Share any thoughts on anything

ECE329: Tutorial Session 3

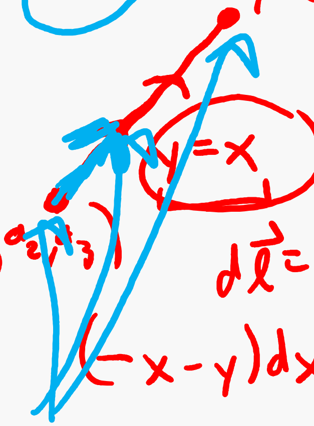
September 18th, 2025

General line integrals

$$\oint_C \vec{E}\cdot d\vec{\ell} = \iint (\nabla\times\vec{E})\cdot d\vec{s}$$

$$V = -\int_a^b \vec{E}\cdot d\vec{\ell}$$

$\vec{b}$ (b_1, b_2, b_3)



$x=\lambda$ $y=\lambda$ $z=$

$$\vec{r} = \vec{a} + \lambda\vec{d} = \vec{a}+\lambda(\vec{b}-\vec{a}) \qquad d\vec{\ell} = \hat{n}\,d\ell$$

$$x = \cancel{a_1} + \lambda d_1 = a_1 + \lambda_1(b_1 - a_1)$$
$$y = \cancel{a_2} + \lambda d_2$$
$$z = \cancel{a_3} + \lambda d_3$$

$$\hat{n} = \frac{(b_1-a_1)\hat{x} + (b_2-a_2)\hat{y} + (b_3-a_3)\hat{z}}{\cancel{\sqrt{(b_1-a_1)^2 + (b_2-a_2)^2+\cdots}}}$$

$$d\vec{\ell} = dx$$
$$(-x-y)\,dx$$

$$d\ell = \sqrt{\left(\frac{dx}{d\lambda}\right)^2 + \left(\frac{dy}{d\lambda}\right)^2 + \left(\frac{dz}{d\lambda}\right)^2}\,d\lambda$$

$$= \sqrt{d_1^2 + d_2^2 + \cancel{d_3^2}}\,d\lambda$$

$x=y$

$x = \lambda d_1$
$y = \lambda d_2$

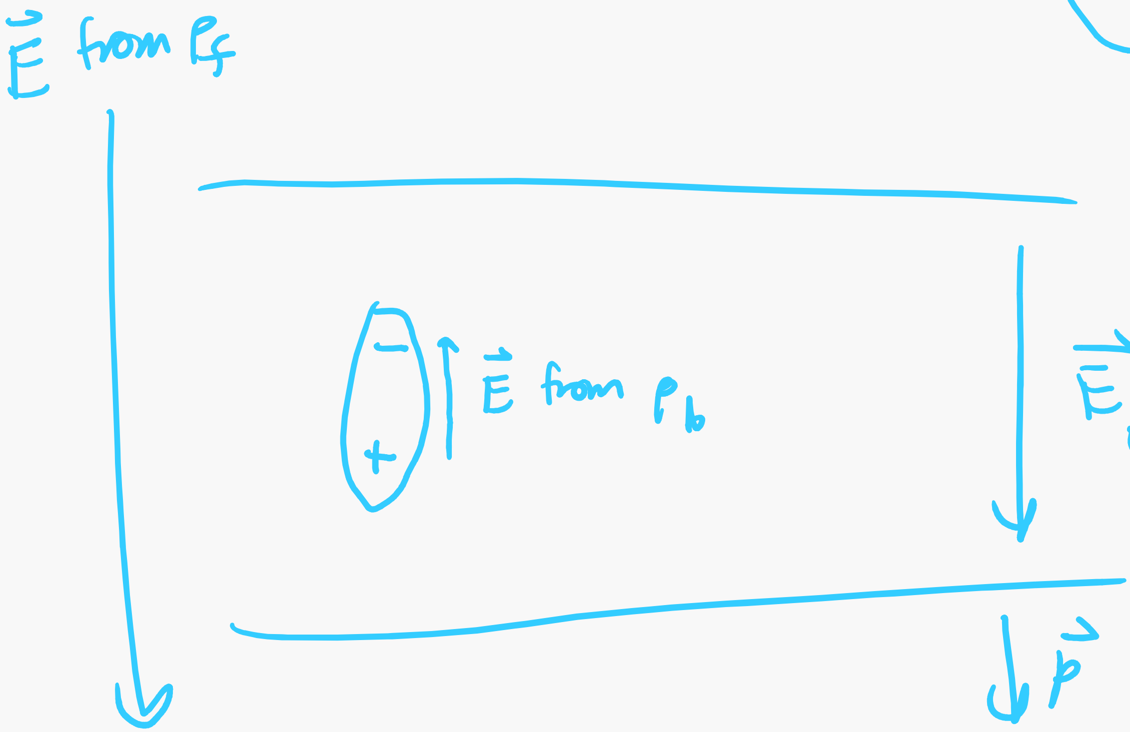
$$d\vec{\ell} = \left((b_1-a_1)\hat{x} + \cdots\right)d\lambda$$

