Share any thoughts on anything

Hello :)

ECE329: Tutorial Session 1

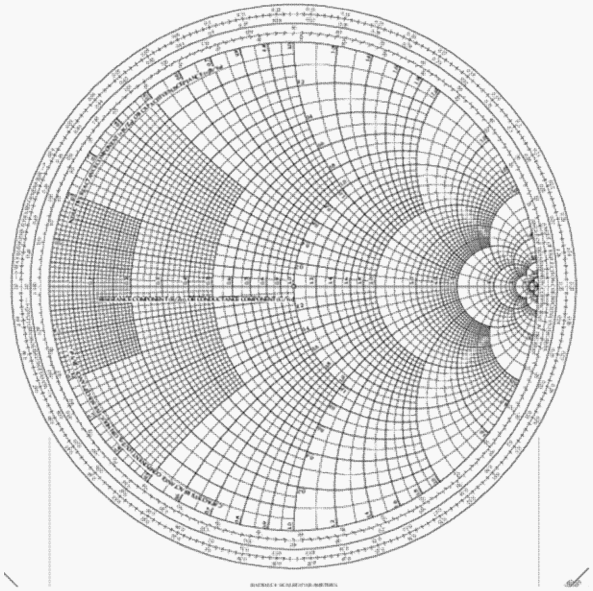
September 4^{th}, 2025

Course Logistics

Any questions on any of these?

- Lectures
- Homeworks
- Campuswire
- Canvas
- Gradescope
- Textbooks
- Exams
- Course Website
- Office Hours
- Tutorial Sessions
- …
- Life in general

<u>You will *need* a compass and a ruler later in the semester.</u>



Some Tips

- The course ramps in difficulty and complexity as the semester progresses.
- Do not fall behind conceptually.
- Don’t forget the physics behind the math.

- Attempt the homeworks on your own first before heading to office hours.

- Thursday office hours will be crowded.
- Go to professor’s office hours for conceptual help, not homework help.

1.

Foundations

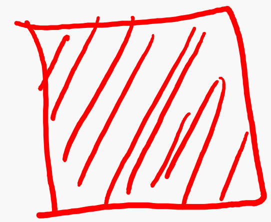
Notation

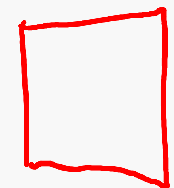
Scalars: $0, \quad 1, \quad 2, \quad \rho_S, \quad x$
Vectors: $\vec{x}, \quad \vec{y}, \quad \vec{z}, \quad \vec{E}, \quad \vec{B}, \quad \vec{0}$
Directions: $\hat{x}, \quad \hat{y}, \quad \hat{z}, \quad \hat{\theta}, \quad \hat{\phi}$

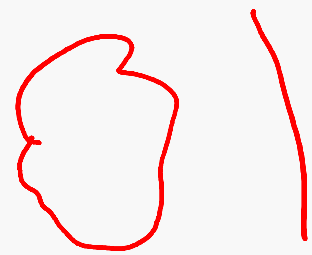
Infinitesimal changes: $dx, \quad dy, \quad dz, \quad d\theta, \quad dr$

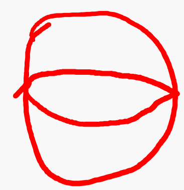
Infinitesimal changes w/ direction: $d\vec{l}, \quad d\vec{S}$

Integrals: $\int \iint \iiint$. Closed integrals: $\oint \oiint$









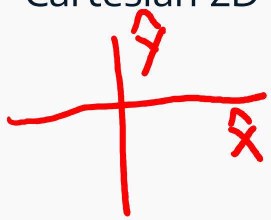
Coordinate Systems

$\delta(\theta-3)\delta(\phi-1)$ ←

$r=3$, $z=1$

$\delta(r-3)\delta(z-1)$

Cartesian 2D



$dx\,dy$

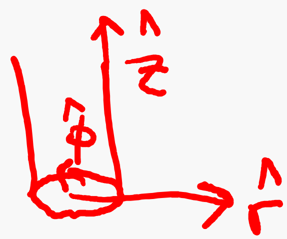
Polar

$r\,dr\,d\phi$

Cartesian 3D

$dx\,dy\,dz$

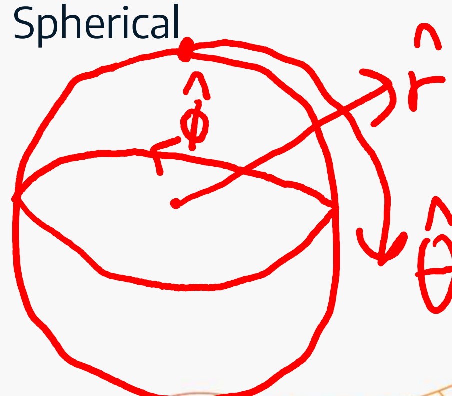
Cylindrical



$r\,dr\,d\phi\,dz$

Spherical

$r^2\sin\theta\,dr\,d\theta\,d\phi$



Points, Lines, Surfaces, & Volumes (3D)

