ECE329: Tutorial Session 2

September 11th, 2025

1.

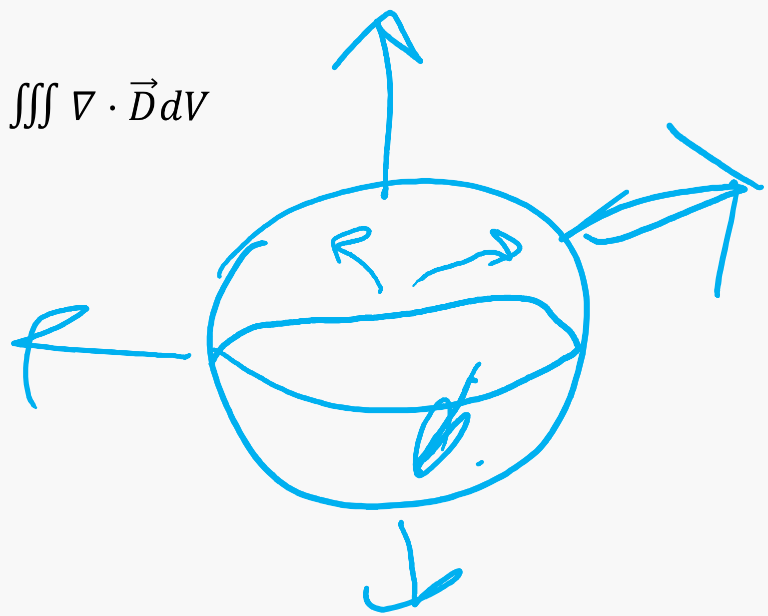
Math (again ☹)

Divergence & Divergence Theorem

Divergence = How much the field is DIVERGING at a certain point.

- Notation: $\nabla \cdot \vec{D}$
- Input: Vector field
- Output: Scalar field

Divergence Theorem: $\oiint \vec{D} \cdot d\vec{S} = \iiint \nabla \cdot \vec{D} dV$

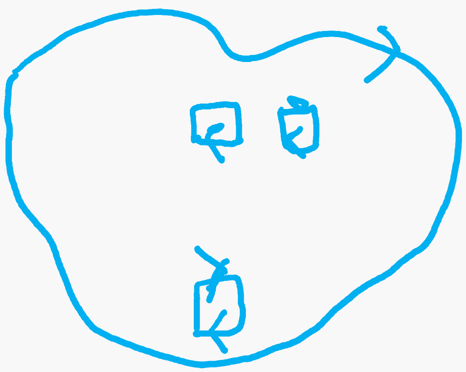


Curl & Stoke's Theorem

Curl = How much the field is CURLING around a certain point.

- Notation: $\nabla \times \vec{E}$
- Input: Vector field
- Output: Vector field, with direction indicating how the field right-hand curls around.

Stoke's Theorem: $\oint \vec{E} \cdot d\vec{l} = \iint \left(\nabla \times \vec{E}\right) \cdot d\vec{S}$



Gradient & Gradient Theorem

Gradient = How much the scalar function changes (or its GRADE).

- Notation: ∇V
- Input: Scalar field
- Output: Vector field, with direction indicating steepest **uphill**.

Fundamental theorem of calculus: $\int_a^b \mathrm{f}'(\mathrm{t}) \cdot dt = f(b) - f(a)$

Similarly:

Gradient theorem: $\int_a^b \nabla V \cdot d\vec{l} = V(b) - V(a)$

Laplacian

Laplacian (scalar) = How much the scalar function [illegible]APL[illegible]C[illegible]S.

Laplacian (scalar) = How much the rate of change of the scalar function varies (aka stress).

- Notation: $\nabla^2 V = \nabla \cdot \nabla V$
- Input: Scalar field
- Output: Scalar field

Problem 1

Find the divergence and curl of $\vec{E} = xy\hat{x} + yz\hat{y} + xz\hat{z}$.