Polarization noises

$\nabla \cdot \vec{D} = \rho_f \leftarrow \vec{D} \leftarrow \vec{D} = \varepsilon_0 \vec{E}$

$$\vec{E}_{\rho_f} = \vec{E}_{tot} + \vec{P}$$

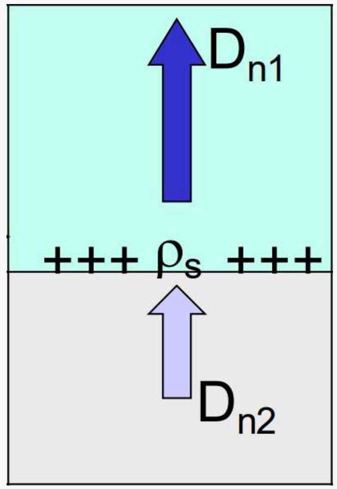
$$\vec{P} = \varepsilon_0 \underbrace{\chi_\varepsilon}_{\text{dielectric}} \vec{E}_{tot}$$



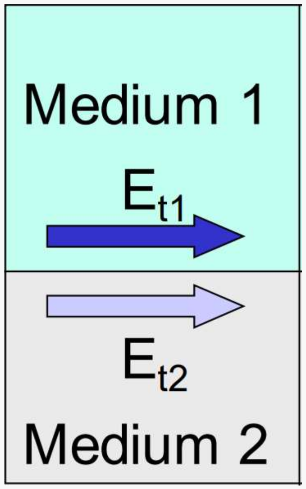
$$\vec{E}_{tot} = \vec{E}_{\rho_f} + \vec{E}_{\rho_b}$$

Boundary Conditions

$$\hat{n} \cdot \left(\vec{D}_1 - \vec{D}_2\right) = \rho_s$$



$$\hat{n} \times \left(\vec{E}_1 - \vec{E}_2\right) = 0$$



Problem 1

Suppose the yz-plane in free space holds a surface charge density of $\rho_s = 5$ C/m². The electric displacement field on the $-x$ side is given as $\vec{D} = \hat{x} + \hat{y} + \hat{z}$. Find the electric displacement field on the $+x$ side using boundary conditions.

$\hat{x} \times (\vec{D}_1 - (\hat{x} + \hat{y} + \hat{z}))$

$\hat{n} \cdot (\vec{D}_1 - \vec{D}_2) = \rho_s$

$\hat{n} \times (\vec{E}_1 - \vec{E}_2) = 0$

$\hat{n} = \hat{x}$

$+x$

ε_0

$\rho_s = 5$

$\vec{D} = \hat{x} + \hat{y} + \hat{z}$

ε_0

$\hat{x} \times \left(\frac{\vec{D}_1}{\varepsilon_0} - \frac{(\hat{x} + \hat{y} + \hat{z})}{\varepsilon_0} \right) = 0$

$\hat{x} \times \frac{\vec{D}_1}{\varepsilon_0} - \frac{(\hat{z} - \hat{y})}{\varepsilon_0} = 0 \quad (\hat{z} - \hat{y}) \quad -\hat{z} + \hat{y}$