Points

(x, y, z) (r, θ, ϕ)

Lines

$\vec{x} + \lambda \vec{y}$

$\begin{cases} x = \lambda d_x + x_0 \\ y = \lambda d_y + y_0 \\ z = \lambda d_z + z_0 \end{cases}$

(x_0, y_0, z_0) (d_x, d_y, d_z)

Surfaces

$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

point: (x_0, y_0, z_0)

$\hat{n} = (a, b, c)$

$ax + by + cz + d = 0$

$z = 0$ $y = 0$ $x = 0$

Volumes

$\int_{z=-1} \delta(z)\, dz$

$\delta(z)$

$\delta(z - z_0)$

Line, Surface, & Volume integrals

Volume

$$\iiint \rho \, dV = \iiint \rho \, dx \, dy \, dz$$

Surface

$$d\vec{S} = \hat{n} \, dA$$

$$\iint \vec{E} \cdot d\vec{S}$$

$dx\,dy$

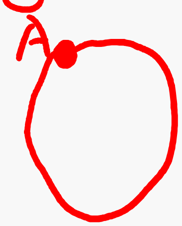
$r\,dr\,d\theta$

$$\oiint_S \vec{E} \cdot d\vec{S}$$

Line

$$\int \vec{E} \cdot d\vec{l}$$

$$\oint \vec{E} \cdot d\vec{l}$$



$$d\vec{l} = \hat{l} \, dl$$

$$\int \vec{E} \cdot d\vec{l}$$

Ex 1:

Given a vector field $E = x\hat{\phi} + z\,\hat{z}$ find the line integral of that field along the arc of a circle going from 0 to $\frac{\pi}{2}$, with a radius of r = 2.

Ex 1:

$r=2$

int. $\phi=0 \downarrow \phi=\frac{\pi}{2}$

$x\hat{\phi}+2\hat{z}$

$$\int_{\phi=0}^{\phi=\frac{\pi}{2}} (x\hat{\phi}+2\hat{z})\cdot d\vec{l}$$

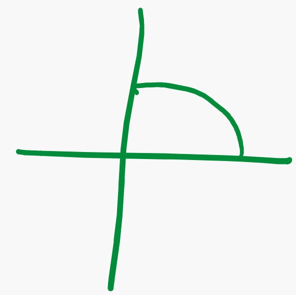
↳ $r\hat{\phi}\,d\phi$

$x = r\cos\phi$

$y = r\sin\phi$

$$\int_{\phi=0}^{\phi=\frac{\pi}{2}} (r\cos\phi\,\hat{\phi} + \cancel{2\hat{z}})\cdot r\hat{\phi}\,d\phi$$

$$\int_{\phi=0}^{\phi=\frac{\pi}{2}} r^2\cos\phi\,d\phi \rightarrow r^2=4$$



Ex 2:

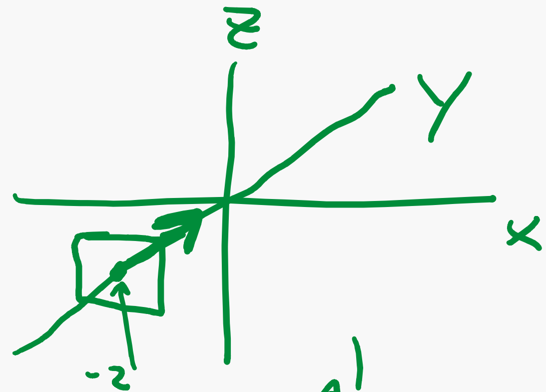
Given a vector field $F = x\,\hat{x} + x^2 y\hat{y} - 2z\hat{z}$ find the flux of that field in the $+\hat{y}$ direction through the rectangle with corners at:
(-1, -2, -1), (-1, -2, 1), (1, -2, 1), (1, -2, -1).

Ex 2:

$\vec{F} = x\hat{x} + x^2 y\hat{y} - 2z\hat{z}$

flux: $+\hat{y}$

$y = -2$



$\iint \vec{F} \cdot d\vec{S}$

$\hookrightarrow \hat{n}\, dA$

$\hookrightarrow = \hat{y}\, dx\, dz$

$\int_{x=-1}^{1} \int_{z=-1}^{1} x^2 y \,(\hat{y} \cdot \hat{y})\, dx\, dz$

$y =$

$-\frac{8}{3}$

2.