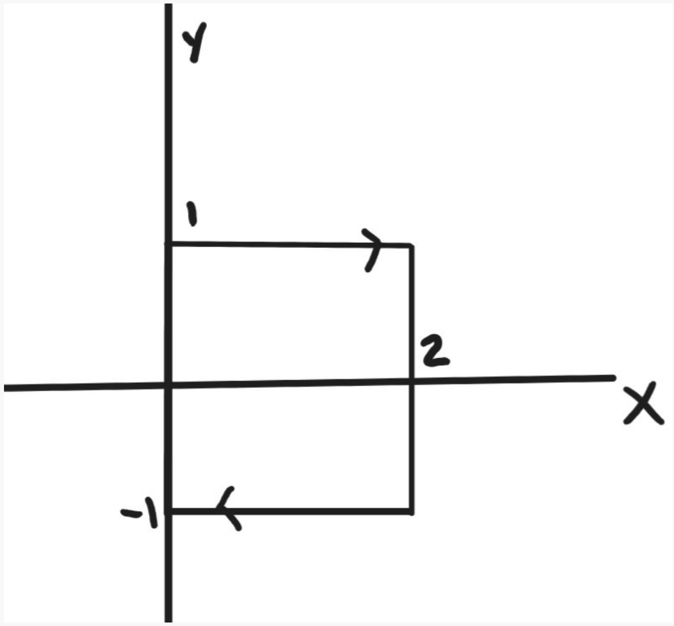
Problem 2

Find the gradient and Laplacian of $f = x^2 + y^2 + z^2$.

Problem 3

Stoke: $\oint \vec{E} \cdot d\vec{l} = \iint (\nabla \times \vec{E}) \cdot d\vec{S}$

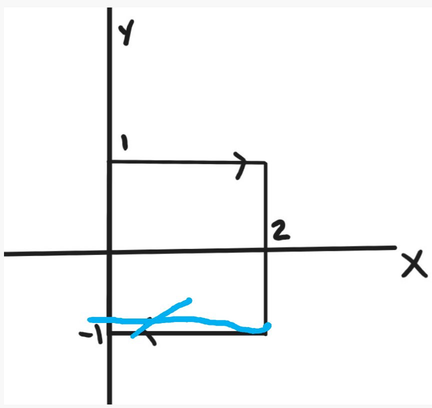
Suppose $\vec{E} = \frac{x^2}{2}\hat{y} - x\hat{z}$ V/m². What is the circulation for the closed square loop picture below?



Problem 3: Blank slide

$\oint \vec{E}\cdot d\vec{\ell}$

Suppose $\vec{E} = \frac{x^2}{2}\hat{y} - x\hat{z}$ V/m². What is the circulation for the closed square loop picture below?



$$\left[\int_{x=2}^{0}\vec{E}\cdot\hat{x}\,dx\Big|_{y=-1} + \int_{y=-1}^{1}\vec{E}\cdot\hat{y}\,dy\Big|_{x=0} + \int_{x=0}^{2}\vec{E}\cdot\hat{x}\,dx\Big|_{y=1} + \int_{y=1}^{-1}\vec{E}\cdot\hat{y}\,dy\Big|_{x=2}\right]$$

$$= \int_{y=1}^{-1} 2\,dy = 2y\Big|_{1}^{-1} = -4$$

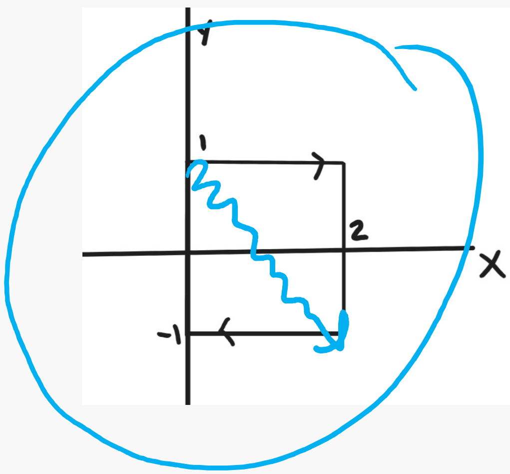
Problem 3: Blank slide

$$\oint \vec{E}\cdot d\vec{l} = \iint (\nabla\times\vec{E})\cdot d\vec{S}$$

Suppose $\vec{E} = \frac{x^2}{2}\hat{y} - x\hat{z}$ V/m^2. What is the circulation for the closed square loop picture below?