$\hat{x} \cdot (\vec{D}_1 - (\hat{x} + \hat{y} + \hat{z})) = \rho_s = 5$

$(\vec{D}_1 \cdot \hat{x}) - 1 = 5$

$\vec{D}_1 = 6\hat{x} \ \frac{C}{m^2}$

$D_{1x} = 6 \quad D_{1y} = 1 \quad D_{1z} = 1$

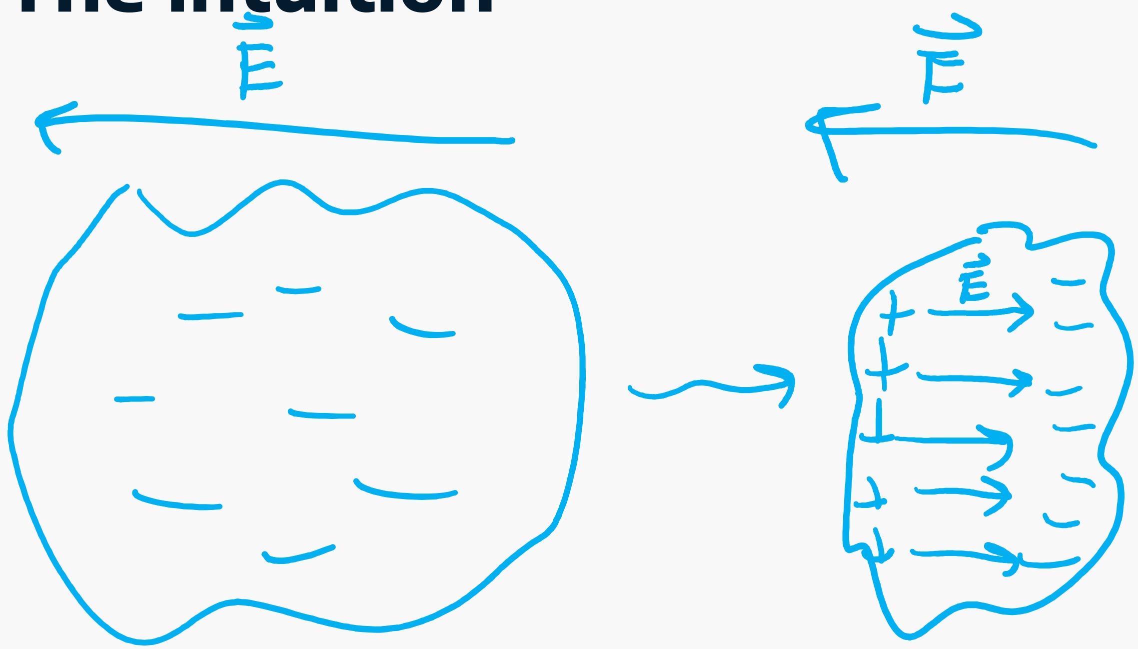
Problem 1: Blank slide

Suppose the yz-plane in free space holds a surface charge density of $\rho_s = 5$ C/m^2. The electric displacement field on the $-x$ side is given as $\vec{D} = \hat{x} + \hat{y} + \hat{z}$. Find the electric displacement field on the $+x$ side using boundary conditions.

$$\hat{n} \times (\vec{E}_1 - \vec{E}_2) = 0$$

$$\vec{E}_{2t} = \frac{\hat{y} + \hat{z}}{\varepsilon_0} \longrightarrow \vec{E}_{1t} = \frac{\hat{y} + \hat{z}}{\varepsilon_0}$$

Conductors: The intuition



Conductors: The math

Described by σ, aka conductivity (units: Siemens/meter)

What to know:

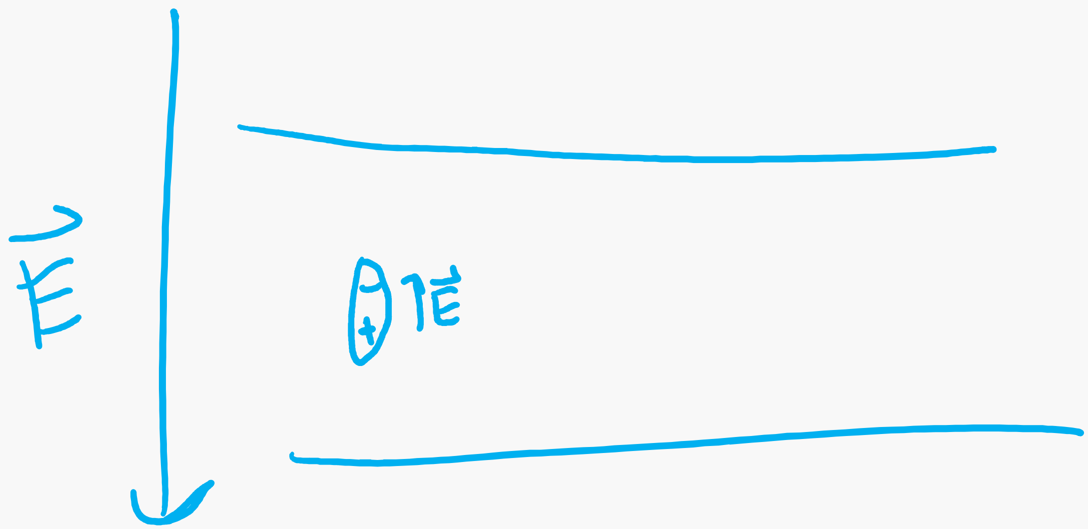
- $\vec{J} = \sigma \vec{E}$ (Ohm's Law)

Assumption: We are dealing with electrostatics.

- If $\sigma \neq 0$, material is an equipotential with zero internal fields and finite surface charge densities.

$\vec{E} = 0 \quad \vec{D} = 0 \quad \vec{P} = 0$

P fields: The intuition



P fields: The math

$\vec{p} = c\vec{E}$

$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$: The defining equation.

Assumption: Dielectric is 'isotropic', so $\vec{P}$ is collinear to $\vec{E}$. Then:

- $\vec{P} = \epsilon_0 \chi_e \vec{E}$ with electric susceptibility $\chi_e \geq 0$ nearly always in this class.
- Let electric permittivity be $\epsilon = \epsilon_0(1 + \chi_e)$.
- Let relative electric permittivity be $\epsilon_r = 1 + \chi_e$
- $\vec{D} = \epsilon \vec{E}$

P fields: The math

~~Assumption:~~ Dielectric is 'isotropic', so $\vec{P}$ is collinear to $\vec{E}$. Then:

Divergences:

- Gauss's Law: $\nabla \cdot \epsilon_0 \vec{E} = \rho_f + \rho_b$
- Gauss's Law: $\boxed{\nabla \cdot \vec{D} = \rho_f} = \rho$

