\partial Q_{\text{enclosed}}}{\partial t}$$

$$\epsilon = \epsilon_0(1+\chi_e)$$

$$\vec{P} = \epsilon_0\chi_e\vec{E}$$

$$\vec{D} = \epsilon_0\vec{E} + \vec{P} = \epsilon\vec{E}$$

$$\vec{J} = \sigma\vec{E}$$

$$\rho_b = -\nabla\cdot\vec{P}$$

$$\nabla\cdot\epsilon_0\vec{E} = \rho_f + \rho_b$$

$$\nabla\times\vec{E} = 0$$

$$\vec{E} = -\nabla V$$

$$\oint \vec{E}\cdot d\vec{l} = 0$$

$$V_{ab} = V(b) - V(a) = -\int_a^b \vec{E}\cdot d\vec{l}$$

$$\oiint \vec{D}\cdot d\vec{S} = \iiint \nabla\cdot\vec{D}dV$$

$$\oint \vec{E}\cdot d\vec{l} = \iint \left(\nabla\times\vec{E}\right)\cdot d\vec{S}$$

$$\int_a^b \nabla V\cdot d\vec{l} = V(b) - V(a)$$

Units

Charge Q: C
Electric field $\vec{E}$: N/C or V/m
Displacement field $\vec{D}$: C/m^2
Polarization field $\vec{P}$: C/m^2
Electric potential V: V
Magnetic field $\vec{B}$: T or Wb/m^2
Charge density ρ: C/m^3
Surface charge density ρ_s: C/m^2
Current density $\vec{J}$: A/m^2

Electric permittivity ϵ: F/m
Magnetic permeability μ: H/m
Conductivity σ: Si/m

Office Hours

Any questions?

Office Hours

Start Time	Mon	Tue	Wed	Thurs	Fri
9am					
10am					
11am	Prof. Shao 5034 ECEB			TA Office Hour 5034 ECEB	Prof. Chen 5040 ECEB
12pm				TA Office Hour 5034 ECEB	
1pm					
2pm					Prof. Mitchell 3015 ECEB
3pm	TA Office Hour 5034 ECEB		TA Office Hour 5034 ECEB		TA Office Hour 3015 ECEB
4pm	TA Office Hour 5034 ECEB		TA Office Hour 5034 ECEB	TA Office Hour 5034 ECEB	TA Office Hour 3015 ECEB
5pm				TA Office Hour 5034 ECEB Tutorial Session 3017 ECEB	
6pm				Tutorial Session 3017 ECEB	
7pm					