$$\nabla\times\vec{E} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial_x & \partial_y & \partial_z \\ 0 & \frac{+x^2}{2} & -x \end{vmatrix} = \hat{y} + x\hat{z}$$

$$d\vec{S} = \hat{n}\,dA = (-\hat{z})\,dx\,dy$$



$$\iint (\hat{y} + x\hat{z})\cdot(-\hat{z}\,dx\,dy)\Big|_{z=0} = \int_{x=0}^{2}\int_{y=-1}^{1} -x\,dx\,dy = -4$$

2.

Physics ☺

Fluxes

Recall electric and current flux equations from last time.

$$\oiint \vec{D} \cdot d\vec{S} = \iiint \rho_{\text{enclosed}} dV \qquad \oiint \vec{J} \cdot d\vec{S} = -\frac{\partial}{\partial t} \iiint \rho_{\text{enclosed}} dV$$

Fluxes

$$\oiint \vec{D} \cdot d\vec{S} = \iiint \rho_{\text{enclosed}} dV \qquad \oiint \vec{J} \cdot d\vec{S} = -\frac{\partial}{\partial t} \iiint \rho_{\text{enclosed}} dV$$

$$= \iiint \nabla \cdot \vec{D} \, dV = \iiint \rho \, dV \qquad \iiint \nabla \cdot \vec{J} \, dV = \iiint -\frac{\partial \rho}{\partial t} dV$$

$$\nabla \cdot \vec{D} = \rho$$

$$\nabla \cdot \vec{B} = 0$$

Maxwell's Eqs
Gauss Law

$$\nabla \cdot \vec{J} = \frac{-\partial \rho}{\partial t}$$

Conservative Fields

The following are equivalent for vector field $\vec{E}$:

- $\nabla \times \vec{E} = 0$
- $\vec{E}$ is conservative
- $\oint \vec{E} \cdot d\vec{l} = 0$ ∀ closed loops
- $\int_a^b \vec{E} \cdot d\vec{l}$ is path-independent ∀ a, b
- $\vec{E} = \nabla V$ for some scalar field V
- E field arises from electrostatics

Electrostatic Potential

Work done by you to move a charge from a to b, causing change in electrostatic potential U:

$$W = U(b) - U(a) = \int_a^b \vec{F}_{\text{applied}} \cdot d\vec{l}$$

$$= -\int_a^b \vec{F}_E \cdot d\vec{l} = -\int_a^b q\vec{E} \cdot d\vec{l}$$

Volts = work per unit charge (take $q = 1$).
U/q = electrostatic potential energy per unit charge = V = **electrostatic potential**:

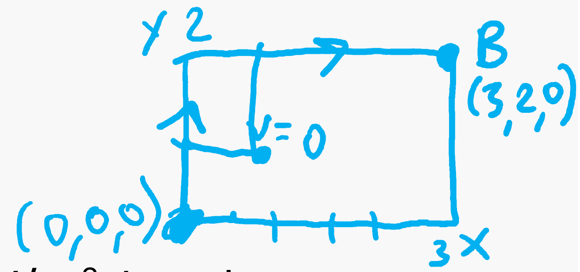
$$V_{ab} = V(b) - V(a) = -\int_a^b \vec{E} \cdot d\vec{l}$$

V to E

$$V_{ab} = V(b) - V(a) = -\int_a^b \vec{E} \cdot d\vec{l}$$

$$V(b) - V(a) = \int_a^b \nabla V \cdot d\vec{l} = -\int_a^b \vec{E} \cdot d\vec{l}$$

$$\vec{E} = -\nabla V$$



Problem 5

Suppose $\vec{E} = 2(x-1)\hat{x} + 3(z+y^2+1)\hat{y} + 3(y+1)\hat{z}$ V/m². Let the point $(0,0,0)$ be grounded with $V = 0$. Find the voltage difference from the origin to point B at $(3,2,0)$.