- Therefore, $\rho_b = -\nabla \cdot \vec{P}$

$\nabla \cdot \vec{D} = \rho$

Problem 2

$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$

$\hookrightarrow \vec{P} = \vec{D} - \varepsilon_0 \vec{E}$

$\vec{D} = \varepsilon \vec{E}$

ρ

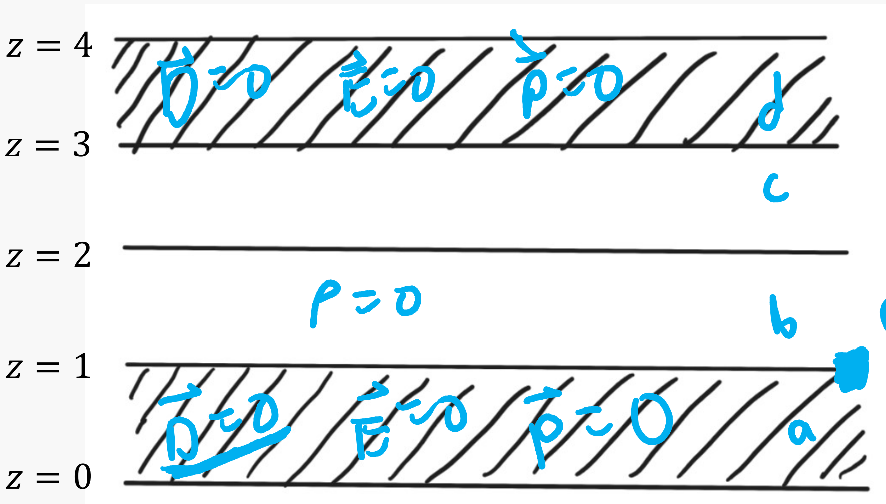
$\hat{n} \times (\vec{E}_1 - \vec{E}_2) = 0$

Find $\vec{D}$, $\vec{E}$, and $\vec{P}$, in all volumes. Find ρ_s at each material boundary. What is the voltage drop from $z = 0$ to $z = 4$?

$\hat{z}$ ① ②

$\hat{z} \cdot (\vec{D}_1 - \vec{D}_2) = \rho_s$

$\hat{z} \cdot \vec{D}_1 = 6$ $\quad D_{1z} = 6$



$\epsilon = 2\epsilon_0, \sigma = 10^6$

$\epsilon = 5\epsilon_0, \sigma = 0$

$\epsilon = 3\epsilon_0, \sigma = 0$

$\rho_s = 6$ C/m²

$\epsilon = \epsilon_0, \sigma = 10^3$

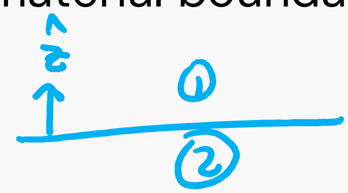
$\vec{D}_b = 0\hat{x} + 0\hat{y} + 6\hat{z}$ $\frac{C}{m^2}$

$\vec{E}_b = 0\hat{x} + 0\hat{y} + \frac{6}{3\varepsilon_0}\hat{z}$ $\frac{V}{m}$

$\vec{P}_b = 4\hat{z}$ $\frac{C}{m^2}$

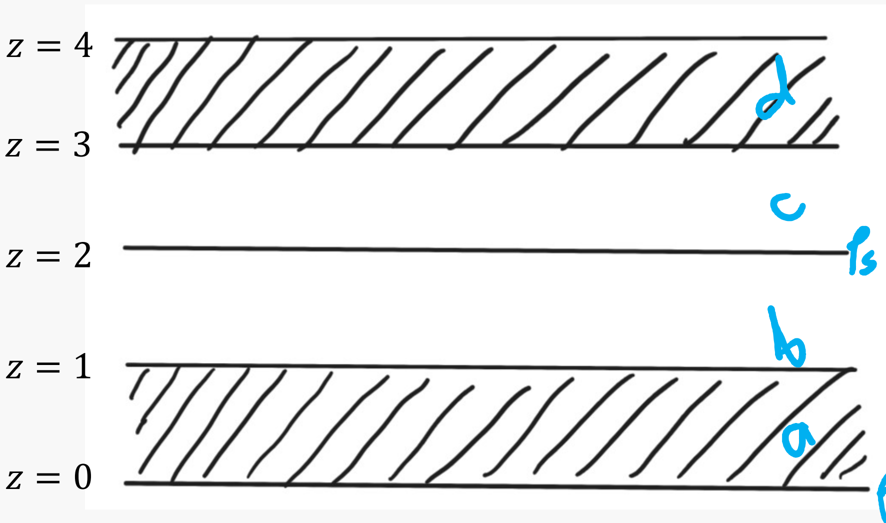
Problem 2: Blank slide

Find $\vec{D}$, $\vec{E}$, and $\vec{P}$, in all volumes. Find ρ_s at each material boundary. What is the voltage drop from $z = 0$ to $z = 4$?