Physics

In the beginning…

There exists positive electric charge and negative electric charge.

Concept: **charge** [Coulombs]

Charges interact with each other through **Coulomb's Law**.

$$\vec{F} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}\hat{r}$$

Concept: **Force** [Newtons]

Fields

We want to describe one charge's effect on any other point charge. However, force relies on knowing what the other point charge is!

Introduce **electric field** [Newtons/Coulomb] OR [Volts/meter]: the effect that a charge has on its surroundings.

Coulomb's Law vs. electric field: point charges

$$\vec{F} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}\hat{r} \qquad \vec{E} = \frac{q_2}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\vec{F} = q_1\vec{E}$$

Superposition

What if there are multiple charges?

Force from one charge is independent of force from another charge.
We call this **superposition**.

Therefore, electric fields can also superpose.

$$\vec{F} = q_1 \sum^{n} \frac{q_n}{4\pi\epsilon_0 |\vec{r}_n|^2} \frac{\vec{r}_n}{|\vec{r}_n|} \qquad \vec{E} = \sum^{n} \frac{q_n}{4\pi\epsilon_0 |\vec{r}_n|^2} \frac{\vec{r}_n}{|\vec{r}_n|}$$

Electric Fields of Geometries

Some geometries are very popular in this class. You will use these fields constantly.

Point charge q: $\vec{E} = \frac{q}{4\pi\epsilon_0 r^2}\hat{r}$

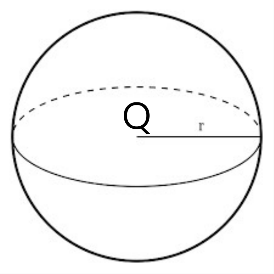
Constant line charge density λ: $\vec{E} = \frac{\lambda}{2\pi\ {}_0 r}\hat{r}$

Constant surface charge density ρ_s @ $x = 0$: $\vec{E} = \frac{\rho_s}{2\epsilon_0}\mathrm{sgn}(x)\hat{x}$

Electric Field Flux

$$\vec{E} = \frac{q_2}{4\pi\epsilon_0 r^2}\hat{r}$$

What if there are very many point charges?
Calculating electric field requires knowledge of all point charge locations.



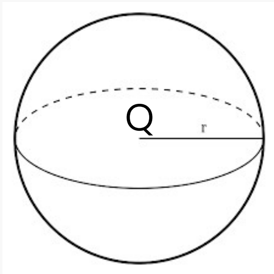
That's annoying.
But we don't usually need this level of detail.

Suppose I have a *closed* surface. I want to calculate how much electric field **flows** through this surface.
Call this flow 'electric field flux', or '**flux**' [N*m^2/C] OR [V*m], the Latin word for flow.

Use surface integrals! $$\oiint \vec{E} \cdot d\vec{S}$$

Gauss's Law: Electric

If there is more electric field flux,
Then there is more electric field flowing out of the surface,
Then, more charge must be enclosed in the surface.



This is **Gauss's Law**!

$$\epsilon_0 \oiint \vec{E} \cdot d\vec{S} = Q_{\text{enclosed}}$$

This law holds for any distribution of charges.
$d\vec{S}$ points OUT of the surface to calculate the flux going OUT.
This does not work for open surfaces.

Gauss's Law: Magnetic

Magnetic fields are defined similarly as electric fields, except for one 'small' problem:

There do not exist positive magnetic charges or negative magnetic charges!

Gauss's Law still works, but no magnetic charge can ever be enclosed because they do not exist!

$$\oiint \vec{B} \cdot d\vec{S} = 0$$

Displacement Field

$$\epsilon_0 \oiint \vec{E} \cdot d\vec{S} = Q_{\text{enclosed}}$$

In class, vector field $\vec{D} = \epsilon_0 \vec{E}$ was introduced. This is a notational convenience for now.

Vector field $\vec{D}$ is known as the **electric displacement** field.

$$\oiint \vec{D} \cdot d\vec{S} = Q_{\text{enclosed}}$$

Charge Density

We find it convenient to introduce **charge density** ρ [C/m^3]. It is pretty much what it sounds like.

In Gauss's Law, Q_{enclosed} can now be written as a volume integral of the charge density over the volume enclosed by the closed surface.

- Add up all the charges in the volume to get the charges in the volume.

$$\epsilon_0 \oiint \vec{E} \cdot d\vec{S} = \oiint \vec{D} \cdot d\vec{S} = \iiint \rho dV = Q_{\text{enclosed}}$$

Current Density

Charges move, which is called a current, denoted I [Amps] = [Coulomb/second].