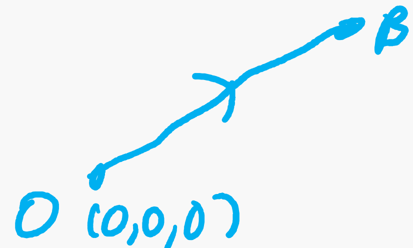
$$V(3,2,0) - V(0,0,0)$$

$$-\int_{(0,0,0)}^{(0,2,0)} \vec{E}\cdot\hat{y}\,dy - \int_{(0,2,0)}^{(3,2,0)} \vec{E}\cdot\hat{x}\,dx = -\int_{y=0}^{2} 3(z+y^2+1)\,dy\Big|_{x=0,\,z=0}$$

$$-\int_{x=0}^{3} 2(x-1)\,dx\Big|_{y=2,\,z=0}$$

$$= -17\ \text{V}$$

Problem 5: Blank slide



Suppose $\vec{E} = 2(x-1)\hat{x} + 3(z+y^2+1)\hat{y} + 3(y+1)\hat{z}$ V/m². Let the point $(0{,}0{,}0)$ be grounded with $V = 0$. Find the voltage difference from the origin to point B at $(3{,}2{,}0)$.

$$\vec{d} = (0,0,0) + \lambda(3,2,0)$$

$$x = 3\lambda \qquad y = 2\lambda \qquad z = 0$$

$$-\int \vec{E}\cdot d\vec{l}$$

$$d\vec{l} = \hat{n}\,dl$$

$$\hat{n} = \frac{(3-0)\hat{x} + (2-0)\hat{y} + (0-0)\hat{z}}{\sqrt{(3-0)^2 + (2-0)^2 + (0-0)^2}} = \frac{1}{\sqrt{13}}(3\hat{x} + 2\hat{y})$$

$$dl = \sqrt{\left(\frac{dx}{d\lambda}\right)^2 + \left(\frac{dy}{d\lambda}\right)^2 + \left(\frac{dz}{d\lambda}\right)^2}\,d\lambda = \sqrt{13}\,d\lambda$$

Problem 5: Blank slide

Suppose $\vec{E} = 2(x-1)\hat{x} + 3(z+y^2+1)\hat{y} + 3(y+1)\hat{z}$ V/m². Let the point $(0,0,0)$ be grounded with $V = 0$. Find the voltage difference from the origin to point B at $(3,2,0)$.

$$-\int_{\lambda=0}^{1} \vec{E} \cdot \frac{1}{\sqrt{13}}(3\hat{x}+2\hat{y})\sqrt{13}\,d\lambda$$

$$-\int_{\lambda=0}^{1} 6(x-1) + 6(z+y^2+1)\,d\lambda = -17V$$

$x = 3\lambda \qquad y = 2\lambda$

Problem 5: Blank slide

Suppose $\vec{E} = 2(x-1)\hat{x} + 3(z + y^2 + 1)\hat{y} + 3(y+1)\hat{z}$ V/m². Let the point $(0,0,0)$ be grounded with $V = 0$. Find the voltage difference from the origin to point B at $(3,2,0)$.