σ ρ

$\hat{z} \cdot (\underbrace{\vec{D}_c} - \underbrace{\vec{D}_b}) = \underline{0}$



$\epsilon = 2\epsilon_0, \underline{\sigma = 10^6}$

$\epsilon = 5\epsilon_0, \sigma = 0$ $\sigma = ?$

$\epsilon = 3\epsilon_0, \sigma = 0$

$\rho_s = 6$ C/m²

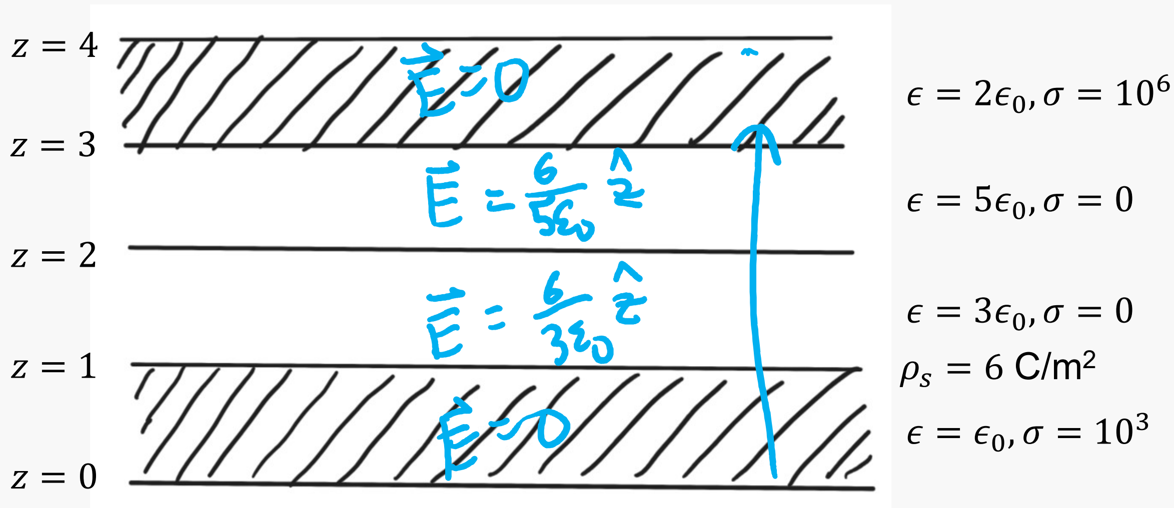
$\epsilon = \epsilon_0, \sigma = 10^3$

$\rho_s = -6$

Problem 2: Blank slide

Find $\vec{D}$, $\vec{E}$, and $\vec{P}$, in all volumes. Find ρ_s at each material boundary. What is the voltage drop from $z = 0$ to $z = 4$?

$d\vec{l} = \hat{z}\, dz$



Problem 3

Let $\rho = 6\epsilon_0\delta(z) + \rho_s\delta(z-4)$ C/m³. The displacement field in the $0 < z < 4$ region is given as $\vec{D} = \epsilon_0\hat{x} + 3\epsilon_0\hat{z}$ and the electric permittivity is known to be $\epsilon_2 = 4\epsilon_0$. It is known that $D_z = 2\epsilon_0$ and $\epsilon_3 = 2\epsilon_0$ for the $z > 4$ region, while $\epsilon_1 = \epsilon_3$ for the $z < 0$ region.
Find ρ_s. Find $\vec{D}$ and $\vec{E}$, in all volumes. Is the plane at $z = 4$ an equipotential surface?

$z=4$ ——————— ρ_s

$z=0$ ——————— $6\epsilon_0$

Problem 3: Blank slide

Find ρ_s. Find $\vec{D}$ and $\vec{E}$, in all volumes. Is the plane at $z = 4$ an equipotential surface?

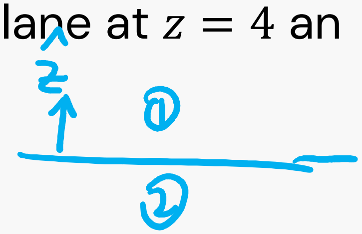
$D_z = 2\epsilon_0$ $\qquad$ $\epsilon_3 = 2\epsilon_0$

$z = 4$ $\qquad$ ρ_s

$\vec{D} = \epsilon_0\hat{x} + 3\epsilon_0\hat{z}$ $\qquad$ $\epsilon_2 = 4\epsilon_0$

$z = 0$ $\qquad$ $6\epsilon_0$

$\epsilon_1 = 2\epsilon_0$



$$\hat{z} \cdot (\vec{D}_1 - \vec{D}_2) = \rho_s$$

$$\hat{z} \cdot \left(\cancel{?\hat{x} + ?\hat{y}} + 2\epsilon_0\hat{z} - \cancel{\epsilon_0\hat{x}} - 3\epsilon_0\hat{z}\right) = \rho_s$$

$$2\epsilon_0 - 3\epsilon_0 = \rho_s$$

$$\rho_s = -\epsilon_0 \frac{C}{m^2}$$

Problem 3: Blank slide

$\longrightarrow \hat{n} \times (\vec{E}_1 - \vec{E}_2) = 0$

tan of $\vec{E}$

Find ρ_s. Find $\vec{D}$ and $\vec{E}$, in all volumes. Is the plane at $z = 4$ an equipotential surface?