Current is flowing charge. If current is punching through a surface, then the surface has **charge flux** through it.

We find it convenient to define **current density** $\vec{J}$ [Amps/meter^2]. Integrating current density over a surface yields charge flux!

$$\epsilon_0 \oiint \vec{E} \cdot d\vec{S} = Q_{\text{enclosed}} \qquad I = \oiint \vec{J} \cdot d\vec{S} = -\frac{\partial Q_{\text{enclosed}}}{\partial t}$$

P.S.: Lorentz Force

Currents create magnetic fields for some reason.
Magnetic fields also exert force on moving charges for some reason.

So, here's an equation to describe that.
Don't worry about where it came from.

Lorentz Force equation (includes Coulomb's Law, so it's just better):

$$\vec{F} = q_1\vec{E} + q_1(\vec{v}_1 \times \vec{B})$$

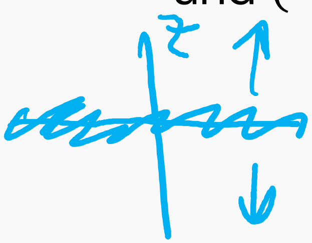
Problem 1

$\text{flux} = \iint \vec{E} \cdot d\vec{S} \rightarrow \hat{z}\, dx\, dy$

@ $x = 0$ $\text{sgn}(x)\,\hat{x}$

In free space, an infinite sheet sits at $z = 0$ and contains a uniform surface charge density of $\rho_s = 5$ C/m^2.

1. How much electric field flux passes through a square in the $+\hat{z}$ direction at $z = 2$ with corners at $(1, 1, 2)$ and $(-1, -1, 2)$?
2. Repeat but the square is now at $z = -2$ with corners at $(1, 1, -2)$ and $(-1, -1, -2)$.



$$\vec{E} = \frac{\rho_s}{2\varepsilon_0}\,\text{sgn}(z)\,\hat{z}$$

$$\int_{x=-1}^{1}\int_{y=-1}^{1} -\frac{\rho_s}{2\varepsilon_0}\, dx\, dy = -\frac{10}{\varepsilon_0}\ [\text{Vm}]$$

Problem 2

Q $\vec{E}$ φ

In free space, a total charge of Q is distributed uniformly in a sphere with radius R, centered at the origin.

1. Write an expression for the electric field $\vec{E}$ as a function of r.
2. How does the answer to the above change if the charge is distributed uniformly on a spherical shell?

$r > R$ $\oint \vec{E} \cdot d\vec{S} = \frac{Q}{\varepsilon_0}$

$\oint \vec{E} \cdot d\vec{S} = \frac{Q_{encl}}{\varepsilon_0}$

$\vec{E} = E(r)\hat{r}$

$d\vec{S} = \hat{r}\, r^2 \sin\theta \, d\phi \, d\theta$

$r < R$ $\frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3}$

$\oint \vec{E} \cdot d\vec{S} = \frac{Q r^3}{\varepsilon_0 R^3}$

Problem 2: Blank slide

In free space, a total charge of Q is distributed uniformly in a sphere with radius R, centered at the origin.

1. Write an expression for the electric field $\vec{E}$ as a function of r.
2. How does the answer to the above change if the charge is distributed uniformly on a spherical shell?

$$E(r) = \begin{cases} \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r} & r > R \\ \frac{Qr}{4\pi\varepsilon_0 R^3}\hat{r} & r < R \end{cases}$$

$$\vec{E} = E(r)\hat{r} \qquad \vec{dS} = \hat{r}\, r^2 \sin\theta\, d\theta\, d\phi$$

$$\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} E(r) r^2 \sin\theta\, d\theta\, d\phi = \frac{Q}{\varepsilon_0}$$

Week 1 equations, in one place

$$\vec{F} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\vec{F} = q_1\vec{E} + q_1(\vec{v}_1 \times \vec{B})$$

$$\vec{E} = \frac{q_2}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\epsilon_0 \oint\!\!\oint \vec{E} \cdot d\vec{S} = Q_{\text{enclosed}}$$

$$\vec{D} = \epsilon_0 \vec{E}$$

$$\oint\!\!\oint \vec{D} \cdot d\vec{S} = Q_{\text{enclosed}}$$

$$\oint\!\!\oint \vec{B} \cdot d\vec{S} = 0$$

$$I = \oint\!\!\oint \vec{J} \cdot d\vec{S} = -\frac{\partial Q_{\text{enclosed}}}{\partial t}$$

$$\iiint \rho dV = Q_{\text{enclosed}}$$

Share any thoughts on anything

Office Hours

Any questions?