$\vec{E} = -\nabla V$

$V(x,y,z) = -(x^2 - 2x + 3zy + y^3 + 3y + 3z) + c$

$\frac{\partial V}{\partial x} = -\vec{E}_x = 2(x-1)$

$\frac{\partial V}{\partial y} = -\vec{E}_y$ $\quad$ $\frac{\partial V}{\partial z} = -\vec{E}_z$

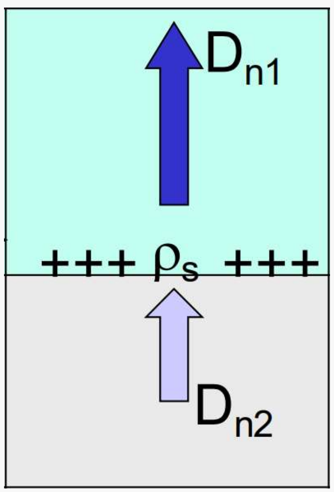
$c = 9$

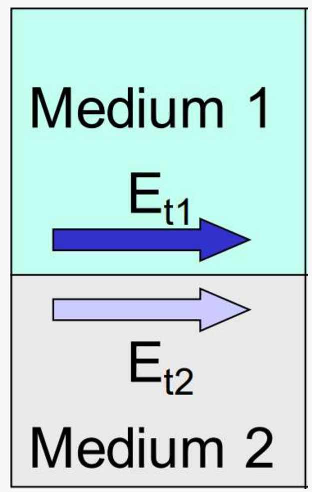
$V(3,2,0) - V(0,0,0) = -17V$

Boundary Conditions

$$\hat{n} \cdot \left(\vec{D}_1 - \vec{D}_2\right) = \rho_s$$

$$\hat{n} \times \left(\vec{E}_1 - \vec{E}_2\right) = 0$$

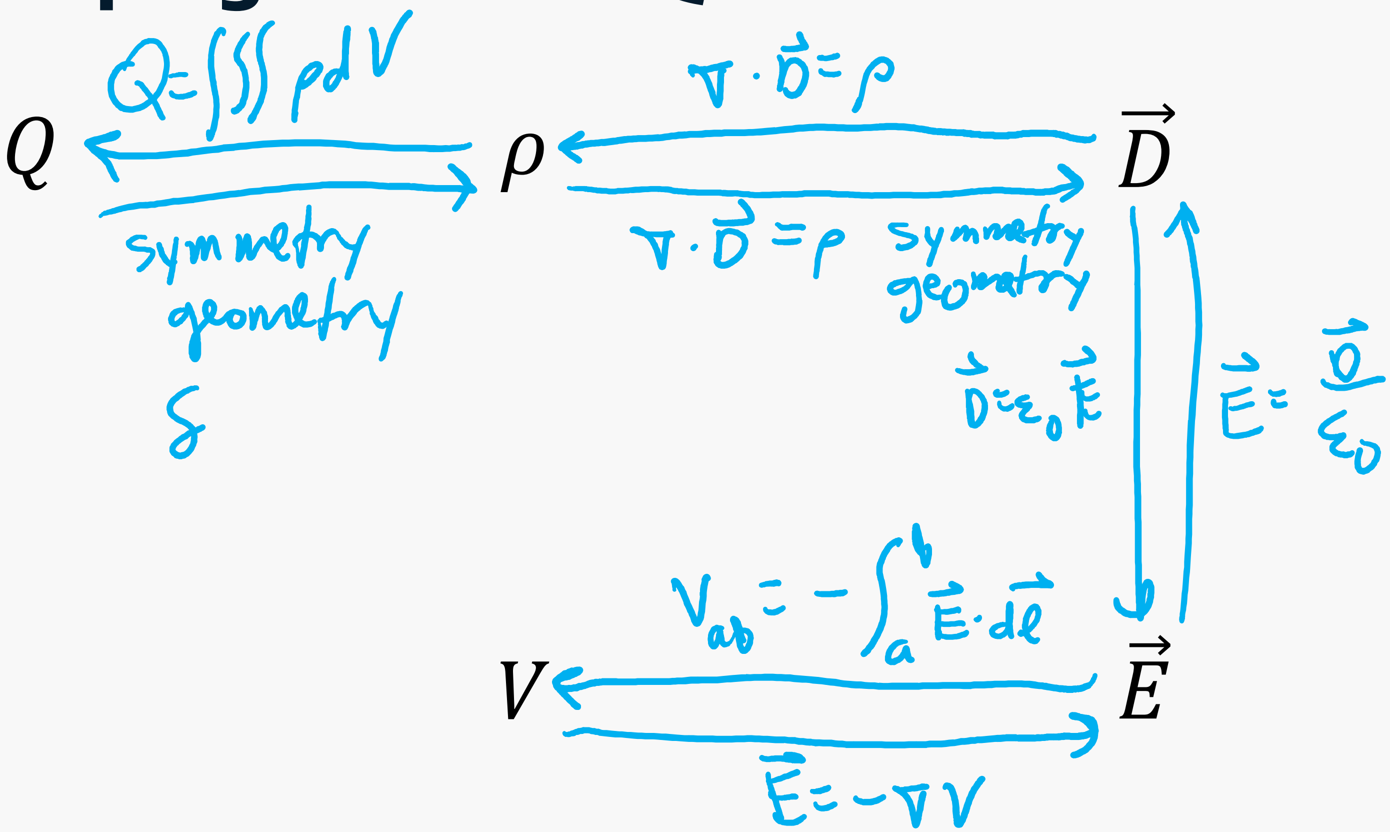




Problem 6

Suppose the yz-plane holds a surface charge density of $\rho_s = 5$ C/m^2. The electric displacement field on the $-x$ side is given as $\vec{D} = \hat{x} + \hat{y} + \hat{z}$. Find the electric displacement field on the $+x$ side using boundary conditions.

Jumping Between Quantities



P.S.: Helmholtz Theorem

A vector field $\vec{E}$ is specified completely by its divergence and curl.

Week 2 equations, in one place

$$\vec{F} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\vec{F} = q_1\vec{E} + q_1(\vec{v}_1 \times \vec{B})$$

$$\vec{E} = \frac{q_2}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\hat{n}\cdot(\vec{D}_1 - \vec{D}_2) = \rho_s$$

$$\hat{n}\times(\vec{E}_1 - \vec{E}_2) = 0$$

$$\epsilon_0 \oiint \vec{E}\cdot d\vec{S} = Q_{\text{enclosed}}$$

$$\vec{D} = \epsilon_0\vec{E}$$

$$\oiint \vec{D}\cdot d\vec{S} = Q_{\text{enclosed}}$$

$$\iiint \rho dV = Q_{\text{enclosed}}$$

$$\oiint \vec{B}\cdot d\vec{S} = 0$$

$$I = \oiint \vec{J}\cdot d\vec{S} = -\frac{\partial Q_{\text{enclosed}}}{\partial t}$$