$D_z = 2\epsilon_0$ $\quad \epsilon_3 = 2\epsilon_0$ $\quad$ a

$z = 4$ $\quad$ $z=4$

$\vec{D} = \epsilon_0 \hat{x} + 3\epsilon_0 \hat{z}$ $\quad \epsilon_2 = 4\epsilon_0$ $\quad$ b

$z = 0$ $\quad$ $6\varepsilon_0$

$\epsilon_1 = 2\epsilon_0$ $\quad$ c

$\hat{z}$ ① ②

$\vec{D}_a = \frac{1}{2}\varepsilon_0 \hat{x} + 2\varepsilon_0 \hat{z}$

$\vec{D}_b = \varepsilon_0 \hat{x} + 3\varepsilon_0 \hat{z}$

$\vec{D}_c = \frac{1}{2}\varepsilon_0 \hat{x}$

$\vec{E}_a = \frac{1}{4}\hat{x} + 0\hat{y} + \hat{z}$

$\vec{E}_b = \frac{1}{4}\hat{x} + \frac{3}{4}\hat{z} + 0\hat{y}$

$\vec{E}_c = \frac{1}{4}\hat{x} + 0\hat{y}$

$V = -\int \vec{E} \cdot d\vec{l}$

$\cancel{\vec{E}_a = \frac{1}{4}\hat{x}}$

$\cancel{\vec{E}_b = \frac{1}{4}\hat{x}}$

$\vec{E}_a = 0 + \hat{z}$

$\vec{E}_b = 0$

Problem 3: Blank slide

$V = \int \vec{E} \cdot d\vec{l}$

Find ρ_s. Find $\vec{D}$ and $\vec{E}$, in all volumes. Is the plane at $z = 4$ an equipotential surface?

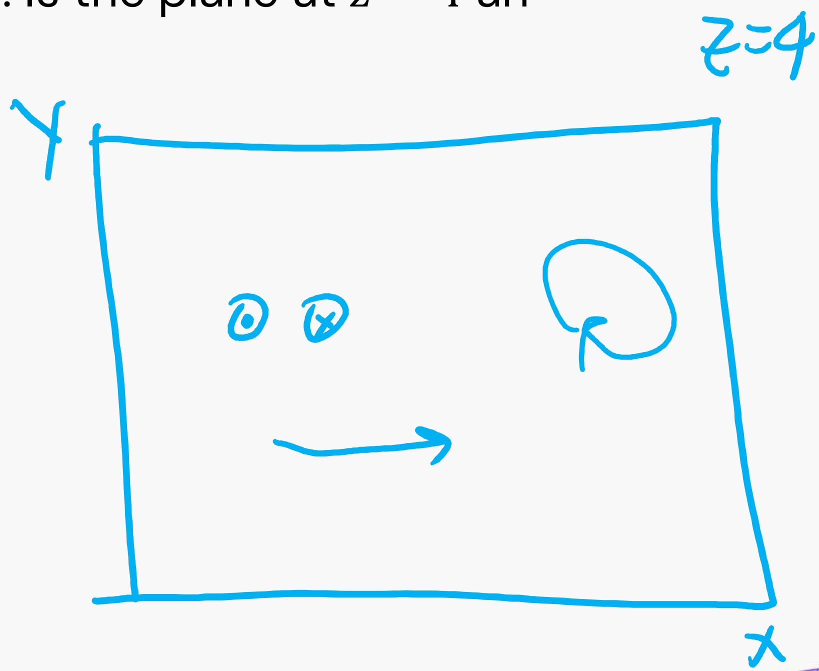
$D_z = 2\epsilon_0$ $\quad \epsilon_3 = 2\epsilon_0$

$z = 4$

$\vec{D} = \epsilon_0 \hat{x} + 3\epsilon_0 \hat{z}$ $\quad \epsilon_2 = 4\epsilon_0$

$z = 0$

$\epsilon_1 = 2\epsilon_0$



Problem 3: Blank slide

Find ρ_s. Find $\vec{D}$ and $\vec{E}$, in all volumes. Is the plane at $z = 4$ an equipotential surface?

$D_z = 2\epsilon_0$ $\quad\quad$ $\epsilon_3 = 2\epsilon_0$

$z = 4$

$\vec{D} = \epsilon_0\hat{x} + 3\epsilon_0\hat{z}$ $\quad\quad$ $\epsilon_2 = 4\epsilon_0$

$z = 0$

$\epsilon_1 = 2\epsilon_0$

Poisson's Equation

$\nabla \cdot \vec{D} = \rho$

$\vec{D} = \varepsilon \vec{E}$ $\quad \nabla \cdot (\varepsilon \vec{E}) = \rho$

$\varepsilon \nabla \cdot \vec{E} = \rho$

ε const

$\vec{E} = -\nabla V$ iff $\nabla \times \vec{E} = 0$

$$\underbrace{\nabla^2 V = -\frac{\rho}{\varepsilon}}$$

Laplace's Equation

$$\nabla^2 V = -\frac{\rho}{\varepsilon} \qquad \text{and} \quad \rho = 0$$

(in dielectric)

$$\nabla^2 V = 0$$

$$\rightarrow V = mx + b$$

Problem 4

$\hat{z}\cdot(\vec{D}_1-\vec{D}_2)=0$

PEC

$z = 1$

$\vec{E}_2 = -\frac{3\epsilon_1}{8\epsilon_0}\hat{z}$ $\quad \vec{D}_2 = -\frac{3\varepsilon_1\varepsilon_2}{8\varepsilon_0}\hat{z}$ $\quad \epsilon_2$ $\quad \sigma = 0$

$z = d$ $\quad \rho_s = 0$

$\vec{E}_1 = -\frac{3\epsilon_2}{8\epsilon_0}\hat{z}$ $\quad \vec{D}_1 = -\frac{3\varepsilon_1\varepsilon_2}{8\varepsilon_0}\hat{z}$ $\quad \epsilon_1 = 2\epsilon_0$ $\quad \sigma = 0$

$z = 0$

$V(0) = 0$

PEC $\leftarrow \sigma$

Verify that the fields given satisfies Maxwell's boundary condition regarding $\vec{D}$ at the boundary between the two dielectric slabs.

✓

Problem 4

$\vec{E} = -\nabla V$

$c = -\nabla V$

$V_{ab} = -\int_a^b \vec{E} \cdot d\vec{\ell}$

$z = 1$ PEC

$\nabla^2 V = 0$ $\quad \vec{E}_2 = -\frac{3\epsilon_1}{8\epsilon_0}\hat{z}$ $\quad V_2 = A_2 z + B_2$ $\quad \epsilon_2$ $\quad \sigma = 0$

$z = d$ $\quad \rho_s = 0$

$V_1 = A_1 z + \cancel{B_1} = 0$ $\quad \sigma = 0$

$\nabla^2 V = 0$ $\quad \vec{E}_1 = -\frac{3\epsilon_2}{8\epsilon_0}\hat{z}$ $\quad \epsilon_1 = 2\epsilon_0$

$z = 0$ PEC

$V(0) = 0$

Write the expression for the electrostatic potential V (z) for $0 < z < 1$ in terms of ϵ_1, ϵ_2, and d.

Determine ϵ_2 if the surface charge density on the top plate $\rho_s = 3\epsilon_0$ C/m².

$V = \underbrace{\frac{3\epsilon_2}{8\epsilon_0}}_{A_1} z + 0 \qquad 0 \le z \le d$

$V_1 = -E_{1z} z + C$

$-\nabla V_1 = E_{1z}$

$V_2 = -\underbrace{E_{2z}}_{= A_2} z + C$

A_1

Problem 4: Blank slide

PEC = perfect electrical conductor = $\sigma = \infty$

$\vec{D}=0 \quad \vec{E}=0 \quad \vec{\rho}=0$ PEC ρ_s

$z = 1$

$\vec{E}_2 = -\dfrac{3\epsilon_1}{8\epsilon_0}\hat{z}$ $\qquad \epsilon_2$

$z = d$

$\vec{E}_1 = -\dfrac{3\epsilon_2}{8\epsilon_0}\hat{z}$ $\qquad \epsilon_1 = 2\epsilon_0$

$z = 0$

$V(0) = 0$ PEC

$\hat{z}$

1

2

Write the expression for the electrostatic potential V (z) for $0 < z < 1$ in terms of ϵ_1, ϵ_2, and $d.$

Determine ϵ_2 if the surface charge density on the top plate $\rho_s = 3\epsilon_0$ C/m^2.