$$\nabla\cdot\vec{D} = \rho$$

$$\nabla\cdot\vec{J} = -\frac{\partial\rho}{\partial t}$$

$$\nabla\times\vec{E} = 0$$

$$\vec{E} = -\nabla V$$

$$\oint \vec{E}\cdot d\vec{l} = 0$$

$$V_{ab} = V(b) - V(a) = -\int_a^b \vec{E}\cdot d\vec{l}$$

$$\oiint \vec{D}\cdot d\vec{S} = \iiint \nabla\cdot\vec{D}dV$$

$$\oint \vec{E}\cdot d\vec{l} = \iint (\nabla\times\vec{E})\cdot d\vec{S}$$

$$\int_a^b \nabla V\cdot d\vec{l} = V(b) - V(a)$$

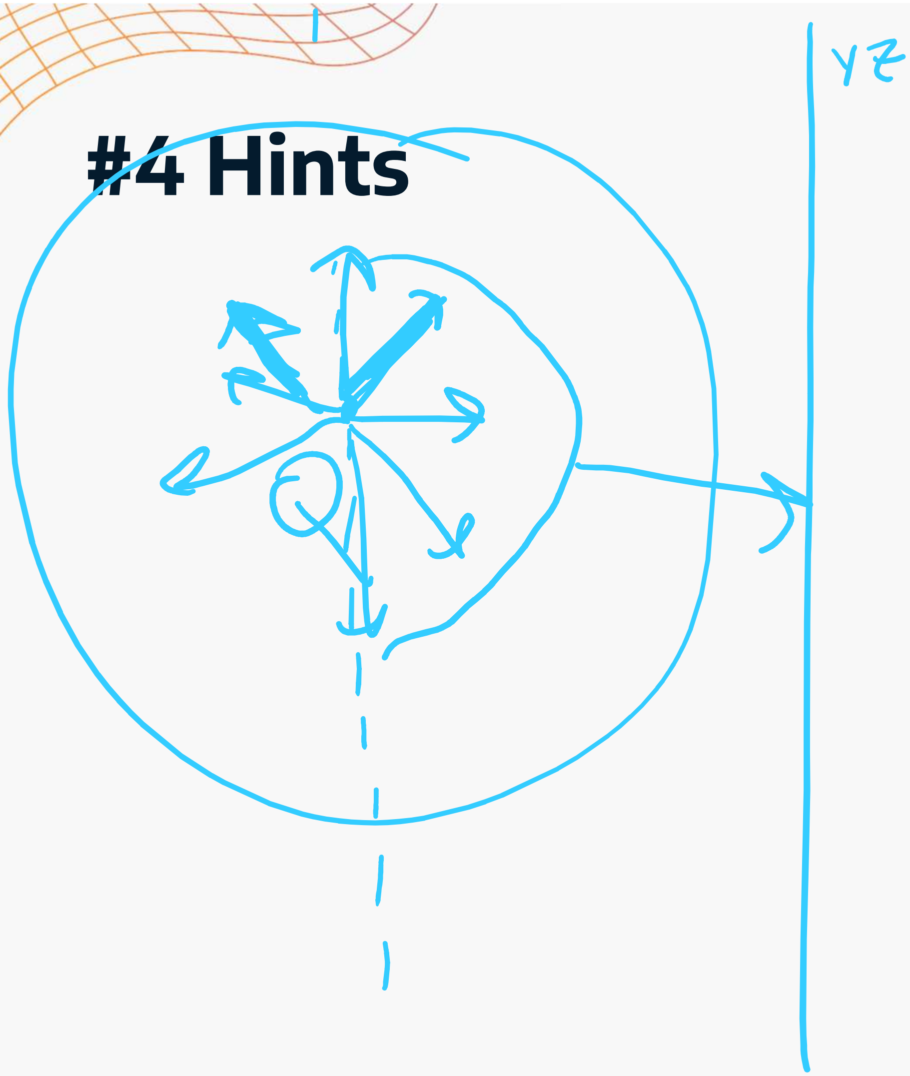
Office Hours

Any questions?

Office Hours

Start Time	Mon	Tue	Wed	Thurs	Fri
9am					
10am					
11am	Prof. Shao 5034 ECEB			TA Office Hour 5034 ECEB	Prof. Chen 5040 ECEB
12pm				TA Office Hour 5034 ECEB	
1pm					
2pm					Prof. Mitchell 3015 ECEB
3pm	TA Office Hour 5034 ECEB		TA Office Hour 5034 ECEB		TA Office Hour 3015 ECEB
4pm	TA Office Hour 5034 ECEB		TA Office Hour 5034 ECEB	TA Office Hour 5034 ECEB	TA Office Hour 3015 ECEB
5pm				TA Office Hour 5034 ECEB Tutorial Session 3017 ECEB	
6pm				Tutorial Session 3017 ECEB	
7pm					

#4 Hints



$$\oiint \vec{D} \cdot d\vec{S} = \iiint \rho \, dV$$

$$\oiint \vec{D} \cdot d\vec{S} = Q$$