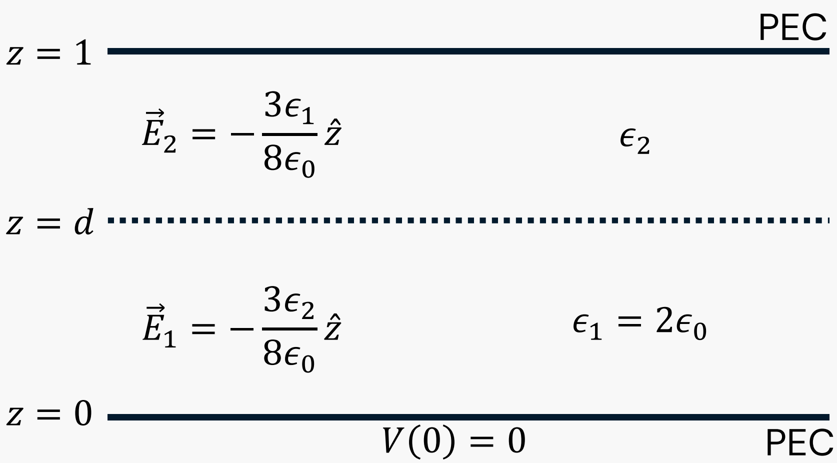
if $\sigma = 0$

$\hat{n} \cdot (\cancel{\vec{D}_1} - \vec{D}_2) = \rho_s = 3\varepsilon_0$

$\hat{z} \cdot \left(-\varepsilon_2 \vec{E}_2\right) = \rho_s = 3\varepsilon_0$

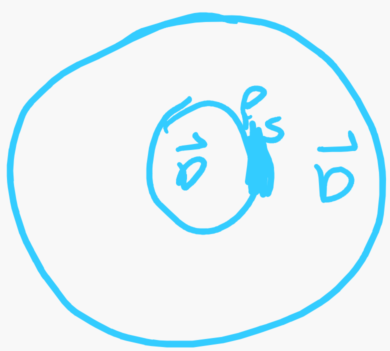
$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$

Problem 4: Blank slide



Write the expression for the electrostatic potential V (z) for $0 < z < 1$ in terms of ϵ_1, ϵ_2, and d.

Determine ϵ_2 if the surface charge density on the top plate $\rho_s = 3\epsilon_0$ C/m^2.



Week 3 equations, in one place

$$\vec{F} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}\hat{r}$$
$$\vec{F} = q_1\vec{E} + q_1(\vec{v}_1 \times \vec{B})$$
$$\vec{E} = \frac{q_2}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\hat{n}\cdot(\vec{D}_1 - \vec{D}_2) = \rho_s$$
$$\hat{n}\times(\vec{E}_1 - \vec{E}_2) = 0$$

$$\nabla\cdot\vec{D} = \rho$$
$$\nabla\cdot\vec{J} = -\frac{\partial\rho}{\partial t}$$
$$-\nabla^2 V = \frac{\rho}{\epsilon}$$

$$\epsilon\oiint \vec{E}\cdot d\vec{S} = Q_{\text{enclosed}}$$
$$\oiint \vec{D}\cdot d\vec{S} = Q_{\text{enclosed}}$$
$$\iiint \rho dV = Q_{\text{enclosed}}$$
$$\oiint \vec{B}\cdot d\vec{S} = 0$$
$$I = \oiint \vec{J}\cdot d\vec{S} = -\frac{\partial Q_{\text{enclosed}}}{\partial t}$$

$$\epsilon = \epsilon_0(1+\chi_e)$$
$$\vec{P} = \epsilon_0\chi_e\vec{E}$$
$$\vec{D} = \epsilon_0\vec{E} + \vec{P} = \epsilon\vec{E}$$
$$\vec{J} = \sigma\vec{E}$$
$$\rho_b = -\nabla\cdot\vec{P}$$
$$\nabla\cdot\epsilon_0\vec{E} = \rho_f + \rho_b$$

$$\nabla\times\vec{E} = 0$$
$$\vec{E} = -\nabla V$$
$$\oint \vec{E}\cdot d\vec{l} = 0$$
$$V_{ab} = V(b) - V(a) = -\int_a^b \vec{E}\cdot d\vec{l}$$

$$\oiint \vec{D}\cdot d\vec{S} = \iiint \nabla\cdot\vec{D}dV$$
$$\oint \vec{E}\cdot d\vec{l} = \iint (\nabla\times\vec{E})\cdot d\vec{S}$$
$$\int_a^b \nabla V\cdot d\vec{l} = V(b) - V(a)$$

Units

Charge Q: C
Electric field $\vec{E}$: N/C or V/m
Displacement field $\vec{D}$: C/m^2
Polarization field $\vec{P}$: C/m^2
Electric potential V: V
Magnetic field $\vec{B}$: T or Wb/m^2
Charge density ρ: C/m^3
Surface charge density ρ_s: C/m^2
Current density $\vec{J}$: A/m^2

Electric permittivity ϵ: F/m
Magnetic permeability μ: H/m
Conductivity σ: Si/m

Office Hours

Any questions?

Office Hours

Start Time	Mon	Tue	Wed	Thurs	Fri
9am					
10am					
11am	Prof. Shao 5034 ECEB			TA Office Hour 5034 ECEB	Prof. Chen 5040 ECEB
12pm				TA Office Hour 5034 ECEB	
1pm					
2pm					Prof. Mitchell 3015 ECEB
3pm	TA Office Hour 5034 ECEB		TA Office Hour 5034 ECEB		TA Office Hour 3015 ECEB
4pm	TA Office Hour 5034 ECEB		TA Office Hour 5034 ECEB	TA Office Hour 5034 ECEB	TA Office Hour 3015 ECEB
5pm				TA Office Hour 5034 ECEB Tutorial Session 3017 ECEB	
6pm				Tutorial Session 3017 ECEB	
7pm					